Attribution Analysis of Bull/Bear Alphas and Betas

with Applications to Downside Risk Management

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Asymmetrical Risk and Return

Asymmetries in risk and return characteristics come in various forms: assets with highly non-linear payoff profiles, correlations which increase in times of market turbulence, successful information-driven market timing strategies and data-driven dynamic portfolio insurance strategies lead to gain and loss sensitivities which can be very different in bull and bear markets.

This contrasts strongly with traditional models, which are dominated by symmetric risk measures such as volatility and beta: A return of 15% below the mean has the same impact on volatility as a return of 15% above the mean; the beta of a portfolio is 0.85, independent of whether the benchmark makes -15% or +15%. To be fair, one has to say that research to enhance the traditional concepts with asymmetrical features began as far back as the 1970's. Interestingly asymmetric modeling still enjoys the status of "frontier research" about 40 years later.

In this research note, we will discuss a specific asymmetrical model and build an attribution framework which allows an analysis of the impact of asymmetry on Alpha and Beta to the traditional single-index model with its symmetric Alpha and Beta. Our attribution methodology has some interesting features which differ from traditional attribution analysis of portfolio returns. For example, in a Brinson decomposition we are used to having an interaction effect and the sum of allocation, selection and interaction is typically positive or negative. In contrast, our attribution of asymmetrical alpha and beta will be free of any interaction effects and we will show that the sum of effects on alpha and beta will always equal zero; asymmetry is a zero sum game when evaluated relative to a symmetrical model.

The zero sum property of relative effects does not mean that asymmetric investment strategies or asymmetric models have "zero value". On the contrary, we will illustrate how asymmetrical models can be used in ex post portfolio analysis to detect "false" alphas caused by "hidden" asymmetrical betas. Additionally, asymmetrical betas can be used in ex ante portfolio construction for the purpose of downside risk management.

Attribution Framework for a Dual alpha/dual beta Factor Model

Single-index models are nothing more than a linear regression of portfolio, fund or asset excess return time series on some benchmark excess return time series.

$$r_{P,t} = \alpha + \beta \cdot r_{B,t} + \varepsilon_t$$

Typically, the slope coefficient Beta is calculated as the ratio of covariance between portfolio and benchmark returns and benchmark variance.

$$\beta = \frac{Cov[r_P, r_B]}{\sigma_B^2}$$

The above linear relationship is exact at one point, namely average portfolio and benchmark return.

$$\bar{r}_P = \alpha + \beta \cdot \bar{r}_B$$

This property can be used to calculate Alpha after we have calculated Beta with the above formula and the average returns.

$$\alpha = \bar{r}_P - \beta \cdot \bar{r}_B$$

We see that Alpha is not calculated directly, but as the residual not explained by the return contribution of Beta. A straight-forward approach to model asymmetry is to partition the time series in the above calculations into two distinct data sets and then compare the Alpha and Beta values. Various criteria to partition the return time series are possible. An obvious choice is to distinguish bear and bull returns. We define bear markets as states of the world in which benchmark returns are negative. Similarly, bull markets are defined as states of the world in which benchmark returns are larger than or equal to zero. Of course, portfolio returns will differ from market returns so not all bear market portfolio returns will necessarily be negative, and not all bull market portfolio returns will be positive.

The bull and bear market alphas and betas can be derived by applying the single-index model calculations to the bear and bull market data sets separately. We run the following regressions.

$$r_{P,Bull,t} = \alpha_{Bull} + \beta_{Bull} \cdot r_{B,Bull,t} + \varepsilon_{Bull,t}$$

$$r_{P,Bear,t} = \alpha_{Bear} + \beta_{Bear} \cdot r_{B,Bear,t} + \varepsilon_{Bear,t}$$

If asymmetric risk and return characteristics are present, the bull parameters will differ from the bear parameters. More formally, we could perform a Chow test, which is usually used to detect structural breaks. In the context of comparing regression parameters across bull and bear markets, we would test whether or not asymmetrical risk and return characteristics are relevant.

It would be very convenient if the parameters are additive. Unfortunately, this is not the case; it can be shown that.

$$\alpha_{Bear} + \alpha_{Bear} \neq \alpha$$

$$\beta_{Bear} + \beta_{Bear} \neq \beta$$

The calculations can be simplified by introducing dummy variables, which are binary indicators of whether the state of the world at time t is a bull or bear market.

$$\mathrm{I}_{Bull,t} = \begin{cases} 0 & r_{B,t} < 0 \\ 1 & r_{B,t} \geq 0 \end{cases}$$

$$\mathrm{I}_{Bear,t} = \begin{cases} & \\ 1 & r_{B,t} < 0 \end{cases}$$

With the help of these indicators, we estimate bull and bear parameters from a single regression, which we call the dual alpha / dual beta asymmetrical index model.

$$r_{P,t} = \alpha_{Bull} \cdot \mathrm{I}_{Bull,t} + \alpha_{Bear} \cdot \mathrm{I}_{Bear,t} + \beta_{Bull} \cdot r_{B,t} \cdot \mathrm{I}_{Bull,t} + \beta_{Bear} \cdot r_{B,t} \cdot \mathrm{I}_{Bear,t} + \varepsilon_t$$

Quantitative Analysis

The statistical properties of the asymmetrical index model would require more detailed discussion than is possible in this paper. For example, note that since bull and bear states are exclusive, the indicator functions are perfectly negatively correlated, something which can cause problems when estimating parameters and judging their significance. It is possible to formulate more suitable models for estimation purposes, from which the parameters of the above dual alpha and dual beta model can be recovered. In order to keep the presentation as clear as possible, we continue using the intuitive dual alpha / dual beta model introduced above.

While the above model is intuitive and convenient to estimate, we are still not able to establish simple additive relationships between the symmetric and asymmetric parameters.

$$r_{P,t} \neq (\alpha_{Bull} + \alpha_{Bear}) + (\beta_{Bull} + \beta_{Bear}) \cdot r_{B,t} + \varepsilon_t$$

Sacrificing the convenience of the one-step estimation procedure, one can derive an additive relationship with a two-step estimation procedure.

First Step: Estimate the symmetric model and calculate the time series of residuals ε.

$$r_{P,t} = \alpha + \beta \cdot r_{B,t} + \varepsilon_t$$

Second Step: Estimate incremental asymmetrical Alphas and Betas by regressing the residuals ε on the dummy variables and benchmark returns.

$$\varepsilon_t = \Delta\alpha_{Bear} \cdot \mathrm{I}_{Bear,t} + \Delta\alpha_{Bull} \cdot \mathrm{I}_{Bull,t} + \Delta\beta_{Bear} \cdot r_{B,t} \cdot \mathrm{I}_{Bear} + \Delta\beta_{Bull} \cdot r_{B,t} \cdot \mathrm{I}_{Bull,t} + \varepsilon'_t$$

The incremental parameters are related to the symmetric parameters as follows.

$$\alpha_{Bear} = \alpha + \Delta\alpha_{Bear}$$

$$\alpha_{Bull} = \alpha + \Delta\alpha_{Bull}$$

$$\beta_{Bull} = \beta + \Delta\beta_{Bull}$$

$$\beta_{Bear} = \beta + \Delta\beta_{Bear}$$

Inserting the definition of the incremental effects into the equation in the second step, and then substituting ε in step one, we get.

$$r_{P,t} = \alpha + \beta \cdot r_{B,t} + (\alpha_{Bear} - \alpha) \cdot \mathrm{I}_{Bear,t} + (\alpha_{Bull} - \alpha) \cdot \mathrm{I}_{Bull,t} + \ldots$$
$$\ldots(\beta_{Bear} - \beta) \cdot r_{B,t} \cdot \mathrm{I}_{Bear,t} + (\beta_{Bull} - \beta) \cdot r_{B,t} \cdot \mathrm{I}_{Bull,t} + \varepsilon_t$$

This simplifies to.

$$r_{P,t} = \alpha_{Bear} \cdot \mathrm{I}_{Bear,t} + \alpha_{Bull} \cdot \mathrm{I}_{Bull,t} + \beta_{Bear} \cdot r_{B,t} \cdot \mathrm{I}_{Bear,t} + \beta_{Bull} \cdot r_{B,t} \cdot \mathrm{I}_{Bull,t} + \varepsilon_t$$

Regressing the residuals of the symmetric index model on the explanatory variables of the asymmetric dual Alpha / dual Beta model, results in incremental bull and bear Alphas and Betas that establish a simple additive relationship between the parameters of the two models.
We now define an Alpha effect a, which measures the return contribution of the incremental alphas.

$$a = p_{Bear} \cdot \Delta\alpha_{Bear} + p_{Bull} \cdot \Delta\alpha_{Bull}$$

The variable p represents the state probability. By substituting the definitions of the incremental Alphas, it is possible to express the Alpha effect in terms of symmetric and asymmetric Alphas.

$$a = p_{Bear} \cdot \alpha_{Bear} + p_{Bull} \cdot \alpha_{Bull} - \alpha$$

Beta is a sensitivity measure. In order to measure its contribution to return, it needs to be multiplied with the expected bull and bear benchmark returns.

$$b = \Delta\beta_{Bear} \cdot \overline{r_B \cdot \mathrm{I}_{Bear}} + \Delta\beta_{Bull} \cdot \overline{r_B \cdot \mathrm{I}_{Bull}}$$

$$b = \beta_{Bear} \cdot \overline{r_B \cdot \mathrm{I}_{Bear}} + \beta_{Bull} \cdot \overline{r_B \cdot \mathrm{I}_{Bull}} - \beta \cdot \bar{r}_B$$

Both the single-index model and the bull/bear model analyze the same portfolio.

$$\bar{r}_P = \alpha + \beta \cdot \bar{r}_B = \bar{\mathrm{I}}_{Bear} \cdot \alpha_{Bear} + \bar{\mathrm{I}}_{Bull} \cdot \alpha_{Bull} + \beta_{Bear} \cdot \overline{r_B \cdot \mathrm{I}_{Bull}} + \beta_{Bull} \cdot \overline{r_B \cdot \mathrm{I}_{Bull}}$$

From the above, it follows that.

$$\bar{r}_{P,SingleIndexModel} - \bar{r}_{P,DualAlphaDualBetaModel} = a + b = 0$$

This is the proof that relative to the single-index model, asymmetries measured in an asymmetrical model are a “zero sum game”: The sum of return contributions from asymmetric Alphas must be offset by the return contribution from asymmetric Betas.

Note what the above does not imply.

$$\Delta\alpha_{Bear} \cdot \bar{\mathrm{I}}_{Bear} + \Delta\alpha_{Bull} \cdot \bar{\mathrm{I}}_{Bull} \neq 0$$

The redistribution of Alpha to bull and bear Alphas is not zero sum, neither is the redistribution of Beta to bull and bear Betas.

$$\overline{r_B \cdot \mathrm{I}_{Bear}} \cdot \Delta\beta_{Bear} + \overline{r_B \cdot \mathrm{I}_{Bull}} \cdot \Delta\beta_{Bull} \neq 0$$

The sum of the return contribution from bull and bear Alphas and Betas add up to what we call the overall “contribution from asymmetry”.

$$p_{Bear} \cdot \alpha_{Bear} + p_{Bull} \cdot \alpha_{Bull} - \alpha = ContrAsymmetry$$

$$ContrAsymmetry = \beta \cdot \bar{r}_B - \beta_{Bear} \cdot \overline{r_B \cdot \mathrm{I}_{Bear}} - \beta_{Bull} \cdot \overline{r_B \cdot \mathrm{I}_{Bull}}$$

The contribution from asymmetry consists of redistribution from Alpha to Beta contributions when the portfolio’s investment strategy, relative to its benchmark, is convex. For concave dependencies, the redistribution takes place in the other direction.

This means that assessing convex investment strategies with the single-index model will always result in overestimated Alphas: The single-index model indicates “false Alphas due to the existence of “hidden asymmetrical betas”. The Alphas of concave strategies will have a downward bias. The purpose of our bull/bear attribution approach is to identify the sign and the magnitude of the bias.

It is also possible to measure the contribution of individual positions to total contribution from asymmetry. This is straight-forward, because the disaggregation of portfolio Alphas and Betas into contributions from portfolio constituents is linear.

Exhibit 1:

	Constituents		Single-Index Model		Dual Alpha Dual Beta Model			
	Weight	Avg Return	Alpha	Beta	Bear Alpha	Bull Alpha	Bear Beta	Bull Beta
A	-	0.0086%	0.1920%	0.7563	3.2176%	0.0931%	1.1935	0.5137
B	-	-0.2038%	0.1119%	1.3021	0.5056%	-1.7307%	1.3004	1.7122
C	-	0.0869%	0.1592%	0.2981	0.7397%	1.2192%	0.4166	-0.0089
D	-	0.8719%	0.9777%	0.4364	0.1021%	-0.8212%	0.2513	0.9477
E	-	0.1757%	0.3461%	0.7026	0.3101%	0.0428%	0.6877	0.7790
F	-	0.3390%	0.4834%	0.5955	-0.0246%	-0.7862%	0.4809	0.9467
G	-	0.4976%	0.3259%	-0.7083	1.9599%	3.7552%	-0.3607	-1.6800
H	-	-0.2307%	0.0109%	0.9963	-0.0185%	0.1185%	0.9955	0.9729
I	-	0.8607%	0.9900%	0.5330	1.0269%	0.2144%	0.5135	0.7170
J	-	0.3100%	0.2804%	-0.1222	-0.3340%	1.7606%	-0.1642	-0.4254
K	20.00%	-0.0707%	0.2887%	1.4824	-1.1216%	-1.3506%	1.2247	2.0023
L	20.00%	0.3961%	0.4063%	0.0809	-1.4317%	0.1192%	-0.1958	0.3121
M	20.00%	0.4531%	0.4672%	0.0583	0.4820%	0.4811%	0.0609	0.0537
N	20.00%	0.5552%	0.6213%	0.2724	-0.9751%	1.2998%	0.0618	0.2492
O	20.00%	0.9074%	1.1619%	1.0495	0.3047%	0.3456%	0.8986	1.3220
P	-	0.9974%	1.0242%	0.1105	0.3438%	2.1648%	0.0480	-0.1049
Portfolio		**0.4482%**	**0.5891%**	**0.5887**	**-0.5483%**	**0.1790%**	**0.4100**	**0.7879**
Benchmark		**-0.2425%**						

Exhibit 2:

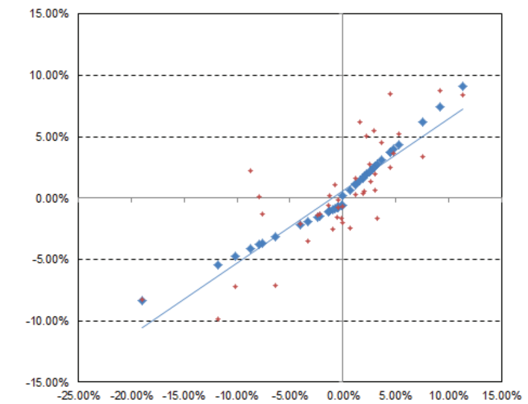


Exhibit 3:

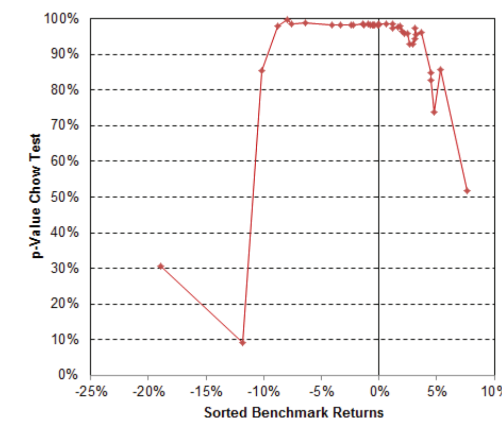


Exhibit 5:

	Constituent Contributions	Single-Index Model			Incremental Dual Alpha Dual Beta Model					
		Alpha	Beta	**Total**	Bear Alpha	Bull Alpha	Beat Beta	Bull Beta	**Total Alpha**	**Total Beta**
A	-	-	-	-	-	-	-	-	-	-
B	-	-	-	-	-	-	-	-	-	-
C	-	-	-	-	-	-	-	-	-	-
D	-	-	-	-	-	-	-	-	-	-
E	-	-	-	-	-	-	-	-	-	-
F	-	-	-	-	-	-	-	-	-	-
G	-	-	-	-	-	-	-	-	-	-
H	-	-	-	-	-	-	-	-	-	-
I	-	-	-	-	-	-	-	-	-	-
J	-	-	-	-	-	-	-	-	-	-
K	-0.0141%	0.0577%	-0.0719%	**-0.0141%**	-0.1343%	-0.1717%	0.1098%	0.1963%	**-0.3061%**	**0.3061%**
L	0.0792%	0.0813%	-0.0039%	**0.0773%**	-0.1750%	-0.0301%	0.1179%	0.0873%	**-0.2051%**	**0.2051%**
M	0.0906%	0.0934%	-0.0028%	**0.0906%**	0.0014%	0.0015%	-0.0011%	-0.0018%	**0.0029%**	**-0.0029%**
N	0.1110%	0.1243%	-0.0132%	**0.1110%**	-0.1520%	0.0711%	0.0897%	-0.0088%	**-0.0810%**	**0.0810%**
O	0.1815%	0.2324%	-0.0509%	**0.1815%**	-0.0816%	-0.0855%	0.0643%	0.1029%	**-0.1671%**	**0.1671%**
P	-	-	-	-	-	-	-	-	-	-
Portfolio	**0.4482%**	**0.5891%**	**-0.1428%**	**0.4463%**	**-0.5416%**	**-0.2148%**	**0.3805%**	**0.3759%**	**-0.7564%**	**0.7564%**
Benchmark	**-0.2425%**									

$$\alpha_P = \sum_{i=1}^{n} w_i \cdot \alpha_i \qquad \beta_P = \sum_{i=1}^{n} w_i \cdot \beta_i$$

where w represents the percentage weight of a constituent in the portfolio.

Ex Post Attribution for a Sample Fund of Hedge Funds

We will now apply the framework to a sample portfolio, which is a fund of hedge funds consisting of 16 single-hedge funds. The table below shows the parameters of the single-index and dual alpha/dual beta model for a particular portfolio.

As we can see, the portfolio outperforms the benchmark. The portfolio seems to exhibit significant Alpha (0.59%) and is positioned rather defensive relative to the benchmark (Beta value of 0.589). The asymmetric model shows that Alpha is distributed very unevenly across bull and bear markets. In fact, the portfolio exhibits a rather significant negative bear Alpha (-0.55%) and only a slightly positive bull Alpha (0.18%). The risk positioning is defensive in both bull and bear markets, but exposure in bull markets is much higher than exposure in bear markets (0.788 versus 0.410). The asymmetric Betas imply that the investment strategy is convex. This can be seen graphically, when plotting actual and fitted portfolio returns against benchmark returns.

The model assumes that the break between the bull/bear regimes occurs at a benchmark return of zero. A Chow test at 95% significance indicates that the use of a dual alpha/dual beta model is indeed justified by the data. Calculating Chow test p-values for all benchmark returns, we see that asymmetry is relevant for breakpoints ranging from -10% to 4%.An important input in the calculation of the attribution effects is the expected factor returns.

Exhibit 6:

	Constituents		Single-Index Model		Dual Alpha Dual Beta Model			
	Weight	Avg Return	Alpha	Beta	Bear Alpha	Bull Alpha	Bear Beta	Bull Beta
A	-	0.0086%	0.1920%	0.7563	3.2176%	0.0931%	1.1935	0.5137
B	-	-0.2038%	0.1119%	1.3021	0.5056%	-1.7307%	1.3004	1.7122
C	-	0.0869%	0.1592%	0.2981	0.7397%	1.2192%	0.4166	-0.0089
D	-	0.8719%	0.9777%	0.4364	0.1021%	-0.8212%	0.2513	0.9477
E	-	0.1757%	0.3461%	0.7026	0.3101%	0.0428%	0.6877	0.7790
F	-	0.3390%	0.4834%	0.5955	-0.0246%	-0.7862%	0.4809	0.9467
G	-	0.4976%	0.3259%	-0.7083	1.9599%	3.7552%	-0.3607	-1.6800
H	-	-0.2307%	0.0109%	0.9963	-0.0185%	0.1185%	0.9955	0.9729
I	-	0.8607%	0.9900%	0.5330	1.0269%	0.2144%	0.5135	0.7170
J	-	0.3100%	0.2804%	-0.1222	-0.3340%	1.7606%	-0.1642	-0.4254
K	-	-0.0707%	0.2887%	1.4824	-1.1216%	-1.3506%	1.2247	2.0023
L	-	0.3961%	0.4063%	0.0809	-1.4317%	0.1192%	-0.1958	0.3121
M	-	0.4531%	0.4672%	0.0583	0.4820%	0.4811%	0.0609	0.0537
N	-	0.5552%	0.6213%	0.2724	-0.9751%	1.2998%	0.0618	0.2492
O	-	0.9074%	1.1619%	1.0495	0.3047%	0.3456%	0.8986	1.3220
P	100.00%	0.9974%	1.0242%	0.1105	0.3438%	2.1648%	0.0480	-0.1049
Portfolio		**0.9974%**	**1.0242%**	**0.1105**	**0.3438%**	**2.1648%**	**0.0480**	**-0.1049**
Benchmark		**-0.2425%**						

Exhibit 7:

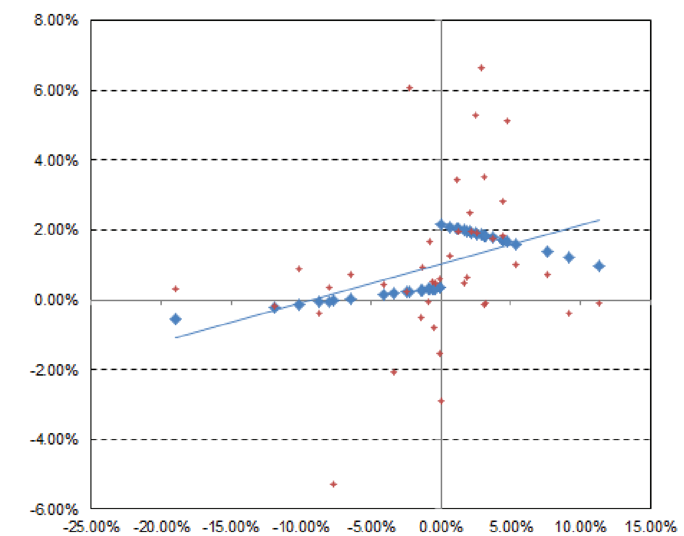


Exhibit 4:

Single-Index Model		Dual Alpha Dual Beta Model			
Alpha	Beta	Bear Alpha	Bull Alpha	Bear Beta	Bull Beta
1.0000	**-0.2425%**	**47.6190%**	**52.3810%**	**-2.1298%**	**1.8874%**

The expected value of the dummy variables can be interpreted as probabilities: the probability of a bear market is 47.6%, the probability of a bull market 1 - 47.6% = 52.4%.

The attribution analysis is summarized in the table below. We see that due to the convexity of the strategy, the return contribution of Alpha was overstated by 0.756%, the largest driver being fund K (due to its bull Beta higher than 2).

The total Alpha effect of -0.756% is the negative of the total Beta effect 0.756%. This reflects the zero sum characteristics of the differences between the symmetrical single-index model and the asymmetrical Dual Alpha / Dual Beta model. The signs of the total effects are indicators for the direction of the redistribution and should not be misinterpreted as "benefits of asymmetric

Exhibit 8:

	Constituents		Single-Index Model		Dual Alpha Dual Beta Model			
	Weight	Avg Return	Alpha	Beta	Bear Alpha	Bull Alpha	Bear Beta	Bull Beta
A	-	0.0086%	0.1920%	0.7563	3.2176%	0.0931%	1.1935	0.5137
B	-	-0.2038%	0.1119%	1.3021	0.5056%	-1.7307%	1.3004	1.7122
C	-	0.0869%	0.1592%	0.2981	0.7397%	1.2192%	0.4166	-0.0089
D	17.89%	0.8719%	0.9777%	0.4364	0.1021%	-0.8212%	0.2513	0.9477
E	-	0.1757%	0.3461%	0.7026	0.3101%	0.0428%	0.6877	0.7790
F	-	0.3390%	0.4834%	0.5955	-0.0246%	-0.7862%	0.4809	0.9467
G	-	0.4976%	0.3259%	-0.7083	1.9599%	3.7552%	-0.3607	-1.6800
H	-	-0.2307%	0.0109%	0.9963	-0.0185%	0.1185%	0.9955	0.9729
I	-	0.8607%	0.9900%	0.5330	1.0269%	0.2144%	0.5135	0.7170
J	-	0.3100%	0.2804%	-0.1222	-0.3340%	1.7606%	-0.1642	-0.4254
K	-	-0.0707%	0.2887%	1.4824	-1.1216%	-1.3506%	1.2247	2.0023
L	-	0.3961%	0.4063%	0.0809	-1.4317%	0.1192%	-0.1958	0.3121
M	-	0.4531%	0.4672%	0.0583	0.4820%	0.4811%	0.0609	0.0537
N	-	0.5552%	0.6213%	0.2724	-0.9751%	1.2998%	0.0618	0.2492
O	78.25%	0.9074%	1.1619%	1.0495	0.3047%	0.3456%	0.8986	1.3220
P	3.86%	0.9974%	1.0242%	0.1105	0.3438%	2.1648%	0.0480	-0.1049
Portfolio		**0.9045%**	**1.1236%**	**0.9036**	**0.2700%**	**0.2072%**	**0.7500**	**1.2000**
Benchmark		**-0.2425%**						

Exhibit 9:

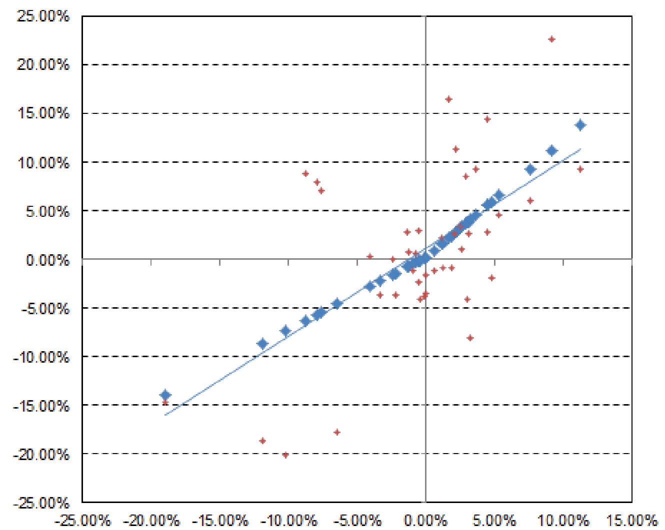
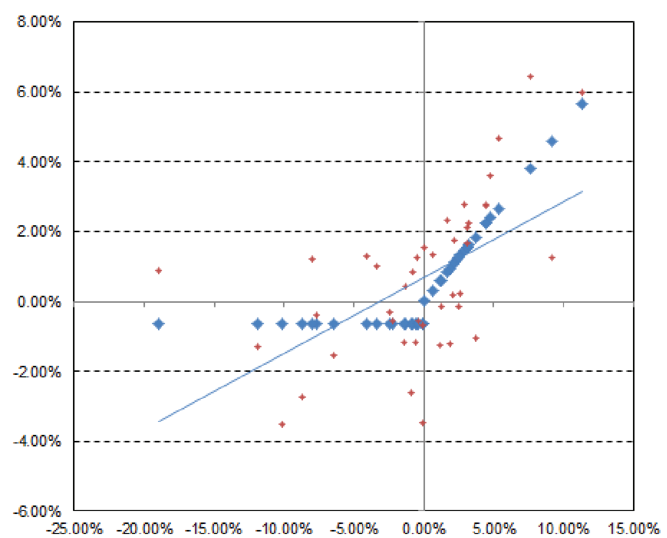


Exhibit 11:



over symmetrical strategies" in absolute terms.

Fund of Hedge Funds Portfolio Construction

The previous analysis was backward looking, explaining realized portfolio results in the context of two factor models and attributing the differences to position-level contributions.

We now would like to illustrate the use of the dual alpha/dual beta model in an ex ante context, in the construction of fund of hedge funds. Assuming that we have all the necessary inputs and that they are representative of future realized values, we can define target overall portfolio characteristics and then solve for consistent weights which result in portfolios which are best aligned with the targets (portfolio optimization). In addition to the targets, we can specify portfolio properties that need to be fulfilled (restrictions).

Let us consider an optimization which aims to maximize the portfolio's return. As restrictions, we define that the portfolio shall not be leveraged (total risk exposure = 100%) and prohibit position-level leverage (each constituent weight <= 100%) as well as short positions (each constituent weight >=0%).

Exhibit 10:

	Constituent Contributions	Single-Index Model Alpha	Beta	Total	Incremental Dual Alpha Dual Beta Model Bear Alpha	Bull Alpha	Beat Beta	Bull Beta	Total Alpha
A	-	-	-	-	-	-	-	-	-
B	-	-	-	-	-	-	-	-	-
C	-	-	-	-	-	-	-	-	-
D	0.3255%	0.3650%	-0.0395%	**0.3255%**	-0.1557%	-0.3518%	0.1471%	0.3603%	**-0.5074%**
E	-	-	-	-	-	-	-	-	-
F	-	-	-	-	-	-	-	-	-
G	-	-	-	-	-	-	-	-	-
H	-	-	-	-	-	-	-	-	-
I	-	-	-	-	-	-	-	-	-
J	-	-	-	-	-	-	-	-	-
K	-	-	-	-	-	-	-	-	-
L	0.2013%	0.2065%	-0.0100%	**0.1965%**	-0.4448%	-0.0764%	0.2995%	0.2218%	**-0.5213%**
M	-	-	-	-	-	-	-	-	-
N	-	-	-	-	-	-	-	-	-
O	-	-	-	-	-	-	-	-	-
P	0.1181%	0.1213%	-0.0032%	**0.1181%**	-0.0384%	0.0708%	0.0158%	-0.0482%	**0.0324%**
Portfolio	**0.6449%**	**0.6928%**	**-0.0526%**	**0.6401%**	**-0.6389%**	**-0.3575%**	**0.4624%**	**0.5339%**	**-0.9963%**
Benchmark	**-0.2425%**								

The bull/bear alpha and beta attribution of the resulting portfolio looks like this.

We see that the return of this portfolio would be solely invested in fund P. The reason fund P is chosen is that it has a very large Alpha next to a very small Beta. Remember, the expected benchmark return is negative (-0.242%), therefore small market exposures (low Beta) and large Alphas are highly desirable.

The maximum return portfolio is presented graphically in Exhibit 7.

The portfolio is not concave and even has a negative bull market beta. Such a portfolio only makes sense for extremely pessimistic expectations, if at all. If we would like to continue working with such pessimistic expectations, but would like to construct portfolios with less extreme features, we can introduce constraints. For example:

- The bull Beta of the portfolio should be at least 1.2
- The bear Beta of the portfolio should be smaller than 0.75

So we are effectively forcing the portfolio to be a convex strategy. Such a portfolio would obviously not perform as well in a scenario with negative expected market returns. On the other hand, it would be a portfolio that performs much better if an "unexpected recovery" takes place and future market returns are positive. The results for such an optimized portfolio are provided in Exhibits 8 and 9.

As we can see from the chart, the strategy is convex, as we required. Interestingly, the return of the portfolio is virtually the same (it is slightly lower) as the return of the optimized portfolio without beta restrictions. Hence, we have constructed a portfolio which performs almost identically to the previous pessimistic portfolio if our return expectations materialize in the future, but which also performs well in a scenario with positive market returns.

Another example would be the construction of a low risk "absolute return product", in the sense of a portfolio with bear beta equal to zero and a bull beta equal to 0.5. We present the results for the optimal absolute return product in Exhibits 10 and 11, leaving the interpretation to the reader.

Summary and Outlook

We have shown how the difference between bull/bear and single-index parameters can be explained in an additive sense in ex post portfolio analysis. We have also illustrated potential uses of asymmetric factor models in portfolio construction for downside risk management purposes.

In order to analyze the potential of a dual alpha/dual beta model to produce superior risk-adjusted returns, empirical work is required which, for example, examines the out-of-sample performance of optimized bull/bear portfolios.

Asymmetries can be interpreted as hidden risk factors. In the presence of asymmetries, single-index Alphas calculated relative to a portfolio benchmark will be distorted performance measures.

An operational version of the dual alpha/dual beta model presented would require refinements to address obvious estimation issues. It would be straight-forward to further generalize the model, for example, by considering asymmetric non-linearities. It would also be interesting to use other regime indicators than the portfolio benchmark (e.g. VIX) or other threshold values than zero. For example, we could distinguish an "extreme bear market" from "normal markets" by setting the threshold to a value much lower than zero.

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