An Introduction to Risk Parity

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Introduction

In the aftermath of the financial crisis, investors and asset allocators have started the usual ritual of rethinking the way they approached asset allocation and risk management. Academic/Practitioner journals are full of articles that are supposed to show investors what went wrong and how they can adjust their models and theories in order to protect themselves against substantial losses the next time equity and credit markets experience significant losses. Most of these recommendations should be viewed with a great deal of skepticism as they are bound to incorporate a healthy dose of data snooping and over fitting biases. For example, both Barclay Capital Global Bond Index and MSCI World Equity Index have earned about 7% annual nominal return since 1990, with volatility of the bond index being about 1/3 of the volatility of the equity index. Clearly, going forward it is all but impossible for the bond index to repeat the performance of the last 20 years.[1] Therefore, any model that would recommend a significant allocation to fixed instruments should be carefully analyzed and its assumptions should be questioned.

The so-called risk parity approach to asset allocation has enjoyed a revival during the last few years because such a portfolio would have outperformed the "normal" portfolios with their typical significant allocations to equities. In this note, we discuss the risk parity approach to asset allocation and examine its underlying assumptions. The central idea of the risk parity approach is that in a well-diversified portfolio all asset classes should have the same marginal contribution to the total risk of the portfolio. For example, as shown below, in a typical 60/40 portfolio, equity risk accounts for almost 90% of the total risk of the portfolio, which is significantly higher than its 60% weight. Under the risk parity approach, there is generally a significant allocation to low risk asset classes. Allocations to equities and other risky assets are typically below what we normally observe for most diversified institutional quality portfolios. Therefore, we want to know if this approach is based on sound economic and financial reasoning or if it is just another attempt to extrapolate the results of the last ten years into the future.

Basics of the Risk Parity Approach

The risk parity approach defines a well-diversified portfolio as one where all asset classes have the same marginal contribution to the total risk of the portfolio. In this sense, a risk parity portfolio is an equally weighted portfolio, where the weights refer to risk rather than dollar amount invested in each asset. This approach highlights three different issues. First, to apply the risk parity approach, we need a definition of the total risk of a portfolio. Second, we need a method to measure the marginal contribution of each asset class to the total risk of the portfolio. Third, to employ this approach, we do not need an estimate of expected returns to implement the risk parity approach. The last point is one of the advantages of this approach because as we have seen during the last two decades, forecasting returns is a risky business. On the other hand, the risk parity approach requires accurate estimates of volatility and other measures of risk, which have been shown to be relatively stable and therefore can be predicted with a good deal of accuracy.

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[1] In theory, bonds could still offer significant returns in real term if one were to assume that a period of significant deflation lies ahead.

Total risk is typically measured by the volatility of the rate of return on the portfolio. This means that risk parity works within the same framework as Harry Markowitz's mean-variance approach. Alternatively, one could use VaR as a measure of total risk. The advantage of using VaR as a measure of total risk is that one can incorporate skewness and kurtosis in the measure of total risk. For the purpose of this introductory note, we will use standard deviation as a measure of total risk.

Once we have decided to use standard deviation as the measure of total risk, the contribution of each asset class to the total risk of the portfolio is well defined and can be easily calculated. The general definition of marginal contribution of an asset class to the total risk of a portfolio is given by the following expression:

$$MC_i = (\text{Weight of Asset Class } i) \times \frac{\Delta \text{Total Risk of Portfolio}}{\Delta \text{Weight of Asset Class } i}$$

Here, MC_i is the marginal contribution of asset class i to the total risk of the portfolio. The last term determines the change in the total risk of the portfolio if there is a very small change in the weight of asset class i. It turns out that the total risk of the portfolio is then equal to the sum of the marginal contributions. That is, if there are N assets in the portfolio, then

$$\text{Total Risk} = MC_1 + MC_2 + \cdots + MC_N$$

To see how this works, let us consider the case of only two risky assets. The rate of return and the standard deviation of the rate of return on this portfolio, $E[R_p]$ and $\sigma[R_p]$, are:

$$E\left[R_p\right] = w_1 E\left[R_1\right] + w_2 E\left[R_2\right]$$

$$\sigma\left[R_p\right] = \sqrt{w_1^2 \sigma\left[R_1\right]^2 + w_2^2 \sigma\left[R_2\right]^2 + 2 w_1 w_2 Cov\left[R_1, R_2\right]}$$

Where, w_1 and w_2 are the weights of the two assets (they add up to one), $E\left[R_1\right]$ and $E\left[R_2\right]$ are expected returns on the two assets, $\sigma\left[R_1\right]$ and $\sigma\left[R_2\right]$ are standard deviations of the rates of return on the two assets, and $Cov\left[R_1, R_2\right]$ is the covariance between the two assets. The marginal contributions of the two assets to the total risk of the portfolio are:

$$MC_1 = w_1 \times \left(\frac{\Delta \sigma\left[R_p\right]}{\Delta w_1} \right) = w_1 \times \left(\frac{w_1 \sigma\left[R_1\right]^2 + w_2 Cov\left[R_1, R_2\right]}{\sigma\left[R_p\right]} \right)$$

$$MC_2 = w_2 \times \left(\frac{\Delta \sigma\left[R_p\right]}{\Delta w_2} \right) = w_2 \times \left(\frac{w_2 \sigma\left[R_2\right]^2 + w_1 Cov\left[R_1, R_2\right]}{\sigma\left[R_p\right]} \right)$$

Exhibit 1 provides all the information we need to calculate the marginal contributions of Barclay Capital Global Bond Index and MSCI World Equity Index to the total risk of a portfolio consisting of 60% in equity and 40% in fixed income.

Exhibit 1: Standard Deviations for Marginal Contribution Calculation

1990-2011	***MSCI World Index***	***Barclays Capital Global Aggregate***	***60/40 Portfolio***
Monthly Standard Deviation	4.50%	1.62%	2.95%
Covariance Between the Two	0.021%		

Given the above table, the marginal contributions are:

$$MC_{MSCI} = 60\% \times \left(\frac{60\% \times (4.50\%)^2 + 40\% \times 0.021\%}{2.95\%} \right) = 2.64\%$$

$$MC_{BarCap} = 40\% \times \left(\frac{40\% \times (1.62\%)^2 + 60\% \times 0.021\%}{2.95\%} \right) = 0.31\%$$

We can see that equity contributes 2.64% to the total risk of 2.95%, while the rest, 0.31%, is contributed by fixed income. In addition, we can see that although the weight of equity is 60%, its contribution to the total risk is 89.34% (2.64%/2.95%). Given the poor performance of equities during the last 10 years, one may wonder if it is sensible to allocate so much of a portfolio's total risk to equity risk.

The general formula for calculating the marginal contribution of each asset to the total volatility of a portfolio when there are more than two assets is:

$$MC_i = w_i \times \frac{\sum_{j=1}^{N} w_j Cov\left[R_i, R_j\right]}{\sigma\left[R_p\right]}$$

$$= w_i \times \beta_i \times \sigma\left[R_p\right]$$

The second line is a rather simple method for calculating the marginal contribution of an asset class. It states that the marginal contribution is equal to the weight of the asset times the beta of the asset with respect to the portfolio times the total risk of the portfolio. Here beta is defined as:

$$\beta_i = \frac{Cov\left[R_i, R_p\right]}{\sigma\left[R_p\right]^2}$$

where $Cov\left[R_i, R_p\right]$ is the covariance between the portfolio and the rate of return on asset i.

In the previous example, the betas of equity and fixed income assets with respect to the portfolio are 1.49 and 0.27, respectively. For instance, the marginal contribution of equity is then equal to:

2.64% = 60% x 1.49 x 2.95%

To create a portfolio using the risk parity approach, we need to adjust the weights until the marginal contributions of the two asset classes are equal.[2] Using trial and error or an optimization package such as Microsoft Excel's Solver, one can show that when 26.45% is allocated to equity and 73.55% to fixed income, risk parity is achieved.

Exhibit 2: Risk Parity Weights

1990-2011	***MSCI World Index***	***Barclays Capital Global Aggregate***	***Total Risk of Risk Parity Portfolio***
Weights	26.45%	73.55%	1.91%
Marginal Contribution in Risk Parity Port	0.955%	0.955%	

As expected, risk parity requires a significant allocation to fixed income and as stated in the introduction, this portfolio would have performed very well during the last 20 years with an annualized rate of return of 7.12%. This is roughly equal to the annualized rate of return on the 60/40 portfolio with a volatility that is 50% smaller than that of the 60/40 portfolio. Given such an impressive result, it is no wonder that several risk parity based investment products have recently appeared in markets.

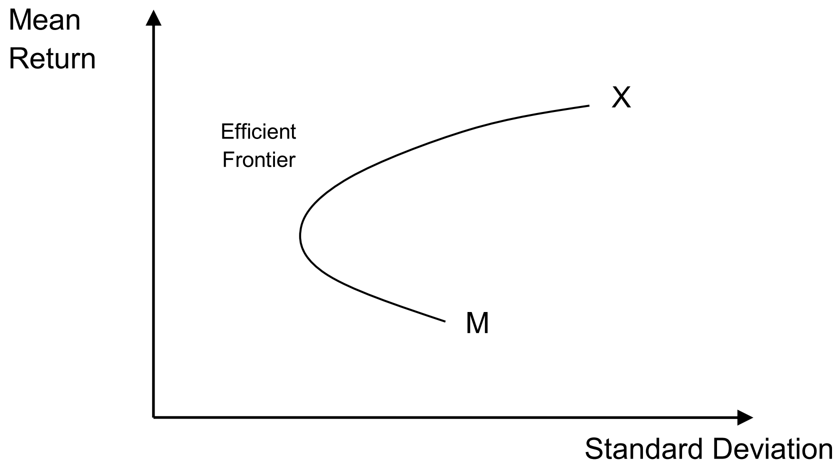
Economic Foundation of Risk Parity Approach

As discussed above, risk parity portfolios make relatively large allocations to low risk asset classes. Notwithstanding the performance of such portfolios over the last 20 years, it is safe to say that going forward a portfolio with a monthly standard deviation of 1.91% is not likely to provide a rate of return required by most investors. Given this, is there a reason to use this approach to asset allocation? It turns out that if one is willing to use leverage, there is a rather strong economic reason to expect a risk parity portfolio to perform rather well and even outperform a typical portfolio where relatively large allocations are made to risky assets.

To see this, we need to go back to the fundamental results of Modern Portfolio Theory and specifically, the results reported by Markowitz and then later by Sharpe and others. According to Markowitz's original results, if investors care only about mean and variance of their portfolios, then they should invest only in portfolios that plot on the efficient frontier. These portfolios have the lowest risk for a given level of expected return. Exhibit 3 displays the familiar efficient frontier.

[2] This can bed done using Solver tool of Microsoft Excel

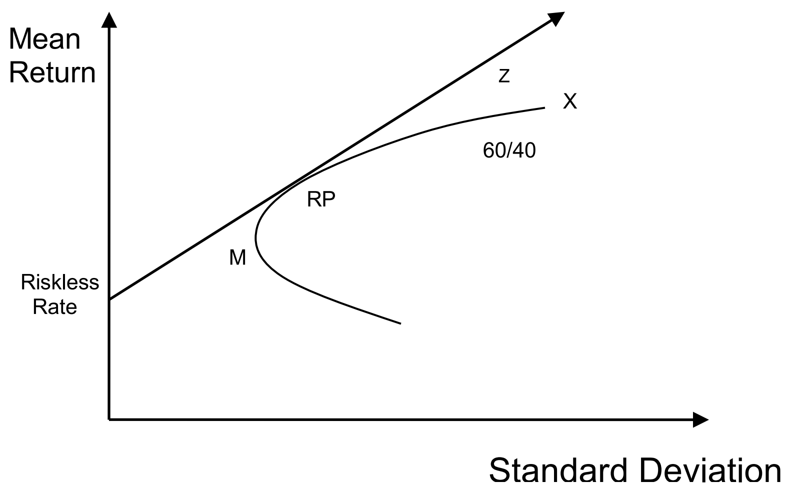
Exhibit 3: Efficient Frontier



According to Markowitz, investors should pick a portfolio that falls on the line segment MX. Investors who are willing to take some risk will pick a portfolio close to X and those who are more risk averse would select a portfolio close to M. Even though the 60/40 portfolio is not likely to be on the efficient frontier, it is likely to be closer to X than to M. On the other hand, the risk parity portfolio is likely to be closer to M. Again, there is no reason to believe that the risk parity portfolio is an efficient portfolio.

In Exhibit 4, we have plotted hypothetical portfolios M, X, 60/40, and risk parity. We have plotted the riskless rate as well.

Exhibit 4: Portfolio Plots and Efficient Frontier



The tangent line originating from the riskless rate is known as the capital market line. It identifies a set of portfolios that can be constructed as the combination of two portfolios/assets: (a) the riskless asset and (b) the efficient portfolio that lies on the tangency point. For example a portfolio that lies between points RP and the riskless rate can be created using a combination of investments in these two assets. On other hand, a portfolio that lies above RP can be created through borrowing at the riskless (using leverage) and investing the proceeds in portfolio RP.

Now that the riskless rate has been introduced, we can see that one can make a case for investing a low risk portfolio and then using leverage to increase the risk and hopefully the expected return on the portfolio. In the above figure, we have assumed that both the risk parity portfolio and the 60/40 portfolio are on the original efficient frontier. We can see that portfolio Z, which is a combination of the risk parity portfolio and leverage, has the same risk as the 60/40 portfolio but with a higher expected rate of return. This appears to present a compelling reason for using a risk parity approach to asset allocation. However, there needs to be a word of caution: if the risk parity portfolio is far away from the efficient frontier, the leveraged approach to risk parity asset allocation may lead to poor performance. In other words, it is critical for the risk parity portfolio to be close to the efficient frontier. In addition, leverage represents a source of risk that many institutional investors may not wish to assume. Exhibit 5 summarizes the results for the risk parity portfolio and its leveraged version.

Exhibit 5: Risk Parity Portfolio Performance

1990-2011	***60/40 Portfolio***	***Risk Parity Portfolio (Unlevered)***	***Risk Parity Portfolio (Levered)***
Monthly Mean	0.59%	0.59%	0.73%
Monthly Standard Deviation	2.95%	1.91%	2.95%
Monthly Information Ratio	0.201	0.310	0.247
Monthly Sharpe Ratio	0.085	0.131	0.131

It is important to note that to raise the volatility of the rate of return on the risk parity portfolio to the same level as the volatility of the rate of return on the 60/40 portfolio one needs to employ 154% leverage. That is, for each $100 capital, one needs to borrow $54 and then to invest $154 in the risk parity portfolio. This leverage figure is given by:

$$\begin{aligned} \text{Leverage} &= \frac{\text{Volatility Target}}{\text{Volatility of Unlevered Portfolio}} - 1 \\ &= \frac{2.95\%}{1.91\%} - 1 \end{aligned}$$

This level of leverage may be too high for many institutional investors. However, in practice it may not be necessary to use that much leverage to reach reasonable expected returns. Of course, given historical performances of equity and bond indices, no amount of leverage was needed to achieve the same rate of return as the 60/40 portfolio because the unlevered risk parity portfolio already has the same average return as the 60/40 over 1990-2011 period (both earned 0.59% per month).

Other Related Approaches

The idea of leveraging up a relatively low volatility portfolio to generate a given expected rate of return can be applied to other portfolios as well. Risk parity is one approach to creating a low volatility portfolio. Any approach that leads to a low volatility well-diversified portfolio can be used to create higher expected returns using leverage. The key is for the low volatility portfolio to have a Sharpe ratio that is higher than the 60/40 or other high volatility portfolios. If the Sharpe ratio of the low volatility portfolio is lower than the riskier portfolio, then leverage will actually lead to a portfolio that will be inferior to the riskier portfolio. This is the key: for risk parity to work it has to lead to a relatively high Sharpe ratio and the investor should be able and willing to use some degree of leverage.

One simple approach to creating a low volatility portfolio is to use an equally weighted portfolio. This portfolio is by definition rather well diversified and is likely to have relatively high allocations to less risky assets. The other approach would be to use an optimization package to identify the minimum variance portfolio. This portfolio is created by finding the weights that minimize the volatility of the rate of return on the portfolio. Portfolio M on the efficient frontier displayed in the above graph is such a portfolio. Finally, a volatility-weighted portfolio can be used to create a low volatility portfolio. In this approach the weight of each asset class is given by:

$$w_i = \frac{\frac{1}{\sigma[R_i]}}{\sum_{j=1}^{N} \frac{1}{\sigma[R_j]}}$$

This means the weight of each asset class is proportional to the inverse of its volatility. This approach is in fact identical to the risk parity approach when we have only two assets and it will be the same as risk parity in the more general case if correlations between asset returns are the same. We are going to use our numerical example to demonstrate this approach.

Exhibit 6: Variance Weighted Portfolio

	MSCI World Index	***Barclays Capital Global Aggregate***
Monthly Standard Deviation	4.50%	1.62%
Weights	26.46%	73.54%

Here:

$$26.46\% = \frac{\frac{1}{4.50\%}}{\frac{1}{4.50\%} + \frac{1}{1.62\%}}$$

$$73.54\% = \frac{\frac{1}{1.62\%}}{\frac{1}{4.50\%} + \frac{1}{1.62\%}}$$

Since we have only two asset classes, it can be seen that the weights are the same as in the risk parity portfolio.

Risk Parity and Alternative Investments

To the degree that alternative investments tend to have low volatility and low correlations with other asset classes, the allocations to alternative investments will be relatively high in a risk parity portfolio. However, many institutional investors may have a difficult time accepting relatively large allocations to alternative investments. Let us use a numerical example to demonstrate this. We are going to consider three asset classes: Barclay Capital Global Bond Index, MSCI World Index, and HFR Hedge Fund Index. Exhibit 7 displays the statistics for these three asset classes as well as those of three different portfolios.

A few observations are in order. First, as expected, both the volatility-weighted and the risk parity portfolios require significant allocations to hedge funds and bonds. Second, the volatility-weighted and the risk parity portfolios are rather similar. Third, both the volatility-weighted and the risk parity portfolios have much higher Sharpe ratios than the 10/50/40 portfolio. This means that if these two low volatility portfolios are levered up to have the same volatility as the 10/50/40 portfolio, they will have higher mean return than the 10/50/40 portfolio.

Exhibit 7: Performance Statistics 1990 to 2011

1990-2011	***HFRI Fund Weighted Composite Index***	***MSCI World Index***	***Barclays Capital Global Aggregate***	***10/50/40 Portfolio***	***Volatility Weighted Portfolio***	***Risk Parity Portfolio***
Monthly Mean	0.99%	0.60%	0.59%	0.63%	0.74%	0.73%
Monthly Standard Deviation	2.03%	4.50%	1.62%	2.66%	1.72%	1.64%
Monthly Sharpe Ratio	0.317	0.056	0.154	0.109	0.230	0.236
Weights in 10/50/40 Portfolio	10%	50%	40%			
Weights in Volatility-Weighted Portfolio	37%	17%	46%			
Weights in Risk Parity Portfolio	35%	14%	51%			

Conclusion

In this article, we introduced the basic ideas behind the risk parity approach to asset allocation and examined its economic foundation. It turns out that risk parity approach is a viable approach to asset allocation and is in fact superior to ad hoc asset allocation models employed by the industry. In the absence of a full optimization approach, risk parity appears to provide a close approximation to the original model of Harry Markowitz. The key in using this approach is the willingness to use leverage and the ability to manage the risks posed by the use of leverage. While risk parity is a viable approach to asset allocation, it does not represent a trading strategy that can be employed by active managers. The reasons are that it does not require any estimate of expected return on an asset class (potentially a source of skill for active managers) and it always leads to positive weights for asset classes (long/short strategies cannot be implemented). It is a suitable model for institutional and high net worth investors who do not face significant constraints on their asset allocation policies and are able to use leverage. Finally, investors who are able and willing to use derivatives could use these instruments to lever up their risk parity portfolios.

Additional Reading on Risk Parity

"Constructing Risk Parity Portfolios: Rebalance, Leverage, or Both?"
by Oleg Ruban and Dimitris Melas
The Journal of Investing, Spring 2011, Vol. 20, No. 1, pp. 99-107

ABSTRACT: Typical multi-asset-class portfolios can be dominated by equity risk, even when the allocation to equities is relatively modest. Achieving risk parity between equities and fixed income in an unlevered portfolio would require significant rebalancing towards fixed income. While such rebalancing can lead to a reduction in risk, portfolios with high fixed-income allocations have historically underperformed equity-dominated portfolios. However, achieving risk parity through leverage, while keeping the initial asset allocation constant, would typically require substantial levels of leverage and could lead to a significant increase in portfolio volatility. The authors combine rebalancing and leverage to construct risk parity portfolios that target the same expected return and the same portfolio risk as the initial asset allocation and examine the performance of these portfolios in different market conditions.

http://www.iijournals.com/doi/abs/10.3905/joi.2011.20.1.099

"Leverage Aversion and Risk Parity"
by Clifford Asness, Andrea Frazzini, and Lasse H. Pedersen
Forthcoming in Financial Analysts Journal

ABSTRACT: We show that leverage aversion changes the predictions of modern portfolio theory. It causes safer assets to offer higher risk-adjusted returns than riskier assets. Consuming the high risk-adjusted returns offered by safer assets requires leverage, creating an opportunity for investors with the ability and willingness to borrow. A Risk Parity (RP) portfolio exploits this in a simple way, namely by equalizing the risk allocation across asset classes, thus overweighting safer assets relative to their weight in the market portfolio. Consistent with our theory of leverage aversion, we find empirically that RP has outperformed the market over the last century by a statistically and economically significant amount, and provides further evidence across and within countries and asset classes.

http://www.econ.yale.edu/~af227/pdf/AFP_20110112.pdf

"Risk Parity Portfolio Versus Other Asset Allocation Heuristic Portfolios"
by Denis Chaves, Jason Hsu, Feifei Li, and Omid Shakernia
The Journal of Investing Spring 2011, Vol. 20, No. 1, pp. 108-118

ABSTRACT: In this article, the authors conduct a horse race between representative risk parity portfolios and other asset allocation strategies, including equal weighting, minimum variance, mean–variance optimization, and the classic 60/40 equity/ bond portfolio. They find that the traditional risk parity portfolio construction does not consistently outperform (in terms of risk-adjusted return) equal weighting or a model pension fund portfolio anchored to the 60/40 equity/bond portfolio structure. However, it does significantly outperform such optimized allocation strategies as minimum variance and mean–variance efficient portfolios. Over the last 30 years, the Sharpe ratios of the risk parity and the equal-weighting portfolios have been much more stable across decade-long sub-periods than either the 60/40 portfolio or the optimized portfolios. Although risk parity performs on par with equal weighting, it does provide better diversification in terms of risk allocation and thus warrants further consideration as an asset allocation strategy. The authors show, however, that the performance of the risk parity strategy can be highly dependent on the investment universe. Thus, to execute risk parity successfully, the careful selection of asset classes is critical, which, for the time being, remains an art rather than a formulaic exercise based on theory.

http://www.iijournals.com/doi/abs/10.3905/joi.2011.20.1.108

"Balancing Asset Growth and Liability Hedging through Risk Parity"
by Edgar E. Peters
The Journal of Investing Spring 2011, Vol. 20, No. 1, pp. 128-136

ABSTRACT: In this article, the author shows that risk parity strategies offer liability-hedging benefits in addition to exposure to growth assets. Risk parity portfolios are typically levered so that risks can be balanced across asset classes. The effect of doing so effectively levers low-risk assets like bonds and de-levers high-risk assets like stocks. The effective leverage of the bond component increases its duration, giving many risk parity portfolios durations similar to those of defined-benefit plans. This is not liability-directed investment, which targets a particular liability, but it does give liability-hedging properties, which other investment strategies typically do not offer. In addition, this study shows that periods of high and low volatility can affect the ability of a static mix of assets to effectively hedge liabilities.

http://www.iijournals.com/doi/abs/10.3905/joi.2011.20.1.128

"Risk Parity and Diversification"
by Edward Qian
The Journal of Investing, Spring 2011, Vol. 20, No. 1: pp. 119-127

ABSTRACT: Traditional 60/40 asset allocation portfolios are not truly diversified because they have an unbalanced risk allocation to high-risk assets. As a result, their expected risk-adjusted returns are low. Risk parity is a new way to construct asset allocation portfolios based on the principle of risk diversification, achieving both higher risk-adjusted returns and higher total returns than traditional asset allocation approaches. The diversification benefits of risk parity portfolios also include balanced correlations to underlying asset classes and stronger downside protection against severe losses. Risk parity portfolios can also incorporate active views on risk-adjusted returns of different asset classes. All of these features make risk parity an attractive alternative to traditional asset allocation approaches.

http://www.iijournals.com/doi/abs/10.3905/joi.2011.20.1.119

"The Dangers of Risk Parity"

by Ben Inker

The Journal of Investing, Spring 2011, Vol. 20, No. 1, pp. 90-98

ABSTRACT: Risk parity is a portfolio construction methodology that is extremely attractive if standard deviation is a good estimate of the risk of asset classes and if there is a wide variety of asset classes that are likely to offer fairly uncorrelated risk premia. In reality, however, standard deviation is a dangerously limited estimate of the true risk of an asset class and there may well be very few risk premia that are truly available to be exploited. Adding to the problem is the fact that bond yields today are at generational lows and sovereign debt loads are at extremely high levels, making the risk of significantly negative bond returns or even sovereign default much higher than history would suggest. The traditional 65/35 portfolio, while far from ideal, at least seems overwhelmingly likely to offer a decent premium over cash in the long run and should be able to survive either economic depression or sovereign default.

http://www.iijournals.com/doi/abs/10.3905/joi.2011.20.1.090

"Risk Parity: Rewards, Risks, and Research Opportunities"

by S. Ramu Thiagarajan and Barry Schachter

The Journal of Investing Spring 2011, Vol. 20, No. 1, pp. 79-89

ABSTRACT: Mean–variance optimization has recently come under great criticism based on the poor performance experienced by asset managers during the global financial crisis. In response, an alternative approach, called risk parity, which proceeds by equalizing risk contributions, has garnered much interest. The authors summarize the work of a group of leading researchers on risk parity chosen for this special issue. They survey more generally what is known about this approach. Although risk parity has intuitive appeal and has performed well over some historical time periods, it is premature to claim the superiority of risk parity over other asset allocation approaches. The authors raise several conceptual and practical questions about risk parity that they think are worthy of additional research.

http://www.iijournals.com/doi/abs/10.3905/joi.2011.20.1.079

"Beyond Risk Parity"

by Vineer Bhansali

The Journal of Investing Spring 2011, Vol. 20, No. 1, pp. 137-147

ABSTRACT: Risk parity is an approach to portfolio construction that focuses on the balance of risks within a portfolio. In this article, the author explores the benefits and shortcomings of the traditional way risk parity is implemented and suggests extensions using a risk-factor based approach.

http://www.iijournals.com/doi/abs/10.3905/joi.2011.20.1.137