Risk Parity for the Long Run:
Building Portfolios Designed to Perform Across Economic Environments

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1. Introduction

This document extends our earlier study which provides a comparison of ex ante risk parity allocations with the ex post optimal portfolio. Our prior work showed that when the investment opportunity set consists of two assets—stocks and bonds—ex ante risk parity performs on par with the optimal allocation, despite being allocated using less information. This finding has profound implications for asset allocation because it implies that risk parity may offer a proxy for the long-run optimal portfolio and serve as the basis for an implementable version of Modern Portfolio Theory (MPT).

We start with a quick review of MPT, which motivates us to focus on risk-adjusted returns as the proper metric for quality of an asset allocation. We then discuss the conditions under which a risk parity allocation in fact provides the highest possible risk-adjusted return available to market participants. Using a long historical dataset of U.S. Equity, 10-Year U.S. Treasury, and Commodity Futures returns, we show that one of these two conditions likely does not hold and statistically reject the possibility that risk parity generates precisely the maximum Sharpe ratio portfolio (MSRP). However, risk-adjusted returns on the parity allocation are quite similar to the optimal portfolio. Having placed risk parity into its proper theoretical context, we next examine the historical performance of an implementable risk parity strategy, which is allocated historically based on data that would have been available at the time of each rebalance. We find that this implementable version of risk parity also performs on par with the MSRP, confirming findings in our earlier work, which considered a shorter time period and fewer asset classes.

In section 3 we examine the economic intuition for why a risk parity portfolio might be expected to perform on par with the long-run optimal portfolio. We show that the three asset classes we are considering here each perform differently in different macroeconomic environments, making them solid building blocks for a risk parity strategy. Because the macro economy went through phases that favored each asset class during the historical period we study, we find that the average implementable risk parity allocation is very similar to the ex post optimal MSRP.

2. Financial Theory and Risk Parity

MPT, which is due to Markowitz (1952), Sharpe (1964), Lintner (1964), and Merton (1972), shows that there is only one efficient portfolio of risky assets—the maximum Sharpe ratio portfolio (MSRP)—which is unique and offers the highest level of excess return per unit of risk possible given the mix of assets you are considering. Exhibit 1 shows a hypothetical efficient frontier for two assets: stocks and bonds. The Capital Market Line (CML) is a ray drawn from the risk-free rate that achieves tangency with the efficient frontier precisely at the MSRP. Assuming that investors are free to borrow and lend at the risk-free rate, the CML represents the best possible set of risk-return combinations available to them. Every point along the CML represents the
Exhibit 1 The Efficient Frontier and the Capital Market Line
Notes: Exhibit illustrates the concepts of an efficient frontier and CML based on data from 1/1978-10/2011. Please see Patridge, et al. (2011) for further information. Past performance is no guarantee of future results.
Source: Bloomberg, Salient Capital Advisors, LLC as of 10/31/2011.

same level of risk-adjusted return.

If the CML offers superior risk-return combinations, then why are the vast majority of investors holding equity-dominated portfolios? One obvious reason is that we do not know what will be in the MSRP ahead of time—only after we have observed returns over the investment horizon is the efficient portfolio known. This complicates attempts to invest based on MPT, because it means the seed for the CML is undetermined. Attempts to implement MPT to date have involved subjective estimates on forward returns, which can be influenced by behavioral biases such as over- or under-estimating returns on an asset class due to recent performance or over- or under-estimating risk due to investor familiarity with an asset class. Another approach is to derive expected returns from capitalization weights based on the assumption that asset markets are efficient and in equilibrium. The poor quality of these forecasts makes the prospect of levered exposure to such portfolios somewhat unappealing.

In a prior whitepaper we showed that risk parity offers a potential solution to problems identifying the optimal portfolio. A risk parity portfolio is one in which each asset in the portfolio is designed to contribute an equal amount of the portfolio’s risk, where risk is measured by the variance of monthly returns. Portfolios constructed according to risk parity will naturally have a higher allocation to (1) assets with lower volatility and (2) assets with lower correlation to other portfolio constituents. It follows that risk parity portfolios will be diversified relative to their capitalization–weighted and 60% Equity, 40% Treasury (60/40) counterparts.

Maillard, Roncalli, and Teiletche (2010) show that the MSRP exactly coincides with a risk parity portfolio when (1) all assets have the same Sharpe ratio and (2) all assets have the same cross-correlations with one another.

Using return data for U.S. Equities, U.S. 10-year Treasuries, and Commodity Futures from October 1958 to December 2011, we compute the historical Sharpe ratio of each asset and the correlations among the assets. Panel (a) of Exhibit 2 shows the long-run Sharpe ratio for each asset. They are quite similar to one another: Treasuries have a Sharpe ratio of 0.238, Commodities have a Sharpe ratio of 0.276, and Equities have a Sharpe ratio of 0.283. Panel
(b) of Exhibit 2 shows the correlation matrix for monthly returns on the assets. It suggests that the cross-correlations across asset classes are not all equal. Statistically speaking, we reject the hypotheses that: (1) the Equity-Treasury correlation is equal to the Treasury-Commodity Future correlation and (2) that the Equity-Commodity Future correlation is equal to the Treasury-Commodity Future correlation.

Exhibit 2 Sharpe Ratios and Correlations
Notes: Panel (a) of this exhibit shows Sharpe ratio of each asset over the period from October 1958 to December 2011. Panel (b) of this exhibit shows a correlation matrix for monthly returns on the assets. U.S. Equity returns are value-weighted returns from the NYSE, Amex, and NASDAQ downloaded from CRSP. 10-year treasury returns are also from CRSP. Commodity Futures returns are given by the Continuous Commodity Index and are downloaded from Bloomberg. The risk-free rate is the return on 3-month T-bills, downloaded from Bloomberg. Data accumulated December 2011. Past performance is not indicative of future results.
Source: Salient Capital Advisors, LLC, December 2011.

Exhibit 3 Feasible Combinations of Return and Risk
Notes: Exhibit uses a fine grid of allocations to Equities, Treasuries, and Commodity Futures to illustrate the combinations of expected return and volatility available to investors over the period of October 1958 to December 2011. The efficient frontier is the outside edge of the area plotted in blue, while the CML is the black line drawn from the risk free rate on the Y-axis through the mean-variance optimal (or MSRP), which is marked with a red asterisk. The risk parity portfolio is plotted in green immediately adjacent to the mean-variance optimal portfolio. Optimal and risk parity portfolio weights are computed using a nonlinear solver in MATLAB. U.S. Equity returns are value-weighted returns from the NYSE, Amex, and NASDAQ downloaded from CRSP. 10-year treasury returns are also from CRSP. Commodity Futures returns are given by the Continuous Commodity Index and are downloaded from Bloomberg. The risk-free rate is the return on 3-month T-bills, downloaded from Bloomberg. All data accumulated December 2011.
Source: Salient Capital Advisors, LLC, December 2011.
correlation is equal to the Treasury-Commodity Future correlation. These statistical findings indicate that the MSRP is distinct from the risk parity portfolio, but the similarity of Sharpe ratios and mild differences in cross-correlation suggest that risk-adjusted returns for the two allocations should be similar.

To illustrate the similarity of risk parity to the MSRP, we plot the two portfolios in return/volatility space in Exhibit 3. The risk parity portfolio is immediately adjacent to the MSRP, with expected excess annual return of 2.88% (versus 2.85% for the MSRP) and annual volatility of 6.69% (versus 6.58% for the MSRP). These results match our prior results from the case of two assets. However, this risk parity portfolio is still un-implementable—just like the MSRP—because the allocation that achieves exact parity is only known after the fact, once the full statistical distribution of returns has been realized. Unlike returns, however, we believe historical correlation and volatility are reasonably informative about future correlation and volatility.

The last thing we do in this section is consider an implementable risk parity strategy which we will call Ex Ante Risk Parity, where allocations are made using only information that would have been available at the time each position was initiated. We simulate the strategy by computing the covariance of the assets during the two years prior to the date they will be held. Each of these covariance estimates implies a risk parity allocation, which we solve for using a nonlinear optimizer in MATLAB. By combining these weights with returns in the subsequent month, we arrive at a monthly return for the portfolio. Next, we roll forward one month in time, re-estimate covariance, re-compute weights, and compute returns for the following month. We repeat this process each month.

Exhibit 4 is identical to Exhibit 3 except that the risk parity return/risk pair has been replaced by the implementable Ex Ante Risk Parity strategy. The exhibit shows that Ex Ante Risk Parity slightly outperformed the static MSRP over our long sample period. In the next section, we will try to formulate some economic intuition for these results by studying the relationship between asset returns, optimality, and the macroeconomic environment.

3. Asset Returns and the Economic Environment
In the previous section, we showed that risk parity generated performance similar to the MSRP over the period of October 1958 to December 2011. In this section we will look more carefully at returns in order to supply some intuition for this result. The remainder of section 3 will explain why risk parity leads to an allocation so similar to the MSRP in the long run while deviating significantly from it in the short run.

An investor on October 1, 1958 would have achieved an optimal outcome over the next 53 years (through 2011) if she had held a portfolio of 16.6% Equity, 48% Treasuries, and 35.4% Commodity Futures, generating excess returns of 2.85%/year, volatility of 6.58%/year, implying a Sharpe ratio of 0.43. This result is somewhat surprising in light of institutional investors’ equity-heavy allocations. Looking at shorter periods of time may shed some light on why a portfolio of 60% equities and 40% bonds is the starting point for many institutional allocations.

In order to provide some intuition for the finding that the long-run efficient portfolio has such a small allocation to equities (16.6%), we calculate efficient portfolios over every overlapping 5-year period between October 1958 and December 2011. Examining the evolution of the short-run optimal portfolio through time offers insight into why the long-run optimal allocation has so much non-equity exposure. Plotted in blue in Exhibit 5 is the optimal short-run allocation to Equities, plotted in green is the optimal short-run allocation to Treasuries, and plotted in red is the optimal short-run allocation to Commodity Futures. As you can see, the asset allocation weights of the MSRP are highly dependent on the time period you examine.

Equities played a dominant role in short-run optimal portfolios over three time periods: from the late 1960s to the
early 1970s, from the early to mid-1980s, and the late 1990s. Treasuries were dominant in short-run optimal portfolios in the late 1950s, briefly in the mid-1970s, from the mid-1980s through the late 1990s, in the early 2000s, and between 2009 and 2011. Commodity Futures were a dominant portion of short-run optimal portfolios throughout the 1970s, the early 1980s, and from the early 2000s through today. In other words, equities produced higher risk-adjusted returns than the other asset classes over a relatively small fraction of the historical period we looked at, and thus make up a small component in the optimal portfolio for the long period we are studying.

Why did these three assets behave so differently over different historical periods? In our view, it is a function of their very different responses to macroeconomic conditions. Exhibit 6 shows correlations between annual returns for each asset and some metrics of macroeconomic conditions over the same time period. The exhibit shows that, on average, Equity returns respond positively to economic growth but negatively to inflation, that Commodity Futures returns respond positively to both inflation and economic growth, and that Treasury returns respond negatively to both inflation and economic growth. Put another way, equities thrive during periods of low inflation and high economic growth, commodities thrive during periods of high growth and high inflation, and Treasuries thrive during periods of low growth and low inflation. Because each of these economic scenarios makes up a substantial portion of the long-run period we are studying, the ex post optimal allocation for the period of October 1958 to December 2011 is spread across all three of these assets. The risk parity portfolio attempts to equalize its...
risk exposure to each of these asset classes, arriving at a result quite similar to that of the long-run MSRP using only information that was available at the time each allocation was made.

We should state explicitly here that if one could have dynamically allocated to the MSRP through time, that portfolio would have outperformed an allocation to risk parity. But it should also be apparent from Exhibit 5 that it is very unlikely one could have predicted ahead of time what allocation would produce the highest Sharpe ratio based on past returns—the short-run optimal allocation is simply too volatile. We are addressing what is, in

Exhibit 5 5-year Ex post Efficient Portfolio Weights

Notes: Exhibit plots the portfolio weights that would have produced the highest ratio of risk to excess return (maximum Sharpe ratio) over the previous five years on a rolling basis. For example, the values plotted at 1970 indicate that the optimal portfolio over the period of February 1965 to January 1970 consisted of 100% Equities, 0% Treasuries, and 0% Commodity Futures. U.S. Equity returns are value-weighted returns from the NYSE, Amex, and NASDAQ downloaded from CRSP. 10-year treasury returns are also from CRSP. Commodity Futures returns are given by the Continuous Commodity Index and are downloaded from Bloomberg. The risk-free rate is the return on 3-month T-bills, downloaded from Bloomberg. Data accumulated April 2012. Past performance is no guarantee of future results. Source: Salient Capital Advisors, LLC, April 2012.

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Exhibit 6 Correlations between Asset Returns and Macroeconomic Variables

Notes: Exhibit shows the correlation between annual asset returns and macroeconomic variables. Macroeconomic data is from the Federal Reserve Bank of St. Louis FRED database. U.S. Equity returns are value-weighted returns from the NYSE, Amex, and NASDAQ downloaded from CRSP. 10-year treasury returns are also from CRSP. Commodity Futures returns are given by the Continuous Commodity Index and are downloaded from Bloomberg. The risk-free rate is the return on 3-month T-bills, downloaded from Bloomberg. Data accumulated April 2012. Source: Salient Capital Advisors, LLC, April 2012.
our view, the more relevant question for strategic asset allocators: how to achieve results on par with the optimal long-run allocation.

Next we want to compare the short- and long-run optimal portfolios to the results we could have achieved using Ex Ante Risk Parity. In Exhibit 7, we plot the portfolio weights generated by Ex Ante Risk Parity. Comparing them with the short-run optimal weights in Exhibit 5 suggests that Ex Ante Risk Parity has little in common with short-run optimal allocations. Risk parity allocations evolve slowly to reflect changes in individual asset volatilities and inter-asset correlation, while the short-run optimal portfolio is largely dependent on which asset class performed best over the past five years. Notice, however, that the risk parity portfolio increased its allocation to commodities in advance of periods of commodity outperformance—identified by observing large allocations to commodities in Exhibit 5—in the 1970s, 1980s, and early 2000s. Similarly, interest rate exposure rose concurrently with long periods of Treasury Bond outperformance during the early 1970s, early 2000s, and late 2000s. The intuition for this result is that risk parity increases allocations to assets as their volatility and correlation to other assets fall. Declining volatility is generally accompanied by positive asset returns, so risk parity produces portfolio weights that gently drift toward assets with improving performance. Some critics of risk parity point to this tendency to “buy high and sell low” as a reason to avoid the strategy. Next, we will use our data to gauge the merit of this criticism.

In order to study the effectiveness of risk parity as an asset allocation strategy over the period of 1958 to 2011, we will compare risk-adjusted risk parity and 60/40 returns with those generated by the optimal portfolio. Exhibit 8 depicts these relationships in a scatter plot. The X-value of each coordinate in the exhibit indicates the Sharpe ratio of the optimal portfolio, while its Y-value indicates the Sharpe ratio of either risk parity (blue for short-run,

Exhibit 7 Ex Ante Risk Parity Portfolio Weights
Notes: Exhibit plots ex ante risk parity portfolio weights. For example, the weights plotted for January 1970 are based on the covariance of monthly asset returns from January 1958 to December 1959. U.S. Equity returns are value-weighted returns from the NYSE, Amex, and NASDAQ downloaded from CRSP. 10-year treasury returns are also from CRSP. Commodity Futures returns are given by the Continuous Commodity Index and are downloaded from Bloomberg. The risk-free rate is the return on 3-month T-bills, downloaded from Bloomberg. Data accumulated April 2012.
Past performance is no guarantee of future results.
Source: Salient Capital Advisors, LLC, April 2012.
green for long-run) or the 60/40 portfolio (red for short-run, yellow for long-run). Points along the 45° line indicate that the portfolio—whether it be 60/40 or risk parity—had a Sharpe ratio equal to that of the optimal portfolio over that same time period. Points below the 45° line indicate that the portfolio produced lower risk-adjusted returns than the optimal portfolio, while points above the 45° line indicate that a portfolio produced risk-adjusted returns better than the optimal portfolio. It is important to note that, because the risk parity strategy changes allocations within each period during which these statistics are calculated, it is possible for it to achieve risk-adjusted returns in excess of the optimal portfolio.

Three primary results emerge from study of Exhibit 8: First, neither risk parity nor the 60/40 consistently generates returns on par with short-run optimal allocations, which would appear as marks along the line. This is exactly what we should expect given that the short-run optimal allocation is chosen with full knowledge of risk and return for each

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**Exhibit 8 Sharpe Ratio Scatter Plot**

Notes: Exhibit plots Sharpe ratio of the short-run ex post efficient (un-implementable optimal) portfolio on the x-axis versus the short-run ex ante (implementable) risk parity (blue) and 60/40 (red) Sharpe ratios from the same time period on the y-axis. Exhibit also plots the Sharpe ratio of the long-run ex post efficient (un-implementable optimal) portfolio on the x-axis versus the Sharpe ratio generated by risk parity (green) and the 60/40 portfolio (yellow) over the long run on the y-axis. The 45° line indicates perfect equality between implementable and non-implementable strategies. Points above the 45° line indicate implementable strategies that outperform the optimal allocation, while points below the 45° line indicate strategies that underperform the optimal static allocation.

Past performance is not indicative of future results.

Source: U.S. Equity returns are value-weighted returns from the NYSE, Amex, and NASDAQ downloaded from CRSP. 10-year treasury returns are also from CRSP. Commodity Futures returns are given by the Continuous Commodity Index and are downloaded from Bloomberg. The risk-free rate is the return on 3-month T-bills, downloaded from Bloomberg. Data accumulated April 2012.
asset. Second, the 60/40 portfolio gets close to the maximum Sharpe ratio more frequently in the short run than does risk parity, shown by the larger number of red versus blue marks in the vicinity of the 45 degree line. However, the 60/40 portfolio also produces the largest outliers—it misses by a very wide margin more often. To understand the intuition for this result, first recognize that the 60/40 portfolio behaves similarly to an all-equity portfolio (87% of the variance of 60/40 returns came from equity during the period from October 1958 to December 2011).

Because equities performed well during most of the 5-year overlapping periods used to generate Exhibit 8, the 60/40 portfolio generally resides near the line. Perhaps this helps explain the prevalence of 60/40 strategies among institutional investors. However, because the 60/40 portfolio is so equity-dominated, it also produced more large drawdowns relative to the optimal portfolio, which by construction held no equity during periods when stocks did very poorly. Third, when we consider our entire sample, Ex Ante Risk Parity outperforms both the ex post optimal static portfolio and the 60/40 portfolio. This mirrors what we found in the case of two assets. It also suggests that the benefits of consistent, systematic diversification may outweigh the costs of “buying high and selling low,” or at least they have over the last 53 years.

This third point, that risk parity dominates both the long-run ex post optimal and 60/40 portfolio deserves further consideration. Recall from earlier in the discussion that the ex post optimal portfolio over the period of October 1958 to December 2011 was 16.6% Equity, 48% Treasuries, and 35.4% Commodity Futures. This allocation is very different from most of the allocations shown in Exhibit 2, and most of the short-run optimal portfolios are very different from the long-run optimal allocation. Ex Ante Risk Parity generates allocations that are, on average, more similar to the long-run optimal allocation than they are to most of the short-run optimal allocation. Also note that because the Ex Ante Risk Parity risk parity allocation changes over time, it is possible for it to outperform the optimal static allocation as it does in our analysis.

4. Conclusions

Financial theory shows that the risk parity portfolio will be identical to the MSRP if all assets have identical Sharpe ratios and equal cross-correlations with one-another. We showed that the first of these conditions may be true, but that the second is most likely not. While this implies that the risk parity portfolio will be slightly different from the MSRP, we find that this difference was relatively minor in practice, even when considering an implementable Ex Ante Risk Parity strategy.

No naïve strategy—Ex Ante Risk Parity included—can rival the performance of portfolios formed with full knowledge of what assets will outperform in the near future. Over the long run, however, because the economic environment is continually in flux, risk parity presents itself as a potential strategy with which to approximate the optimal static portfolio because it provides equal risk exposure to assets that typically perform well in different economic environments.
What a CAIA Member Should Know

1See Partridge, Croce, and Kellert (2011).
2Here ex-ante means a portfolio was allocated using only information that was available at the time an investment would have been made and where ex post means allocations chosen using information that would not have been available at the time the investment was made.
3The maximum Sharpe ratio portfolio is a unique portfolio in the investor’s opportunity set that offers the highest level of expected excess return per unit of annualized monthly return standard deviation; the Sharpe ratio is our metric for risk-adjusted return. In this document, maximum Sharpe ratio will be used interchangeably with optimal portfolio.
4Our return data covers more than 53 years and will be discussed in detail in later sections of this document.
5Excess return is asset return less return on the risk-free asset.
6In order to achieve risk-return combinations on the CML to the right of the tangency point, investors would borrow at the risk-free rate and invest both their own capital and the loan proceeds in the maximum Sharpe ratio portfolio. Investors who prefer risk-return combinations on the CML to the left of the tangency point would hold less than 100% of their assets in the maximum Sharpe ratio portfolio and would lend at the risk-free rate with what remains.
7U.S. Equity returns are value-weighted returns from the NYSE, Amex, and NASDAQ downloaded from CRSP. 10-year treasury returns are also from CRSP. Commodity Futures returns are given by the Continuous Commodity Index and are downloaded from Bloomberg. The risk-free rate is the return on 3-month T-bills, downloaded from Bloomberg. All data accumulated Salient Capital Advisors, LLC, April 2012.
8The statistical test used to reject equal correlations is due to Hotelling (1940).
9See, for example, Poterba and Summers (1987).

References


Author Bios

Lee Partridge, CFA, CAIA is the Chief Investment Officer of Salient Partners, a Houston-based investment firm with over $17 billion in assets under management or advisement. Mr. Partridge spearheaded the development of Salient’s asset allocation funds, credit-related strategies and quantitatively managed funds. He also directly oversees the investment program for one of Salient’s largest institutional clients, a $9+ billion public pension plan, for which he was named Small Public Fund Manager of the Year by Institutional Investor in 2012.
Mr. Partridge holds an MBA degree from Rice University and a Bachelor of Science degree in Psychology from the University of Houston. He also holds both the CFA and CAIA designations.

Roberto Croce is the Director of Quantitative Research for the Investments Group at Salient Partners. Dr. Croce builds and implements the models underlying Salient’s proprietary hedge fund risk monitoring, manager selection, MLP hedging, and asset allocation tools. He also serves as a portfolio manager for Salient’s internally-managed strategies, oversees research into new investment initiatives, and serves on the Risk Parity Index Committee. Roberto holds M.A. and Ph.D. degrees in Economics from Ohio State University.