Research Review

Tail Risk Literature Review

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The Global Financial Crisis brought with it a resurgence of interest in tail risk, both within the financial services industry and the academic world. However, tail risk has been an important topic in financial literature since academic researchers realized that market returns often violate normality assumptions. In this article, we provide a brief literature review of the evolution of tail risk measures, as well as research on tail dependency. We also document a number of academic studies that assess tail risk and tail dependency of hedge fund returns.

The literature related to tail risk and its measurement dates back to the early 1960s. Mandelbrot (1963) challenged the usual assumption of Gaussian return distributions by applying the power law to describe the unconditional tail distributions of financial returns. Consistent with Mandelbrot, Fama (1963) argued that prices in certain markets show large, abrupt movements that one wouldn't expect under a model of Gaussian distributed returns. Blattberg and Gonedes (1974) proposed using the Student (or t) distribution to account for the fat tails of return distributions observed in earlier studies. Akgiray and Booth (1988), Hols and de Vries (1991), and Jansen and de Vries (1991) extended the literature on the shape of fat tails, demonstrating that the tail behavior of returns is fundamentally different from the remainder of the return distribution.

In light of the findings contraindicating the Gaussian assumption, a greater number of economists considered the asymmetry of return distributions in their research. Sortino and Price (1994) advocated the use of downside deviation as a risk measure rather than traditional (Gaussian-based) risk measures such as standard deviation and beta. The Sortino risk measure never achieved the level of acceptance of other measures, such as Value at Risk (VAR), perhaps due to the fact that it does not consider the full distribution of returns. However, many researchers have argued that VaR has several significant drawbacks. Beder (1995) pointed out that VaR is extremely sensitive to parameter choice. Artzner, et al. (1999) demonstrated that VaR is not coherent, i.e., it doesn't possess desirable properties of a risk measure, such as subadditivity, under certain market circumstances. Despite its flaws, VaR remained popular in the financial community, particularly prior to the demise of Long Term Capital Management (LTCM) in 1998.

After the LTCM incident, VaR was criticized as an inaccurate measure of downside risk exposure and researchers began to examine new measures to better estimate the extreme tail. Li (1999) proposed a new approach to estimate VaR based on skewness and kurtosis in addition to volatility. In a similar approach, Favre and

Galeano (2002) developed a new method called Modified Value at Risk in which they use a Corner-Fisher expansion in computing VaR. Rockafellar and Uryasev (2000) proposed another risk measure called expected shortfall (ES) or conditional VaR (CVaR) which has desirable properties of convexity and coherence. Alexander and Baptista (2004) compared VaR and CVaR in their study and demonstrated that CVaR is a more effective constraint on the mean-variance model, especially when a risk-free security is present. Agarwal and Naik (2004) also argued that the left-tail is underestimated in the common mean-variance framework and supported the use of CVaR as an alternative.

Researchers also analyzed quantitative theories in order to provide more accurate estimates of tail risk. Since 2000, an increasing number of studies have used Extreme Value Theory (EVT) to model tail-behavior, based only on the extreme values. Bali (2003) examined the asymptotic behavior of extreme changes in the U.S. Treasury market and claimed that standard VaR approaches can be significantly improved by utilizing EVT. Gencay and Selcuk (2004) demonstrated that EVT based models outperformed classical VaR models in emerging markets. Marimoutou, Raggad, and Trabelsi (2009) applied EVT models in energy markets and found that such models offer significant improvements in estimating tail risk when compared to other traditional techniques such as GARCH, historical simulation and filtered historical simulation.

During the last decade, the topics of tail dependence and time-varying tail distributions have been covered extensively. Because of its effectiveness in capturing different patterns of tail dependence, copula theory has become a popular statistical modeling tool. By conditioning variables with an extension of copula theory, Patton (2006) observed different degrees of correlations in exchange rates during joint appreciations versus joint depreciations. Michelis and Ning (2010) employed a Symmetrized Joe-Clayton (SJC) copula to assess the tail dependence between stock returns and exchange rates. They found a higher dependency of returns in the left-tail of the joint distribution. Litzenberger and Modest (2008) and Billio et al. (2007) extended the literature on tail risk by utilizing Markov regime switching processes to capture time varying risk exposures in different market conditions.

Since alternative investments, particularly hedge funds, display asymmetric return profiles much academic research is aimed at assessing the tail risk of hedge funds. Edward and Caglayan (2001) demonstrated that hedge funds have higher positive correlations with stock returns in bear markets. Agarwal and Naik (2004) analyzed equity-oriented hedge funds and found that hedge funds exhibit short option-like payoffs, bearing significant left-tail risk which is underestimated by a traditional mean-variance framework. In another study, Agarwal, Bakshi and Huij (2008) examined higher moment risks in cross-sectional hedge fund returns. They discovered that hedge funds have considerable exposure to higher moment risks and that these exposures generate significant returns for the funds.

In addition to tail risk, scholars also investigated the tail dependency of returns in the hedge fund industry. Geman and Kharoubi (2003) found that normality assumptions are not appropriate for hedge funds. In addition, they discovered significant left-tail dependence between returns of most hedge funds and traditional assets, suggesting that most hedge funds provide less diversification in large negative market moves than previously thought. They found that the equity market neutral strategy was an exception, providing diversification benefits in down moves. Similarly, Bacmann and Gawron (2004) analyzed return dependency among different hedge fund styles and stocks and bonds. In their study, they claimed a substantial left-tail dependency between funds of hedge funds without managed futures exposure and the stock market, caused by the lack of liquidity

during the LTCM and the Russian crises. On the other hand, Brown and Spitzer (2006) observed a similar left-tail dependency in hedge fund returns with stocks even after the elimination of the financial crisis periods from their analysis and concluded that funds of hedge funds are exposed to significant tail risk. However, Distaso, et al. (2009) criticized previous studies on unconditional tail risk after they found significant conditional time-variation in tail dependency even for hedge funds that display little unconditional tail dependency.

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