Challenging Pension Funds Model Portfolios with Listed Private Equity (LPE)

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Introduction

Pension fund portfolios exhibit major shifts during the last decade with respect to assets and portfolio structure, driven not only by volatile markets but also by regulatory requirements and an enhanced focus on adequate risk measurement.

In search of higher yields, a global trend towards expanded allocation to equities and alternative investments established. Offering a higher risk-return profile, pension funds in smaller markets increased their exposure to equities while decreasing their holdings in bills and bonds (OECD, 2015). Whereas others such as funds in UK reduced their bond and equity positions indicating an expanded allocation to alternative investments (OECD, 2015; Talmod & Vasvari, 2014). Some of the largest pension markets increased their portfolio share in alternative assets more than five percentage points over the period from 2004 to 2014. In the UK, allocation increased by 12.8%, Canadian pension funds increased the share about 8% and funds in Brazil by 8.9%. During the same decade, the US increased the allocation to alternative assets by 4.5% (OECD, 2015). Additionally, those funds in markets with the highest returns in 2014 have switched to alternative investments over the last ten years. The main driver for the allocation shift is that some of the alternative asset classes may exhibit better returns, but also bear higher investment risks than traditional financial assets, hence requiring specialized skills and knowledge of pension fund management (US GAO, 2012).

In Switzerland, pension funds' asset allocations are regulated by the LPP legal framework (BVV2, 2000). The framework not only covers provisions regarding asset categories but also the maximum weights per asset class (BVV2, 2000, art. 54-57). Since its inception in 1985, the LPP framework experienced significant

changes in terms of specification and covered asset classes. One cornerstone to achieve broader diversification was the inclusion of alternative assets. In 1993, the Pictet LPP pension fund reference index consisted mainly of Swiss and international stocks and bonds before the reference index also included private equity and hedge funds in 2005 (Pictet, 1993). The significant change towards alternative assets and the narrower definition of asset classes in the reference index of 2015 give rise to questions regarding portfolio optimization methods and the suitable diversification into alternative assets.

This paper should contribute to the knowledge about listed private equity in the multi-asset portfolio context and the special case of pension funds. The rationale is given by the considerable growth in alternative asset investments of pension funds in the seven largest markets (P7) measured by total pension assets including the US, UK, Japan, Netherland, Australia, Switzerland and Canada (Towers Watson, 2015). The 20% growth in alternative asset investments from 1995 to 2014 has also implications about the importance of pension funds as investors (Preqin, 2011). Thirty-three percent of investments in the top 100 alternative asset managers is made by pension funds. Within this group, private equity funds are the most favored asset type after direct real estate funds (Towers Watson, 2014). However, as there are pension institutions with a preference for liquidity or with a core investment in traditional private equity, which would like to fine-tune the overall exposure with listed instruments, this study mainly focuses on LPE (Brown & Kraeussl, 2012; Cumming, Fleming & Johan, 2011).

Alternative investments, but liquid

This study aims to analyze the model portfolio of the Pictet 2015 LPP-60 index (Pictet, 2015b) in comparison to a portfolio, which follows the general composition of the LPP-60 index and fulfills the LPP weighting requirements but includes an additional asset, listed private equity (LPE).

Given the similarity of LPE to traditional private equity funds' investment strategies, its regulatory treatment as regular stocks with respect to capital requirements and similar characteristics as small cap stocks, LPE could be a beneficial addition to a pension fund's portfolio. Instead of analyzing the influence of LPE on portfolio performance in a mean-variance framework, the applied optimization considers the non-normal return distribution of alternative assets.

The shift towards alternative investments in portfolios of institutional investors mainly considered hedge funds and traditional private equity (see for example Preqin, 2015b; Talmor & Vasvari, 2014; Groh, Liechtenstein & Lieser, 2010; Schneeweis & Martin, 2001).

In 2015, Bain (2015) reported record numbers for private equity with investment values close to USD 250 billion in 2014; a 25% surge in deal values over three years. Part of the capital deployed to participate in the growth of the asset class came from pension funds. Over the last decade, a narrowing gap between target and actual private equity holdings of pension funds could be observed. However, pension funds still have lower holdings in private equity than other institutional investors such as endowments, family offices or sovereign wealth funds (Ang, Ayala & Goetzmann, 2014). Caveats of pension funds towards private equity holdings are directly related to the specifications of limited partnerships (Talmor & Vasvari, 2014). Foremost, its illiquidity and valuation difficulty make the asset class unsuitable for certain institutional investors. The lack of market prices and long-term lock-up of capital also make the determination of optimal portfolio weights more difficult (Woodward & Hall, 2003; see also Woodward, 2004).

Despite the illiquidity, commitment requirements and intransparency, pension funds chose to invest in limited partnerships. In a survey of the US Government Accountability Office (US GAO, 2012), respondents claimed that the reason for the investments were higher risk-adjusted returns than equity. Half of the respondents state that their private equity investments outperformed the equity investments over a five year period. However, pension fund managers note that private equity returns were not shielded from losses during economic crisis. Furthermore, the dependency on co-investors is critized. The ability to actually contribute capital defines the investment strategy and changes due to a lack of capital can be costly (US GAO, 2012). To overcome the before mentioned drawbacks of limited partnerships, e.g. private equity, the inclusion of listed private equity is considered by investors concerned with transparent and regular pricing of their investments (Brown & Kraeussl, 2012; Huss & Zimmermann, 2009). LPE gives the investor the possibility to own a stock of a direct investing fund or fund manager, whose core business, identical to limited partnerships, is to hold investments in private companies. LPE does not require capital commitments, co-investing nor does it apply a lock-up period. Investments can be disposed by a stock sale. Cumming and Johan (2014) analyzed investment behavior among international pension funds and their private equity investments. They note that depending on the mandate, private pension managers have a significantly higher share of funds, which invest in LPE than those that do not. They relate the likelihood of an investment to the size of the investment team and the associated due diligence capacity. Cumming and Johan (2014) also conclude that LPE is a source of diversification and lowers due diligence costs what benefits smaller pension funds the most. Swisscanto (2015), a major Swiss pension fund favors LPE as diversified investments as LPE companies reduce the risks to a considerable extent, while leaving the income opportunities of private equity intact. The LPE investment is attractive because despite its economic allocation to alternative investments, it represents an equity commitment, which is beneficial under risk capital requirements of regulatory accords for pension funds, insurers and banks (Preqin, 2014; IORP, 2014; BIS, 2011; EIOPA, 2015).

Cumming, Hass and Schweizer (2013) presented a benchmark based on the VentureXpert database for venture capital and buyout funds, which is updated monthly and is superior to LPE price indices, transaction based or appraisal value based indices. No weighting restrictions apply except for a 20% threshold to maintain diversification. The authors conducted the optimization based on different risk measures such as lower partial moments, conditional value at risk and variance. In practice, LPE has been

included in institutional investors' model portfolio. The Pictet 2005 reference indices included the LPX50 up to a maximum weight of 7.5% (Pictet, 2005). Pictet removed the LPX50 from the LPP reference indices arguing that daily available price indices only partially reflected the performance of the industry. Despite the high potential, Pictet argued that LPE is not viewed as a separate asset class rather as a sub-category of regular equities (Pictet, 2015b). In contrast, LPEQ, a global association of LPE companies, refers to the fact that some of the LPX50 constituents exhibit a 93% correlation of NAV with unlisted NAV (Preqin, 2015a). Therefore, LPE is not only a proxy for private equity, it is private equity with key advantages such as seasoned portfolios avoiding the J-curve effect and previously mentioned liquidity. LPEQ particularly mentions the liquidity advantage of LPE for defined contribution pension plans which struggle to include alternative assets that lack daily pricing (Preqin, 2015a). Based on the findings about LPE, pension funds' reception of private equity and private equity in the general portfolio context, this work contributes by including LPE in a pension fund's model portfolio. On the one hand, this analysis considers the hybrid characteristics of LPE, its stock-like nature and limited partnerships' related core business by assigning LPE to different investment categories for the optimization. On the other hand, real-world investment limits apply by modeling according to the LPP-60 index provided by Pictet.

Theoretical considerations – The risk is in the tail

For the optimization, return distributions are taken into account. Optimization based on the Markowitz framework only considers mean and variance as objective variables. This bears some drawbacks (Markowitz, 1952). Markowitz optimization, which minimizes variance, assumes that asset returns are normally distributed and the investor has a quadratic utility function (Levy & Markowitz, 1979). With latter, an investor who seeks to maximize the expected portfolio return will only consider mean and variance but not higher moments (Fabozzi, Kolm, Pachamanova & Focardi, 2012). The consequence is that the investor neglects extreme outcomes such as severe losses. The normality of asset return distributions is violated by some asset classes, hence they exhibit higher probabilities to realize returns in the tails of the distribution unlike the assumption that fifty percent of the returns are higher and lower than the mean, which gives the normal distribution its symmetric bell shape (Sharpe, 2007; Favre & Galleano, 2002). In exhibit 1, the histograms of daily returns contrasting with a fitted normal distribution show outliers in and beyond the tails what indicates the positive excess kurtosis and negative skewness of some of the asset returns. Given the previous observations, the assumption of a symmetric return pattern does not hold for certain assets, hence the optimization method must account exactly for that (Xiong & Idzorek, 2010). One measure to consider the skewness and kurtosis of a distribution is Value-at-Risk (VaR), which corresponds to the predicted maximum loss over a pre-specified time period within a given confidence interval (Jorion, 2007).

In the portfolio context, VaR is only of limited use as a risk measure as it is not coherent according to the description offered by Artzner, Delbaen, Eber and Heath (1999). One of the criteria determining the coherence is the subadditivity principle, which does not necessarily hold as the VaR of the entire portfolio can be higher than the sum of the individual assets' VaR for specific portfolio compositions. Moreover, VaR is non-linear what makes optimization of discrete distributions challenging. Additionally, the loss described by VaR gives no indication of the magnitude of a potential loss beyond VaR (Jorion, 2007). The expected tail loss or Conditional Value at Risk (CVaR) overcomes these problems. CVaR is still a simple measure of downside risk, but covers also losses beyond VaR. It gives the average loss of the sum of losses which exceed VaR with a certain probability. It is coherent as shown by Artzner et al. (1999) and Acerbi and Tasche (2001) and is consistent with the mean-variance framework as optimal portfolios based on variance equal optimal portfolios based on CVaR given a normal distribution of asset returns (Uryasev, 2000).

Data and empirical approach

In this study, I follow the approach by Rockafellar and Uryasev (2000) to minimize CVaR as optimization target. To calculate the mean-CVaR efficient frontier, the Pictet LPP-60 portfolio is replicated. As a proxy for stock investments three investable indices are selected. For Swiss equities the Swiss Performance Index (SPI) is included. For international stocks and international small caps, the MSCI World and the MSCI Small Cap are added. Both indices contain stocks of developed countries for liquidity reasons. For the fixed income share of the portfolio, the Swiss Bond Index was added to reflect the performance of Swiss corporate bonds. The BarCap Euro-Aggregate Corporates includes bonds of investment grade quality and therefore has a minimum rating for European Bonds. For sovereign debt of developed countries and emerging markets the Barclays World Government and Barclays Emerging Market Bond Index were added. Corresponding to the sub-indices of the LPP-60 alternative assets are represented by the SXI Real Estate Index and the HFRX Global Hedge Fund Index. The first contains cap-weighted real estate funds listed on stock exchange, the second represents absolute return strategies.

To represent the listed private equity universe, I include an equally weighted buy and hold index of 115 LPE vehicles based on their daily market prices from January 2000 to December 2013. The sample is drawn from Preqin (2012) and the LPX (2015a) universe. In order to put the potential benefits of LPE to a pension fund's portfolio into perspective, the optimization is re-run with holdings in the LPX50 NAV index. This modification allows to contrast risk and return of portfolios with observable market prices and with NAV. As market prices and NAV of most LPE vehicles significantly differ (Lahr & Kaserer, 2010), other optimal portfolios are expected. To complete the analysis, the findings are cross-validated by an optimization of portfolios including LPE based on the NAV of the publicly traded LPE vehicles and portfolios with private equity allocations based on the NAV of limited partnerships. For the NAV of LPE and NAV of unlisted limited partnerships, the LPX50 index and the Thomson Private Equity Buyout index are included.

The mentioned sub-indices have fixed weightings in the Pictet LPP index family. In order to maintain flexibility during the optimization, maximum weightings were included which correspond to the legal constraints imposed by the LPP framework . To replicate the pension portfolio, the following investment caps were introduced:

Weight constraints per asset class

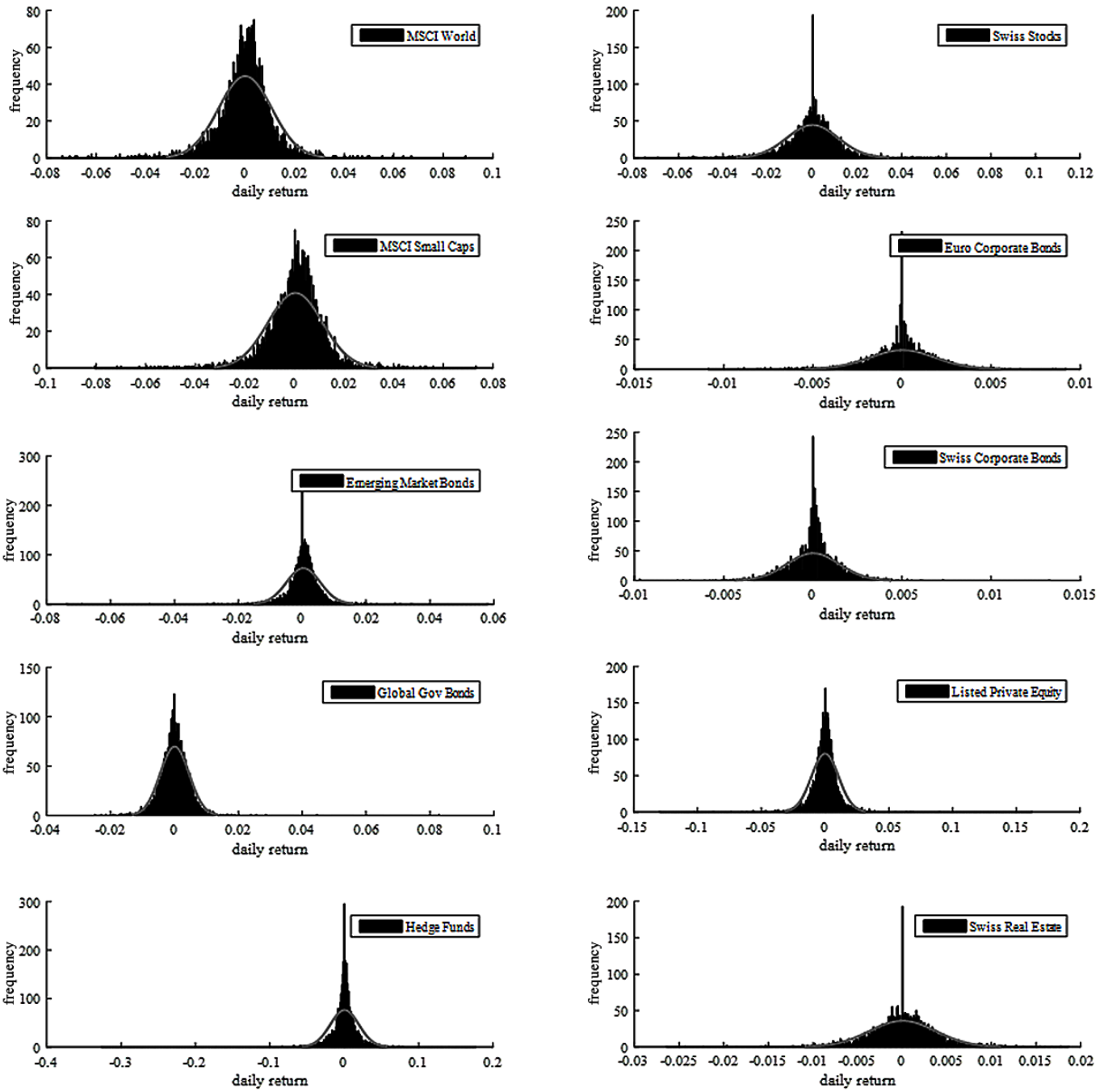

Exhibit 1: This exhibit shows the daily returns and their frequencies over the period of the 1st quarter 2000 through the 4th quarter 2013. The lower four plots show the histograms for bond indices. They exhibit only a few return materializations in the tails. In the mid-section and upper sections three histograms of stock indices and alternative investments/real estate refer the fatter tails.

As LPE is not included in the LPP-60 portfolio, the most reasonable categorization is defined by similarities to the existing LPP-60 asset class definitions. The first and most obvious categorization is into the regular equity bucket. LPE offers ownership rights, which are publicly traded on a stock exchange, therefore fulfilling the criteria of a regular stock. Analyzing the characteristic of LPE stocks more closely, the average company size is similar among the LPE universe identified by Preqin and LPX (LPX, 2015a; Preqin, 2012) Based on the study of Bilo, Christophers, Degosciu and Zimmermann (2005), LPE vehicles have a small market capitalization, which is confirmed by a positive Pearson correlation with the included MSCI Small Caps. Therefore, LPE stocks qualify also as small caps. The core business, holding investments in private companies, also allows categorize LPE as an alternative investment (Swisscanto, 2015). Therefore, LPE can substitute equities (MSCI World, SPI), small caps (MSCI World Small Caps) and alternative investments (HFRX indices).

Hence, to add LPE to the portfolio, we treat it either as a (1) regular stock, as a (2) small cap stock or as an (3) alternative investment . Each scenario (1)-(3) results in a combination

Weight constraints per asset class

	(1) Investment Categories	(2) Index	(3) Pictet LPP-60	(4) Constraint based on LPP-Law (max)	(5) Cumulative Constraint (max)	(6) Applied (Group) Constraint (max)
Bonds	Swiss Corporate Bonds	Swiss Bond Index	10	100	100	100
	Euro Corporate Bonds	BarCap Euro-Aggregate: Corporates	5	100		
	Global Government Bonds	Citi World Government Bond Index	10	100		
	Emerging Market Bonds	BarCap EM LC Government Capped	5	100		
Stocks	Swiss Stocks	Swiss Performance Index	20	50	50	50
	Global Stocks	MSCI AC World	30	50		
	Small Cap Stocks	MSCI Small Cap World	10	50		10
Alternatives	Hedge Funds	HFRX Global Hedge Fund	5	15	15	15
	Private Equity	Market price index of 115 LPE vehicles	0	15		flexible*
Real Estate	Swiss Real Estate	SXI Real Estate Funds	5	30	30	30

Exhibit 2: This table shows the covered investment categories (1) and the indices (2) which represent them. The indices chosen are based on the selected indices of the pictet LPP-60 portfolio. The corresponding weights of the sub-indices are presented in column (3). The individual maximum weights (4) allowed LPP law and the cumulative constraint per asset class (5) lead to the applied weight constraints (6) in the optimization analysis.
***The individual weighting of private equity and the applied cumulative constraint depends on the categorization of LPE.**

weight limit with the respective asset class. Further constraints are imposed. No short positions are allowed and full investment is required.

Methodology

As mentioned earlier, the linearized mean-CVaR optimization (Rockafellar & Uryasev, 2000) is applied for portfolio selection. The CVaR is calculated based on scenarios by simulations of historical return distributions. The following discussion of the methodology follows Rockafellar and Uryasev (2000) and Cornuejols and Tütüncü (2006).

To develop the optimization model, a portfolio of assets with portfolio specification x (portfolio weights) and random events (returns) is considered. This gives a loss function , which does not exceed a certain threshold . For a fixed decision vector , the cumulative distribution function (1) of a loss corresponding to the chosen portfolio specification x is (Tütüncü, 2003):

$$\psi(x,\alpha) = \int_{f(x,y)\le\alpha} p(y)dy \qquad (1)$$

The VaR (2) associated with the portfolio choice x for a specified confidence level α is:

$$VaR_\alpha(x) = \min\{\gamma \in \mathbb{R} : \psi(x,\gamma) \ge \alpha\} \qquad (2)$$

CVaR (3) is therefore the area under the density function which is greater or equal to the VaR divided by 1 minus the confidence level . Working with equation (3) would imply that during optimization VaR has to be calculated first. Simplified by Rockafellar and Uryasev (2000), the calculation of CVaR is detached from the calculation of VaR:

$$CVaR_\alpha(x) = \frac{1}{1-\alpha}\int_{f(x,y)\ge VaR_{\alpha(x)}} f(x,y)p(y)dy \qquad (3)$$

where $f(x,y)$ is still the loss function depending on portfolio weights x and portfolio asset returns y. As CVaR is the average loss beyond VaR, CVaR of a portfolio is at least as large as the VaR, hence a portfolio with a small CVaR will also have a small VaR (Cornuejols & Tütüncü, 2006).

Since the calculation of CVaR depends by definition on the calculation of VaR, processing a CVaR optimization is difficult, a simpler auxiliary version is considered (Cornuejols & Tütüncü, 2006; Tütüncü, 2003):

$$F_\alpha(x,\gamma) = \gamma + \frac{1}{1-\alpha}\int_{f(x,y)\ge\gamma} (f(x,y)-\gamma)p(y)dy$$
$$F_\alpha(x,\gamma) = \gamma + \frac{1}{1-\alpha}\int (f(x,y)-\gamma)^+ p(y)dy \qquad (4)$$

Where $a^+ = \max\{a,0\}$

It follows that VaR is to minimize over y. Hence, to minimize CVaR over the portfolio weights, the function F_α (4) must be minimized with respect to portfolio weights and returns. Instead of optimizing the density function $p(y)$, a handier approach is chosen based on scenarios for different *S*:

$$\tilde{F}_\alpha(x,\gamma) = \gamma + \frac{1}{(1-\alpha)S}\sum_{s=1}^{S}(f(x,y_s)-\gamma)^+ \qquad (5)$$

Equation (5) gives the new optimization problem:

$$\min_{x\in X,\gamma} \gamma + \frac{1}{(1-\alpha)S}\sum_{s=1}^{S}(f(x,y_s)-\gamma)^+ \qquad (6)$$

A lower value of γ in equation (6) leads to a higher weighted sum for a small α and a lower weighted sum for large α. The minimum is found when the decrease in the sum is offset by the increase in γ. Then γ corresponds to VaR.

To simplify the problem further, $(f(x,y)-\gamma)$ is replaced by the artificial variable *z* with the constraint that *z* is larger than zero and smaller than $(f(x,y)-\gamma)$.

$$\min_{x,z,\gamma} \gamma + \frac{1}{(1-\alpha)S} \sum_{s=1}^{S} z_s \qquad (7)$$

The main aspect of equation (7) is that not only the portfolio weights are decision variables but also that the quantile level will be optimized. VaR will be calculated as a by-product. The minimization of CVaR leads to almost optimal VaR levels, as CVaR is never smaller than VaR, hence low CVaR portfolios correspond to low VaR portfolios (Andersson, Mausser, Rosen & Uryasev, 2000).

Results and discussion

In this section, I present the results of the multi-asset portfolio optimization. As the histograms in exhibit 1 showed that the included assets exhibit non-normal return distributions, the optimization was based on minimizing CVaR. This approach considers the higher moments of asset returns. The optimization covers the full time period of January 2000 to December 2013 as well as sub-periods before the economic crisis 1st quarter 2000 through 2nd quarter 2007, during crisis 3rd quarter 2007 through 1st quarter 2009 and post-crisis 2nd quarter 2009 through 4th quarter 2013. I compare the results of the LPE sample of 115 vehicles to the results obtained when optimizing with private equity (limited partnerships) and the LPX50 NAV index. The Thomson Private Equity Buyout index is a proxy for traditional partnerships whereas the LPX50 NAV index captures the NAV performance of the 50 largest and most liquid global LPE stocks.

The presentation of the results is structured in subsections depending on the asset type classification of LPE. The first subsection presents the results of the optimization with LPE categorized as a regular stock, followed by the results when categorized as a small cap stock and as an alternative investment.

Portfolio optimization with different LPE categorization

The efficient frontiers presented in the following sections correspond to the most dominate frontier during each of the analyzed sub-periods and the total observation period. Consequently, the best performing portfolios containing LPE allocations measured by their risk and return are compared to the portfolios without LPE allocations.

Optimization with LPE categorized as stock

The findings on LPE categorized as a regular stock is threefold (see exhibit 4).

First, the inclusion of LPE stocks in a portfolio of LPP-60 index holdings does not add value from a CVaR-return perspective. This is based on the location of the efficient frontier for the overall observation period containing portfolios which include a 2.5% allocation to the self-constructed LPE index. The LPP-60 frontier graphically almost matches the frontier with LPE portfolios. Among the efficient frontiers with LPE index allocations, the efficient frontier with a low allocation of 2.5% was the highest. The low allocation is dominant compared to frontiers containing portfolios with higher LPE allocations, but absolutely weaker than portfolios without LPE allocations.

Second, the findings for the total observation period also hold for the pre-crisis period up until 2nd quarter 2007. However, if only the post-crisis period is considered, the tangency portfolio including a 50% LPE holding, yields a 300 bps higher annual return than a portfolio on the LPP-60 frontier with the same CVaR of 15.42% p.a. In general, the 50% LPE frontier dominates the Pictet LPP-60 frontier when accepting more than a CVaR of 9.6% p.a. The findings for the post-crisis period show that after the crisis, LPE exhibits different characteristics compared to the regular stocks represented by the SPI and MSCI World. A look at exhibit 3 shows that the LPE return distribution has clearly fatter tails than those of the MSCI World and the SPI. Despite the slightly positive skewness of the LPE returns, the consideration of the kurtosis leads to neglection of LPE during pre-crisis. The opposite explanation applies to the post-crisis period where a substantial allocation to LPE benefits the portfolio.

The third finding considers the results on the crisis period. The finding that overall and during pre-crisis period portfolios without LPE allocations (e.g. LPP portfolios) dominate those

Distribution moments of portfolio assets

		Bonds				Stocks			Alternative Investments				Real Estate
		Swiss Corporate Bonds	Euro Corporate Bonds	Global Government Bonds	Emerging Market Bonds	Swiss Stocks	Global Stocks	Small Cap Stocks	Hedge Funds	Listed Private Equity (Prices)	Listed Private Equity (NAV)	Trad. Private Equity	Swiss Real Estate
Post-Crisis	Mean	1.35%	3.34%	-1.70%	9.85%	10.17%	14.31%	19.95%	6.65%	15.02%	4.93%	25.41%	2.95%
	95% CVaR	4.67%	6.43%	17.25%	13.16%	36.11%	38.07%	43.88%	6.13%	20.41%	20.55%	46.35%	15.74%
	Skewness	-0.31	-0.38	3.06	-0.75	-0.30	-0.30	-0.41	1.67	-0.02	2.10	-0.14	-0.14
	Kurtosis	6.99	6.2	10.12	46.57	5.98	5.99	6.47	67.9	6.06	31.95	8.4	5.79
Crisis	Mean	0.64%	-4.32%	4.80%	-1.14%	-35.32%	-37.63%	-42.67%	-12.01%	-54.14%	-33.20%	-24.67%	-3.45%
	95% CVaR	0.35%	10.04%	0.60%	35.40%	5.45%	4.51%	4.82%	23.12%	57.25%	51.89	82.19	14.54
	Skewness	0.62	-0.54	0.27	-1.81	0.25	-0.21	-0.41	-7.69	-0.57	-9.73	-0.27	-0.03
	Kurtosis	7.86	4.06	7.94	33.41	7.1	7.32	5.94	92.23	7.3	141.45	7.72	6.72
Pre-Crisis	Mean	-0.37%	0.14%	1.78%	10.99%	3.41%	2.03%	10.20%	6.02%	-1.67%	19.87%	9.42%	1.43%
	95% CVaR	5.20%	6.46%	12.17%	20.83%	42.04%	31.35%	29.83%	6.64%	44.76%	12.64%	75.10%	11.55%
	Skewness	-0.31	-0.33	-0.21	-0.71	-0.21	-0.07	-0.38	2.90	0.29	4.36	-1.23	-0.43
	Kurtosis	6.03	5.11	4.29	10.43	8.27	5.42	4.26	66.08	33.42	35.09	44.51	7.60
Overall	Mean	0.34%	0.67%	0.98%	9.09%	0.86%	1.23%	6.89%	3.98%	-2.57%	4.99%	10.58%	1.33%
	95% CVaR	5.21%	7.00%	15.16%	20.43%	44.82%	41.37%	43.15%	8.53%	40.94%	21.32%	72.95%	13.07%
	Skewness	-0.05	-0.43	1.93	-1.40	-0.11	-0.33	-0.58	-1.37	0.00	-5.78	-0.97	-0.28
	Kurtosis	7.41	5.42	38.80	32.55	9.29	10.32	8.63	93.77	32.92	46.36	36.08	7.01

Exhibit 3: This table shows the annualized moments of the return distributions for each sub-period and the total (overall) observation period.

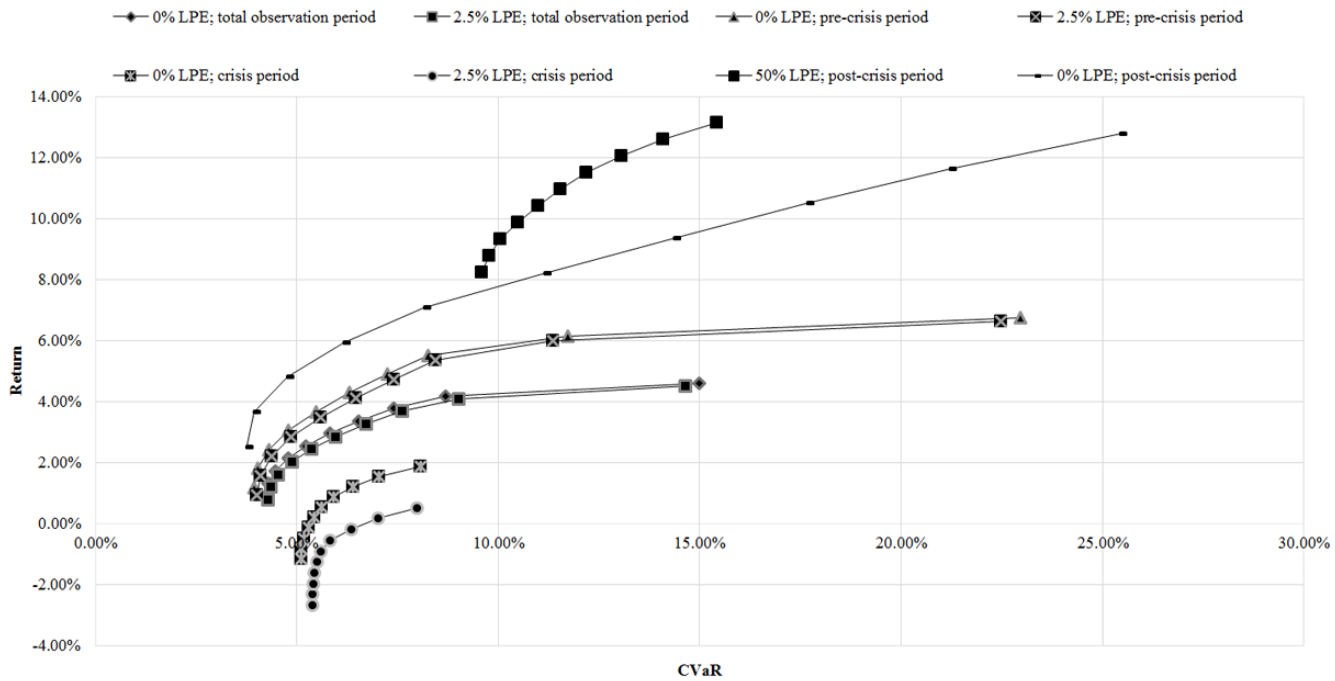

Exhibit 4: This exhibit shows the efficient frontiers for the overall analysed time period from the 1st quarter 2000 through the 4th quarter 2013 as well as for the sub-periods pre-crises (1st quarter 2000-2nd quarter 2007), crises (3rd quarter 2007-1st quarter 2009) and post-crises (2nd quarter 2009-4th quarter 2013). For each time period, an efficient frontier of portfolios containing LPE allocations is presented. For each time period, the chosen frontier is the highest frontier with LPE allocation within the tested range up to the allowed maximum allocation of 50%. For a comparison, the efficient frontier without the LPE allocation is shown only containing indices held by the Pictet LPP-60 reference index. Return and CVaR values are annualized.

with such allocations, accentuates during crisis period. From 3rd quarter 2007 through 1st quarter 2009, LPP and LPE portfolios yield negative returns at low levels of risk. For example, with a 0% LPE allocation, a CVaR of 5.43% is not compensated by a positive return but with a loss of -1.61 percent. Despite the negative returns for low-risk portfolios on both frontiers, portfolios without LPE allocations dominate those with LPE allocations. The tangency portfolio without LPE holdings returns 137 bps with an increase of 7 bps in risk. LPE not only has a bulk of negative returns in the left tail (see exhibit 3) but also shows higher average losses and higher risk than regular stocks.

Optimization with LPE categorized as small cap stock

When assigning LPE to the small cap asset class (see exhibit 5), the findings for the overall, pre-crisis and crisis observation period do not yield significantly different results to the previous analysis (see exhibit 4). A slightly weaker performance of the LPE containing portfolios can be observed. During post-crisis period, the highest efficient frontier allocates 10% to LPE, but does not yield similar returns to the highest efficient frontier with 50% LPE in the stock-replacement optimization. In contrast of the previous scenario, categorization of LPE as regular stock, post-crisis returns for both portfolio types, those with and without LPE, are accompanied by higher risk. The highest risk return portfolios yield 13.61% and 12.77% return with CVaR of 26.79% and 25.44%. This leads to the conclusion that changing the share of actual small cap stocks and replacing it with LPE does not alter the optimal portfolios significantly. In contrast to the previous analysis, the post-crisis results differ as portfolios containing LPE are dominating those without, however to a lesser extent than when the equity portion is substituted.

Optimization with LPE categorized as alternative investment

If the LPP-60 MSCI hedge fund position is replaced with LPE and investments into the SPI, MSCI World and MSCI Small Cap are possible up to 60%, all portfolios for all time segments exhibit a similar risk-return profile than those observed in the small cap-replacement optimization. When comparing to the stock-replacement optimization, portfolios resemble those in the overall, pre- and crisis period and relate to the post-crisis portfolios similarly to those found when replacing hedge funds with LPE. The similar findings for the small cap and alternative investment replacements show that in both cases, the optimal portfolios are determined by the risk and return characteristics of the LPP-60 index holdings and not by the added LPE asset class (see exhibit 5 and 6).

To summarize, the replacement of traditional financial assets and hedge funds covered in the LPP-60 index with LPE does only have a positive effect after the end of crisis, e.g. in the period from 2nd quarter 2009 to the end of 2013. The strongest beneficial effect shows the replacement of regular stocks with a substantial allocation of 50% to LPE. For other time periods and categorization scenarios LPE does not improve optimal portfolios compared to the original LPP-60 portfolio.

To put the findings into perspective and cater to the special properties of LPE with an observable market price and similarity to unlisted private equity limited partnerships, the results of the extended optimization are presented in the following chapter.

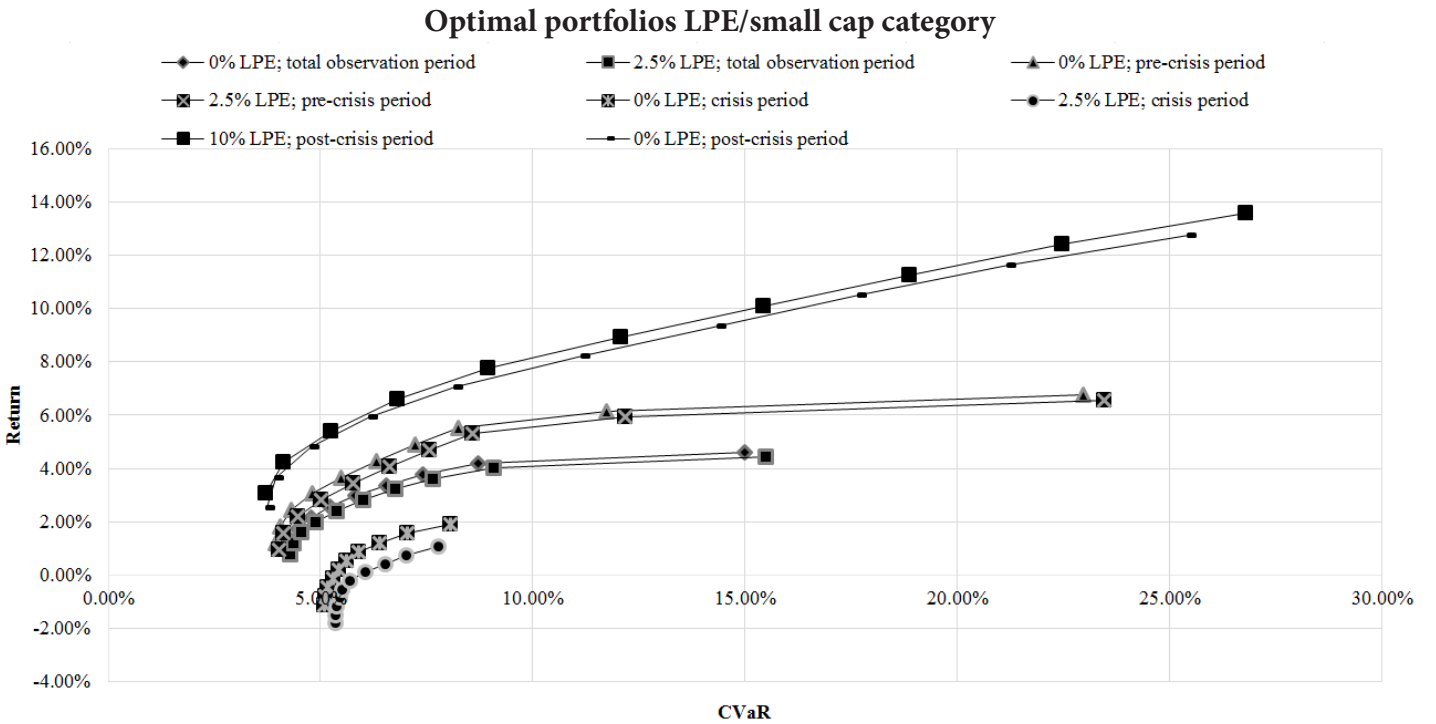

Exhibit 5: This exhibit shows the efficient frontiers for the overall analysed time period from 1st quarter 2000 through 4th quarter 2013 as well as for the sub periods pre-crisis (1st quarter 2000-2nd quarter 2007), crisis (3rd quarter 2007-1st quarter 2009) and post crisis (2nd quarter 2009-4th quarter 2013). For each time period, an efficient frontier of portfolios containing LPE allocations is presented. For each time period, the chosen frontier is the highest frontier with LPE allocation within the tested range up to the allowed maximum allocation of 10%. For comparison, the efficient frontier without the LPE allocation is shown only containing indices held by the Pictet LPP-60 reference index. Retrun and CVaR values are annualized.

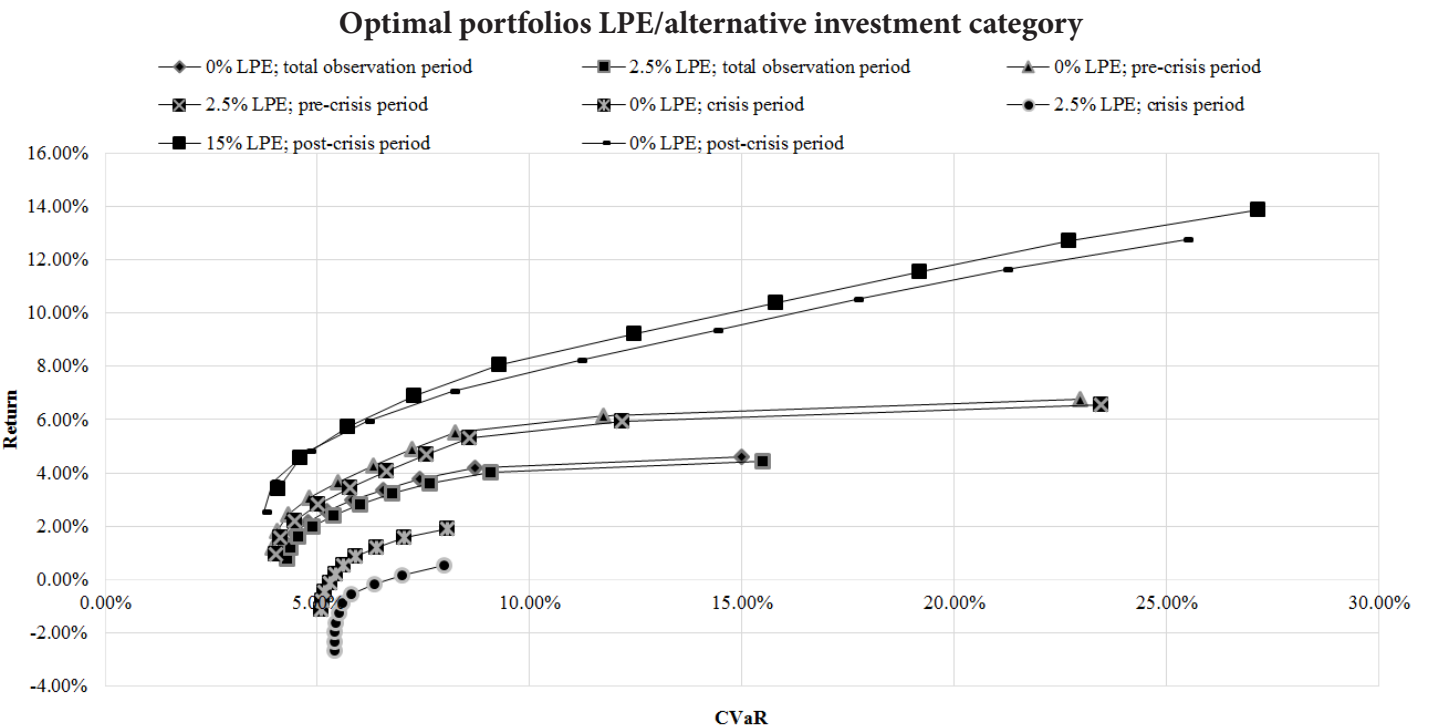

Exhibit 6: This exhibit shows the efficient frontiers for the overall analysed time period from the 1st quarter 2000 through the 4th quarter 2013 as well for the sub-periods pre-crises (1st quarter 2000-2nd quarter 2007), crisis (3rd 2007-1st quarter 2009) and post-crisis (2nd quarter 2009-4th quarter 2013). For each time period, an efficient frontier of portfolios containing LPE allocations is presented. For each time period, the chosen frontier is the highest frontier with LPE allocation within the tested range up the allowed maximum allocation of 15%. For comparison, the efficient frontier without the LPE allocation is shown only containing indices held by the Pictet LPP-60 reference index. Return and CVaR values are annualized.

Portfolio comparison with LPE, LPX50 and Thomson PE allocations

For the first comparison, portfolios for three different weights were compared when investing either in LPE based on market prices (LPE sample), LPE based on NAV (LPX50) and traditional private equity (Thomson Private Equity Buyout index). In this section, the description of the results follows chronological order starting the period before the crisis and discusses the portfolios including three proxies according to their classification as regular stock, small cap stock and alternative investment.

Over the total observation period, LPE could not materially improve pension funds' portfolios. A portfolio in which global and Swiss stocks were replaced with 2.5% LPE yields the best risk-return profile when adding private equity to the model specification. Nevertheless, an unchanged Pictet LPP-60 portfolio exhibits the same risk-return characteristic.

In the next sections, the results for the sub-periods are presented. Regime changes such as the crisis period can influence optimal asset allocations and can show the potential of private equity proxies.

During the pre-crisis period, portfolios holding investments in the LPX50 assigned to the small cap stock and alternative investment portion of the portfolio dominate. Holding portfolios with investments in LPE, hence allocations based on the market price of LPE leads with similar risk but with significantly lower returns. For 15% annual risk (CVaR) a portfolio with 10% LPE holdings returns 8.6% less than the portfolio with the LPX50 holdings. TPE holdings push the frontiers further to the right towards higher risk albeit at the same low return levels as LPE.

The result is not surprising. A look at exhibit 3 shows that the NAV of LPE vehicles not only have higher returns than LPE market prices and traditional private equity, but also higher positive skewness. Additionally, the LPX50 yields strong positive skewness compensating it for the high kurtosis, which otherwise would lead to a reduced representation in the portfolio.

When the indices are assigned to the small cap share or alternative investment share, the findings remain valid for portfolios with LPX50 holdings (see exhibit 8 and 9). LPX50 at low allocations offers the best risk-return relationship, LPE the worst. However, TPE does not increase portfolio risk like in the equity-replacement scenario. The risk-return relationship of TPE and LPE containing portfolios is fairly similar. A direct comparison with the same period analysis but with the proxies categorized as an alternative investment yields similar results. Portfolios with LPX50 holdings compensate with significantly higher returns for the taken risk than portfolios with holdings in TPE or LPE at market prices. Exhibit 3 displays the reason for these findings. The differences of LPE and TPE to LPX50 are striking in terms of higher mean return, lower risk and positive skewness of LPE book value returns (LPX50).

The results for the period up to the second half of 2007 clearly indicate, that book values of LPE had a higher probability to achieve significant positive returns even more pronounced than hedge fund returns. This is an important finding, given that hedge fund returns exhibit skewness and excess kurtosis as well and can be a valuable addition to an institutional investor's portfolio (Till, 2004; Favre & Galleano, 2002). But in contrast to hedge funds, the applied index for LPE book values (LPX50) suffers less from survivorship bias and selection bias than a typical hedge fund index (Pictet, 2014; HFR, 2008; Bilo et al., 2005). As LPE is listed

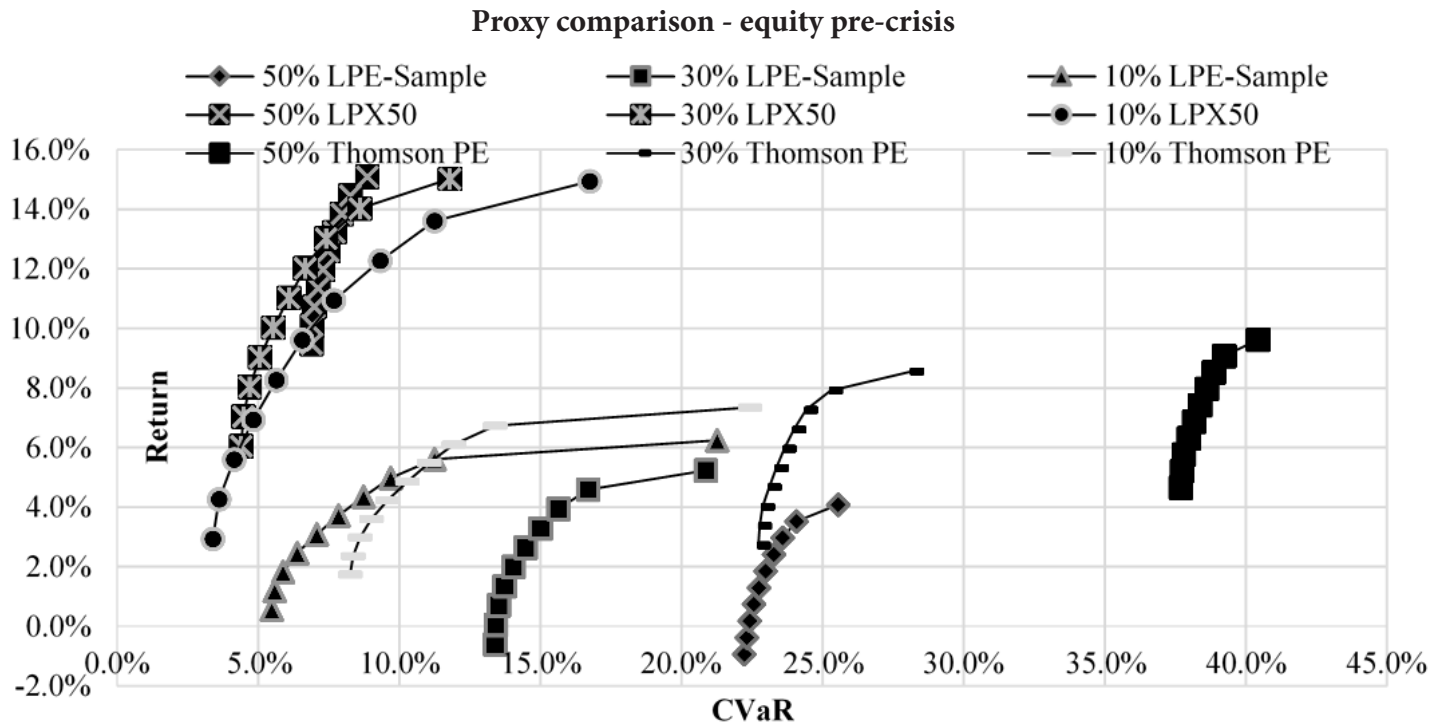

Exhibit 7: This exhibit shows the efficient frontiers for the pre-crisis period with fixed allocations of 10%, 30% and 50% to the self-constructed LPE price index, the LPX50 and the Thompson PE Buyout index. The allocations replace the equity holdings of the portfolio (MSCI World, SPI). Return and CVaR values are annualized.

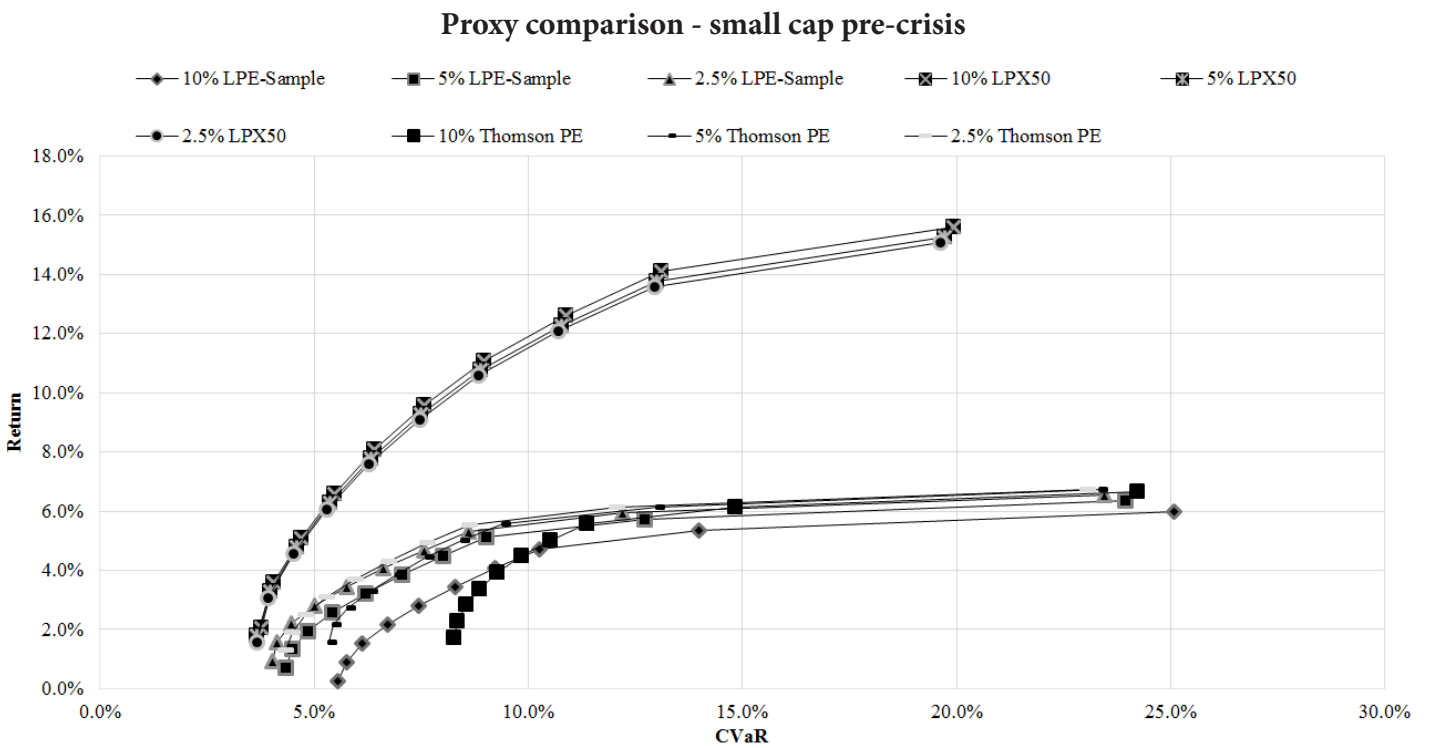

Exhibit 8: This exhibit shows the efficient frontiers for the pre-crisis period with fixed allocations of 2%, 5% and 10% to self-constructed LPE price index, the LPX50 index and the Thomson PE Buyout index. The allocations replace the small cap stock holdings of the portfolio (MSCI Small Cap World). Return and CVaR values are annualized.

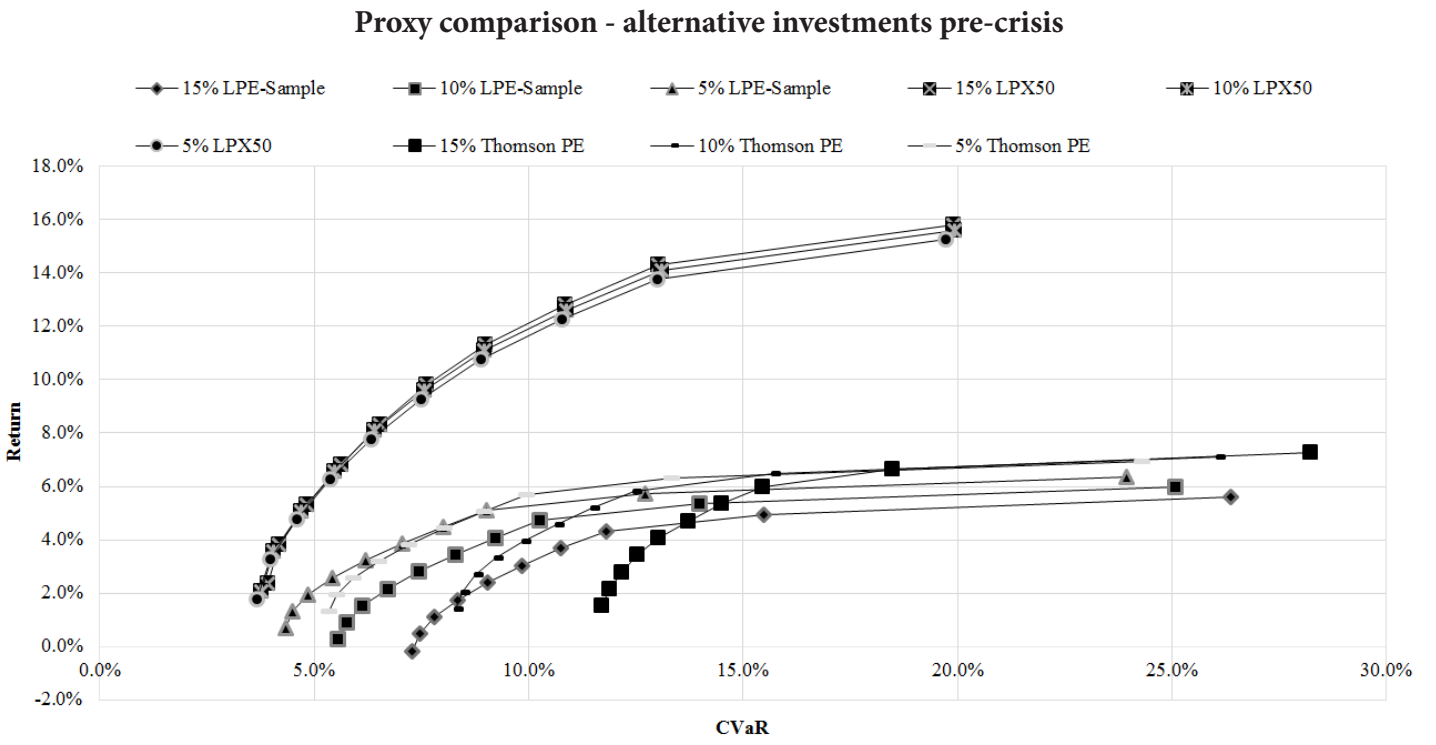

Exhibit 9: This exhibit shows the efficient frontiers for the pre-crisis period with fixed allocations of 5%, 10% and 15% to the self-constructed LPE price index, the LPX50 index and the Thomson PE Buyout index. The allocations replace the hedge fund holdings of the portfolio (HFRX). Return and CVaR values are annualized.

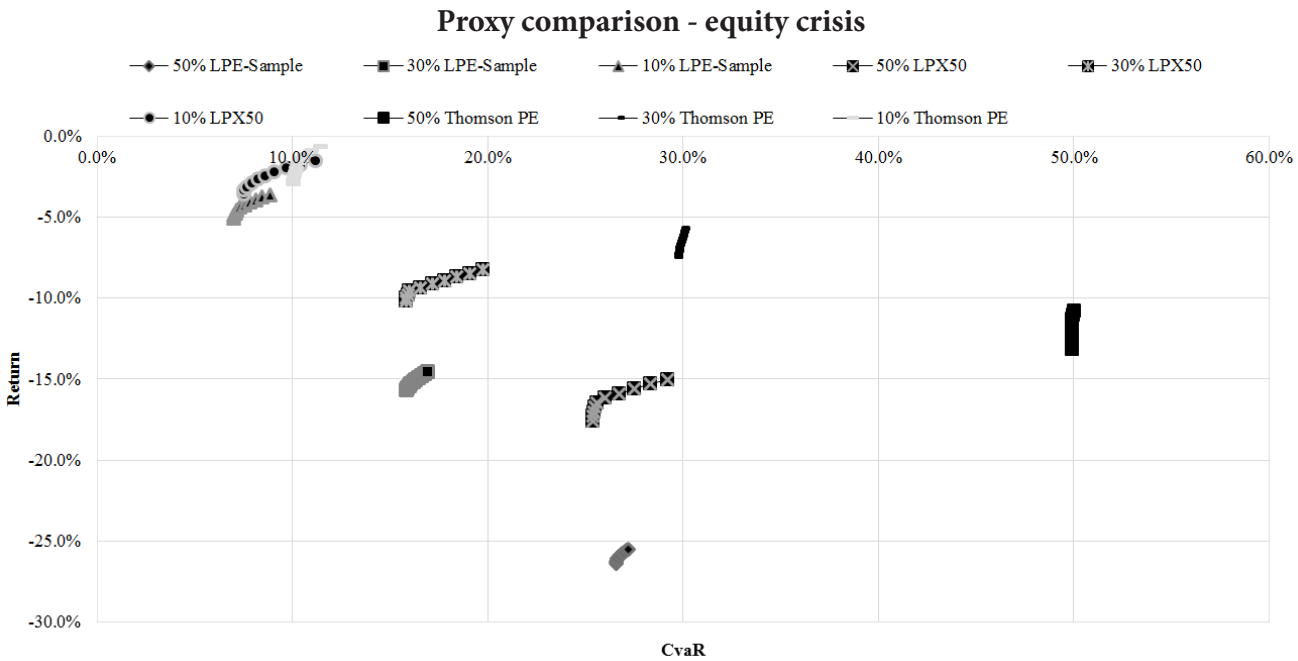

Exhibit 10: This exhibit shows the efficient frontiers for the crisis period with the allocations of 10%, 30% and 50% to the self-constructed LPE price index, the LPX50 index and the Thomson PE Buyout index. The allocations replace the equity holdings of the portfolio (MSCI World, SPI). Return and CVaR values are annualized.

on a stock exchange, reporting and governance requirements lead to higher transparency and reliability of the presented data (LPX, 2015b).

During the crisis, investment possibilities shrinked, leading to short frontiers where the minimum-risk and the tangency portfolio remain close together. For positive allocations to LPE, LPX50 and TPE no positive returns can be achieved in an equity-replacement scenario (see exhibit 10). The best choice is a portfolio with 10% TPE holdings when replacing the equity portion. Traditional private equity lost less in comparison with regard to return and exhibits a smaller change in terms of negative outliers, e.g. skewness compared to previous periods. Contrary to the previous period, LPE holdings based on the book value, e.g. NAV are not a favorable portfolio addition anymore. The highly positive skewness from previous quarters changed to a similarly extreme negative skewness during crisis whereas at the same time returns collapsed.

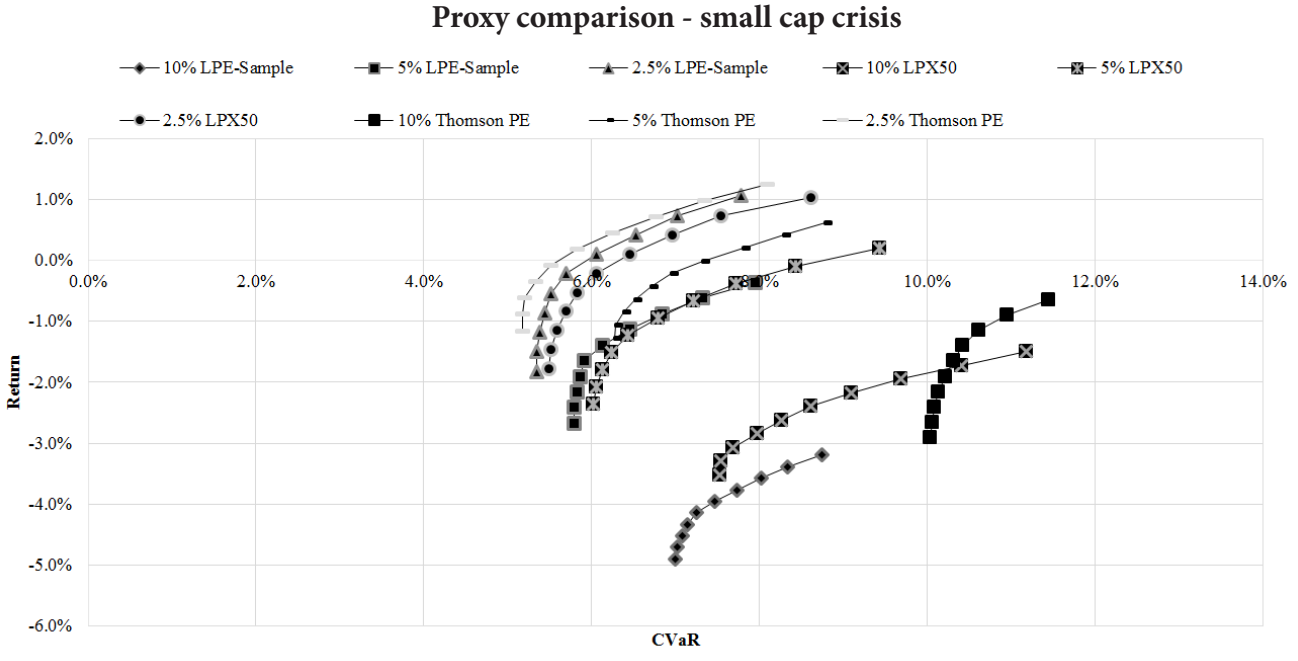

Exhibit 11: This exhibit shows the efficient frontiers for the crisis period with the fixed allocations of 2%, 5% and 10% to the self-constructed LPE price index, the LPX50 index and the Thomson PE Buyout index. The allocations replace the small cap stock holdings of the portfolio (MSCI Small Cap World). Returned and CVaR values are annualized.

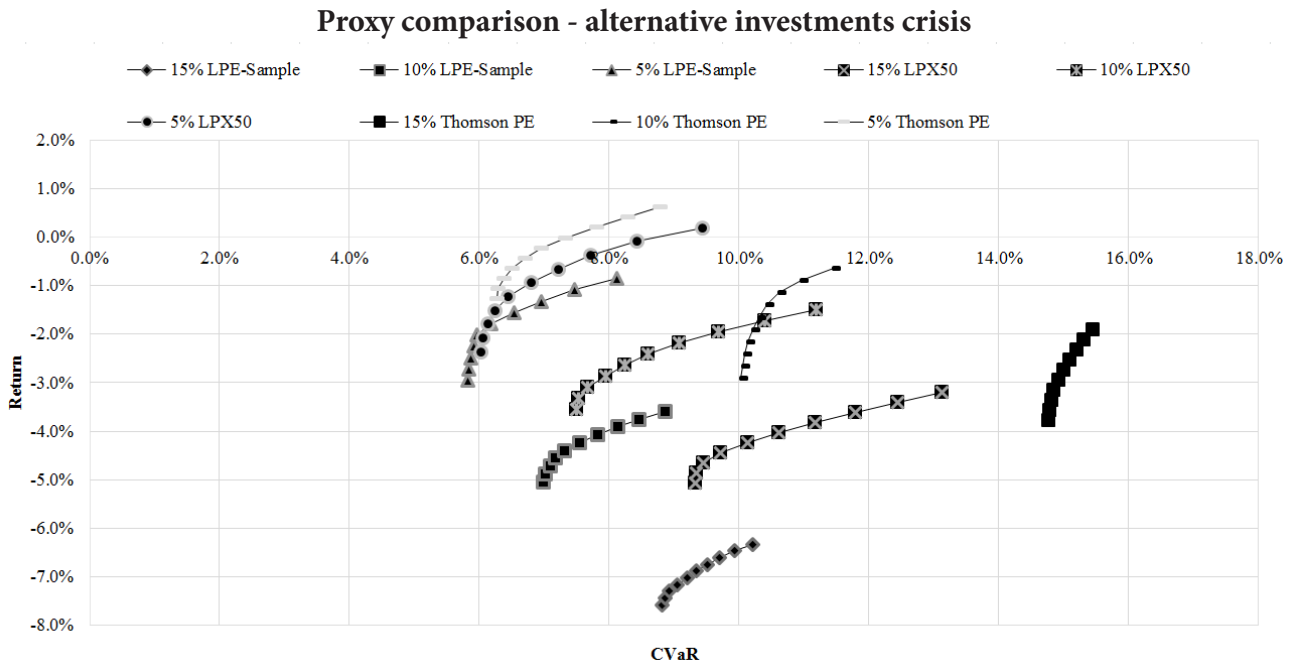

Exhibit 12: This exhibit shows the efficient frontiers for the crisis period with fixed allocations of 5%, 10% and 15% to the self-constructed LPE price index, the LPX50 index and the Thomson PE Buyout index. The allocations replace the hedge fund holdings of the portfolio (HFRX). Return and CVaR values are annualized.

During crisis, TPE is the best substitution for small cap stocks similar to the equity substitution case. But in contrast to the previous analysis, for both cases small cap and alternative investment substitution positive returns are feasible (see exhibit 11 and 12). If categorized as an alternative investment, even an allocation to LPX50 yields a positive return (exhibit 12).

After the crisis, TPE allocations categorized as stock result in the highest efficient frontier (exhibit 13). Due to the far right location, both risk and return are significantly higher than with portfolios containing LPE or LPX50. The highest efficient frontier holding TPE yields an annual return of more than 18.3% at a CVaR of 28%.

For 15% CVaR, LPE holdings of 50% LPE yield 13.1% return p.a., whereas 30% allocations to the LPX50 and TPE offer 9.8% and 11.9% in return (see exhibit 13). Without a target return, from a

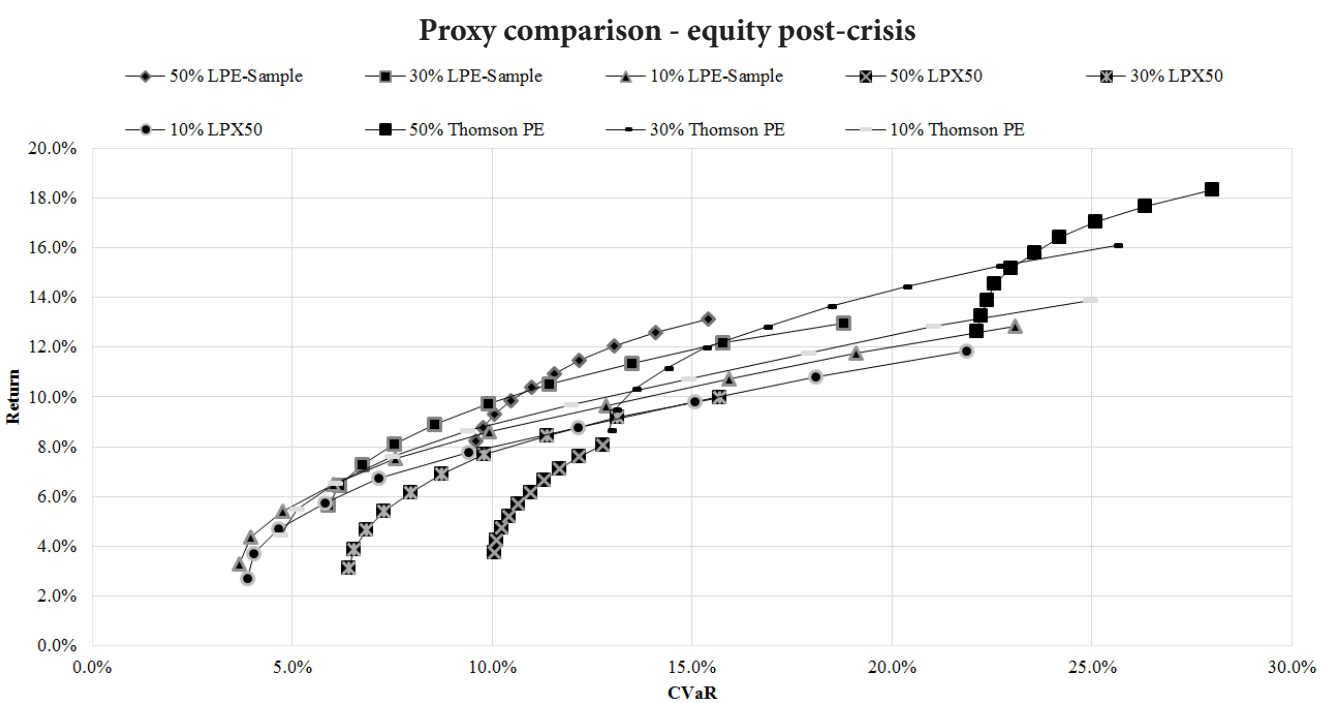

Exhibit 13: This exhibit shows the efficient frontiers for the post-crisis period with fixed allocations of 10%, 30% and 50% to the self-constructed LPE price index, the LPX50 index and the Thomson PE Buyout index. The allocations replace the equity holdings of the portfolio (MSCI World, SPI). Return and CVaR values are annualized.

risk-return perspective, LPE investments leave investors better off. After crisis, more risk-averse and risk-seeking investors can benefit from holding portfolios with LPE allocations. However, with a target return higher than 13.1% p.a., investors must accept annual risk of minimum 18.8% CVaR and shift to mid- and high-range TPE allocations.

Portfolios based on the NAV of LPE lost to LPE and TPE portfolios. Traditional unlisted private equity dominates when included even at a low weight of 10% all variations of LPX50 portfolios. The fact that on each level of risk and with all weight allocations, LPX50 portfolios are dominated by TPE and LPE can be explained by exhibit 1 and exhibit 3. The LPX50 NAV not only has the lowest mean return, but also the highest kurtosis.

In contrast, to previous sub-periods, risk-return ratios increased compared to the other tangency portfolios. For the LPX50 the post-crisis period data shows unfavorable moments compared to pre-crisis values, but LPE and TPE benefit from reduced risk and increased returns. The shift in risk-return characteristics between the LPX50 and TPE and LPE results in new optimal portfolios holding LPE investments at market prices. Optimizations treating proxies as either an alternative asset or a small cap stock highlight again the favorable properties of TPE and the similarities of the distribution moments to LPE (see exhibit 14 and 15). Two findings are noteworthy: First, optimal portfolios and efficient frontiers are closer together, the choice of proxy does lead to different portfolios but with less dispersion than in the stock-replacement scenario. In latter, optimal portfolios differ by the investor's risk taking ability or willingness. Moderate risk takers without a high target return are better off with portfolios containing LPE whereas investors accepting higher risk would ideally invest in a portfolio with traditional private equity. This leads to the second remark: Optimal holdings are clearly identifiable at every level or risk and willingness to take risk. TPE containing portfolios return on average more than portfolios with allocations to LPE and LPX50.

In summary, the results for the post-crisis period differ in terms of dominance of a specific proxy and the asset, which is replaced by it. The highest dispersion could be found when the three proxies replace the equity portion of the portfolio as variations among higher moments of the proxies and the MSCI World as well as the SPI are large. When analyzing the small cap-replacement scenario, frontiers become less dispersed, market prices of LPE and traditional private equity show similar distribution moments than small cap stocks and dominate portfolios with LPX50 holdings. Lastly, when replacing the hedge fund portion only traditional private equity investments are optimal at every risk level.

The findings previously described are based on the most diversified asset allocations of pension funds within the P7 group and correspond to regulations faced by Swiss pension funds whose pension assets to GDP ratio is similar to the US and the UK (Towers Watson, 2015). Although the results about optimal portfolios were derived by application of investment weight ceilings unique to the Swiss pension fund market (see exhibit 2), the findings can be generalized based on the historical asset allocations of pension funds globally. The pension fund study by Towers Watson (2015) showed that stock allocations of pension funds were less than 50% in 2014, a reduction of 10% and 3% in UK and the US over 5 years. In total, pension funds in the largest markets (P7) held on average 51.25% in stocks from 1995 to 2014, which is similar to the stock position ceiling applied (see exhibit 2). With regard to the other constrained asset groups, namely real estate and alternative assets such as hedge funds and private equity, the cumulative weight ceilings of 15% and 30% are ample constraints given the current allocations to alternative assets of 15% and 29% in the UK and the US. Despite the lack of binding investment ceilings, the pension funds of the largest two pension markets showed similar allocations to the main investment categories.

Conclusion

Exhibit 16 displays the best portfolio choice in terms of proxy per time period:

During pre-crisis period, listed private equity would have been a good addition to an LPP-60 portfolio. No matter the categorization, LPE inclusion with maximum weight led to dominant portfolios. However, this dominance could only be achieved when investing in LPE by holding an investment reflecting the net asset values of the LPE vehicles. Hence, the benefit of holding a liquid investment (by a share purchase in an illiquid industry) cannot be obtained. Nevertheless, the best portfolio from a risk-return perspective has the small cap and alternative investment portion replaced by the LPX50.

During crisis, TPE was the best addition to the portfolio. It yielded significantly better risk-return portfolios than when LPE would have been considered. Nevertheless, an unchanged LPP-60 portfolio is the optimal choice. This finding is in line with Goldwhite (2009) who showed that active strategies such as LPE do not strongly diversify in an environment of rising risk aversion. The dominance of TPE compared to LPE and LPX50 during crisis indicate that traditional limited partnerships displace listed private equity in slumping market environments. Clearly, this should be tested for confirmation under the aspect of valuation difficulties with limited partnerships.

After the crisis, the best altered portfolio replaces the stock investment portion to the full extent of 50% with LPE. As the locations of the efficient frontiers depend widely on the selected proxies, strict optimality is only given up to a certain risk threshold of 15.4% and if no return above 13.1% is targeted. Otherwise, only traditional private equity would fulfill the requirements of higher risk-adjusted returns.

The findings differ from previous results in terms of investor type and corresponding investment restrictions, applied optimization model and derived optimal portfolio weights. Firstly, the results presented are based on the assumption that LPE should be tested as a niche-type of private equity, which due to its hybrid character can be classified into investment groups and should not serve merely as a proxy for private equity in the familiar sense of limited partnerships (Preqin, 2015a; Huss & Zimmermann, 2012; Brown & Kraeussl, 2012). In contrast, Cumming et al. (2013) tested LPE as a proxy for private equity and found that LPE is not suitable to track the performance of traditional partnerships in portfolios as it induces unnecessary volatility, which results in low private equity allocations even if considering only return and volatility. Secondly, the results presented in this paper take into account weight caps on individual investment groups. Such investment constraints were not considered in earlier studies (see Cumming,

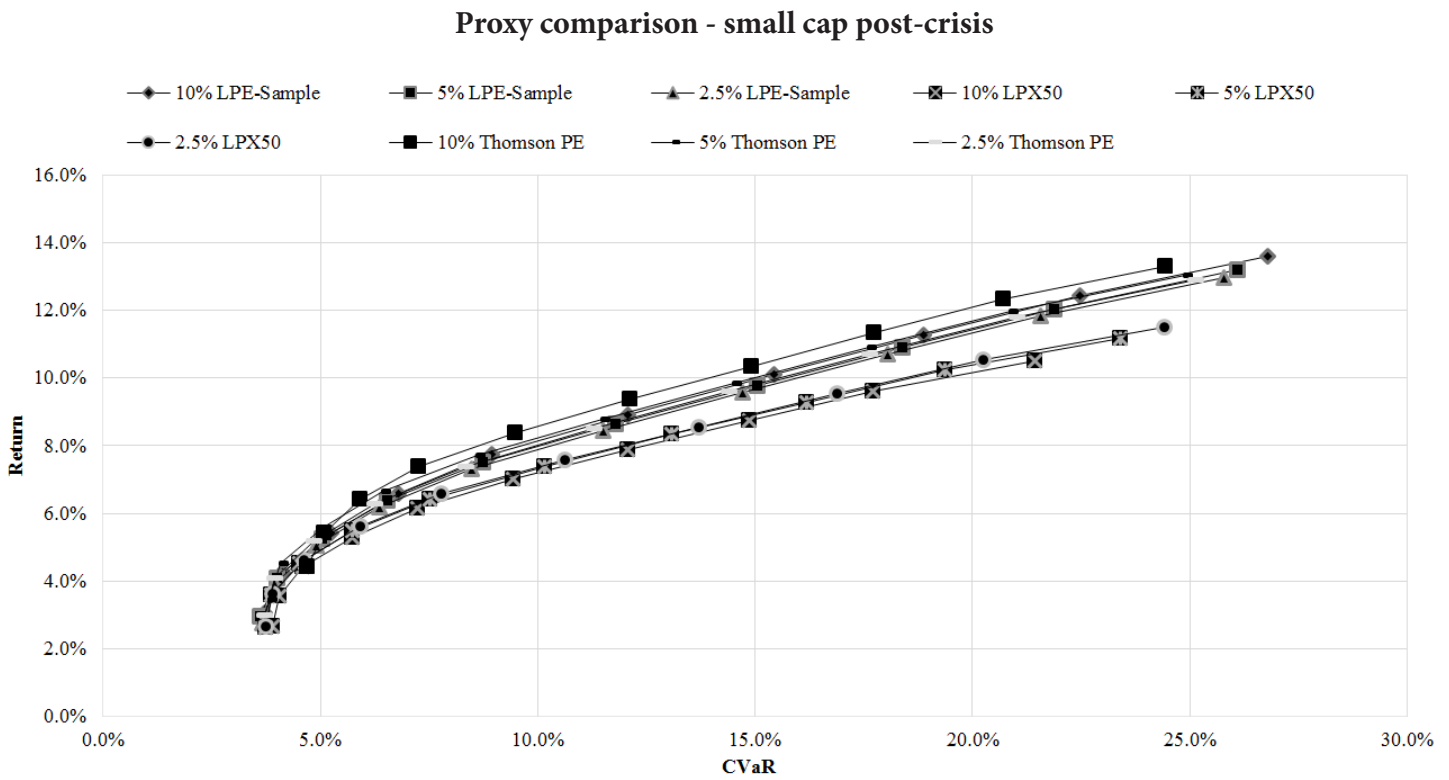

Exhibit 14: This exhibit shows the efficient frontiers for the post-crisis period with fixed allocations of 2%, 5% and 10% to the self-constructed LPE price index, the LPX50 index and the Thomson PE Buyout index. The allocations replace the Small cap stock holdings of the portoflio (MSCI Small Cap World). Return and CVaR values are annualized.

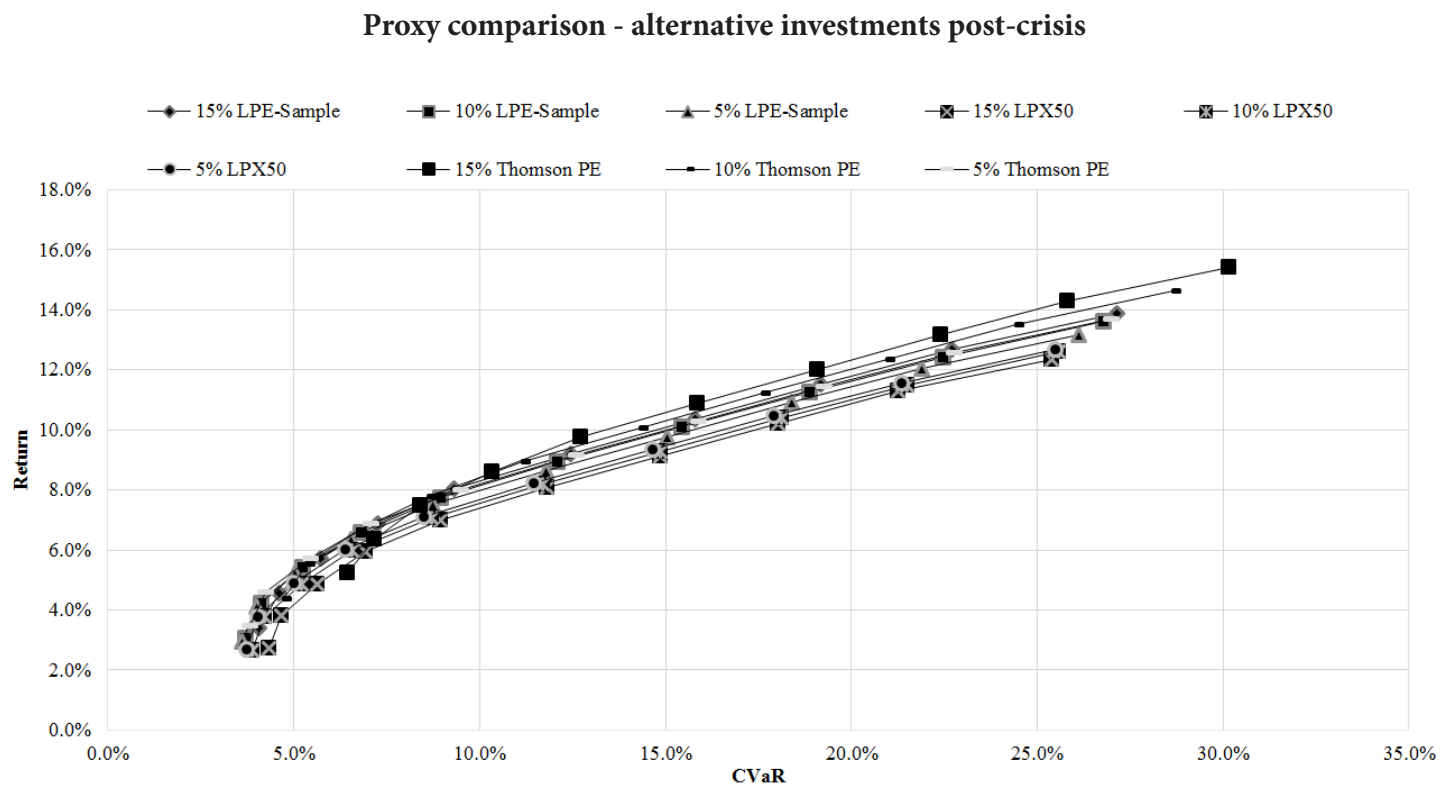

Exhibit 15: This exhibit shows the efficient frontiers for the post-crisis period with fixed allocations of 5%, 10% and 15% to the self-constructed LPE price index, the LPX50 index and the Thomson PE Buyout index. The allocations replace the hedge fund holdings of the portfolio (HFRX). Return and CVaR values are annualized.

Hass & Schweizer, 2014; 2013; Aigner, Beyschlag, Friederich, Kalepky and Zagst, 2012; 2010; Bekkers, Doeswijk & Lam, 2009; Goldwhite, 2009). Aigner et al. (2012) who optimized portfolios consisting of three indices covering global equities, world government bonds and LPE showed that allocations to LPE are strongly driven by an investor's risk aversion (see also Aigner et al., 2010). Contrary to the results presented in this paper, Aigner et al. (2012) do not generalize the optimal LPE weights as they mention the caveat of investment limits for institutional investors regarding alternative investment holdings, but base their analysis on portfolios for unconstrained investors. Thirdly, the portfolios presented in this paper simultaneously consider a realistic number of indices and asset groups and are based on the minimization of CVaR, which adds skewness and tails to the equation. The optimization model applied by Aigner et al. (2012) accounted for higher moments but only considered three broad-market indices. The first-order autoregressive Markov-switching model resulted in moderate allocations to risky assets such as stocks and LPE (up to 32.76%) compared to mean-variance based optimal portfolios (up to 100%) suggested by Bekkers et al. (2009). More specified portfolios considering several asset classes and indices were optimized by Bekkers et al. (2009). However, the chosen optimization framework is based on return variance and does not consider skewness and kurtosis. Bekkers et al. (2009) showed that based on the variance of the tested ten portfolio assets, riskier portfolios contain LPE whose share increases and in the end, LPE ousts bonds, real estate, commodities and stocks. Portfolios consisting entirely of LPE are not feasible in the presented analysis as investment constraints allow for a maximum allocation of 50% when assigning LPE to the stock investment group.

Conclusion

Pension fund managers globally seek to compensate declining performance of traditional financial assets with alternative return sources from hedge funds and private equity. As pension funds in most countries are faced with binding investment constraints in terms of allowed asset classes and maximum weights, not every alternative investment meets the requirements to be included in the portfolio. Listed private equity vehicles as private equity direct investing funds or fund managers provide access to the core business of limited partnerships but with the positive side effect to be a liquid instrument due to public stock market listing.

In this paper, I showed the effects of the addition of listed private equity to a Swiss pension fund model portfolio. This makes the results valid for practitioners, as the model portfolio of the Pictet LPP-60 index is a major reference index for fund managers and adheres to the provisions of LPP law. The approach to base the analysis on the CVaR of portfolio assets caters to the non-normal distribution of asset returns.

The first stage of optimizations only tested the effect of LPE on a pension fund's model portfolio based on their daily market prices without consideration of NAV development or other types of private equity. Empirical results showed that LPE is only a beneficial addition to a pension fund's portfolio in reclining markets as indicated by the post-crisis findings. Specifically, the addition of LPE based on the market prices yields the best results when the LPE investments replaces regular stocks in the portfolio. Under those prerequisites, the weight allocated to LPE should be substantial such as the legally allowed maximum weight of 50%.

Proxy comparison - alternative investments post-crisis

		LPE categorized as:			
		Regular Stock	**Small Cap Stock**	**Alternative Investment**	**without LPE**
Post-Crisis	**Optimal EFF**	50% LPE	10% TPE	15% TPE/2.5% LPE	
	Return	13.10%	13.30%	15.4/13.87%	12.77%
	95% CVaR	15.40%	24.40%	30.2/27.16%	25.44%
	Ratio	0.85	0.55	0.51/0.51	0.5
Crisis	**Optimal EFF**	10% TPE	2.5% TPE	5% TPE	
	Return	-0.50%	1.30%	0.60%	1.89%
	95% CVaR	11.00%	8.10%	8.80%	8.06%
	Ratio	-0.05	0.16	0.07	0.23
Pre-Crisis	**Optimal EFF**	50% LPX50	10% LPX50	15% LPX50	
	Return	8.90%	15.60%	15.60%	6.56%
	95% CVaR	15.10%	19.90%	19.90%	23.45%
	Ratio	0.59	0.78	0.78	0.28
Overall	**Optimal EFF**	2.5% LPE	2.5% LPE	5% LPX50	
	Return	4.50%	4.43%	7.60%	4.60%
	95% CVaR	14.64%	15.50%	28.70%	15.00%
	Ratio	0.31	0.29	0.26	0.31

Exhibit 16: This exhibit shows the efficient frontiers for the post-crisis period with fixed allocations of 5%, 10% and 15% to the self-constructed LPE price index, the LPX50 index and the Thomson PE Buyout index. The allocations replace the hedge fund holdings of the portfolio (HFRX). Return and CVaR values are annualized.

The second stage of optimizations puts LPE in the context of the private equity asset class by testing other types. As one could assume that LPE is only a proxy and potentially not a sufficient one, a comparison to portfolios with traditional private equity and LPE NAV based allocations showed quite the opposite. LPE is a beneficial addition to a pension fund's portfolio albeit not always based on a market price index. The sub-period analysis showed that during the pre-crisis period, portfolios holding LPX50 are substantially better performing on a risk-adjusted basis than the Pictet LPP-60 reference portfolio. Most noteworthy, this finding can be confirmed no matter to which category the LPX50 holding was assigned. The strongest effect on the portfolio is achieved by holding the maximum weight per investment category, hence fully replacing the respective category asset.

During crisis, low allocations to traditional private equity yield the best results of the proxies tested, however leaving the Pictet LPP-60 reference portfolio unchanged was most beneficial.

The results do not surprise based on the distribution parameters of LPE NAV, however, the difference between the higher moments of the LPX50 and TPE is remarkable given that both indices track the performance of private equity entities with the same nature of operations. As due to their listing, LPE vehicles are under scrutiny when it comes to corporate governance and reporting, the same analysis with primary data on limited partnerships might shed light on this specific finding.

In a reclining market environment, all proxies can have a positive effect pension funds' portfolios. Price-based allocations to LPE yield strong results, especially for the stock-replacement scenario. In any case and for each categorization, the portfolio dominate the unaltered Pictet LPP-60 model portfolio.

In summary, the following points can be taken away: In upmarkets and reclining markets, it enhances a pension fund's risk-adjusted portfolio return if the model portfolio altered by private equity allocations. There is no clear indication however, in which investment category private equity proxies would consistently yield the best results. Another component helping to decide in which category to make changes in the asset allocation and based on which type are target returns and risk constraints. A lower willingness to accept risk would indicate allocations to LPE rather than TPE during market recoveries. Furthermore, timing matters in terms of when the model portfolio change should be made. Out-of-sample tests with similar market regime changes would help to build a pattern and strategy for this. Lastly, I showed that LPE adds value to pension fund's model portfolios. This is however arbitrary to the expected benefits flexibility and accessibility to private equity by a share purchase as investor's cannot participate solely in the NAV development unless buying into the LPX50 index. Nevertheless, the good results after crisis indicate future value added of LPE market price indices for pension funds' portfolios.

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