# What is Portfolio Diversification?

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**1. Introduction**

Since the arithmetic average return of a portfolio is simply a linear function of the arithmetic average returns of the portfolio constituents, the benefits of diversification lie not in return enhancement, but in risk reduction.[1] Thus, the true benefits of diversification are sensitive to the choice of risk measure. While there are many alternatives, such as expected drawdown and VaR, most research and financial theory tends to focus on standard deviation or beta as measures of risk.

**2. Market portfolio**

One of the first definitions of a well-diversified portfolio is the market portfolio. Based on the Capital Asset Pricing Model, there exists a linear relationship between systematic risk and portfolio return. In this context, the market portfolio exists and consists of all risky assets traded in the market (Lintner 1965, Mossin 1966), where each asset is weighted by market value. The market portfolio is deemed as being completely diversified and its risk is non-diversifiable. However, the market portfolio can only be approximated by indices like Russell 3000 or MSCI World, since such indices do not contain all tradable assets such as stamps, real estate, and commodities. Furthermore, there are viable alternatives to pure market value weighting, such as fundamental indexing. Proponents of fundamental indexing argue that fundamental analysis can provide a more relevant estimate of firm value for market weighting than the firm's stock price. Fundamental indexing typically considers factors such as sales, earnings, or cash flows in the determination of value.

**3. Number of securities**

Another common way to think about a diversified portfolio is to analyze one that contains a large number of securities N. The return variance of a portfolio of a group of securities is lower than the average variance of the individual securities, unless all of the securities are perfectly correlated.[2] This was first examined in detail by Evans and Archer (1968), who showed the impact on the variance of a portfolio's return as the number of securities increases. Using 470 of the securities listed in Standard & Poor's Index, with semi-annual observations between January 1958 and July 1967, they calculated the geometric mean and standard deviation of the return for each security. They then formulated portfolios by randomly picking securities among the group of 470. Starting with one security and sequentially adding additional securities, they calculated each portfolio's variance and discovered a strong linear relationship between the variance of the formulated portfolios and the inverse of the portfolio size. They noted that the variance of the formulated portfolios asymptotically approached the variance of the market portfolio (consisting of all 470 securities) as the portfolio size increased. The market portfolio variance was well approximated with only 10 securities.

The benefit of holding a large number of securities was clearly demonstrated in a more recent study, where Sankaran and Patil (1999) created a set of portfolios where each portfolio can hold a maximum of *N* stocks. Using a specific algorithm, Sankaran and Patil demonstrated how portfolios with an increasing number of securities are able to achieve higher Sharpe ratios. However, the marginal benefit from diversification decreases with the number of securities. Their findings are based on no constraints on short-selling and the same pairwise correlations.

Focusing on the return profile of multiple stock portfolios, de Vassal (2001) examined the performance of portfolios with an increasing number of stocks. De Vassal calculated the returns of the constituents of the Russell 1000 during the seven-year period between 1992 and 1999, and subsequently used these returns to simulate multiple random portfolios that spanned all sizes between 3 and 100 stocks. De Vassal reported that portfolios with bigger sizes demonstrated returns that had lower variance or downside risk. In particular, single stock portfolios exhibited an 18% probability of a negative return, while portfolios with 10 or more stocks exhibited 0% probability over the bull market period examined. The author confirmed previous findings from Evans and Archer (1968) suggesting that the portfolio variance is inversely related to the number of securities.

The studies mentioned above refer to naïve diversification. While naïve diversification provides benefits by indiscriminately adding additional securities to portfolios, further diversification benefits or more efficient diversification can be achieved by any number of portfolio optimization methodologies, including Modern Portfolio Theory.

**4. Fund of hedge funds**

Denvir and Hutson (2006) mentioned diversification in the context of funds of hedge funds (FOHF) correlation to other indices. Using monthly hedge fund and FOHF returns for the period January 1990 to May 2003 from Hedge Fund Research, they found that although FOHF have lower Sharpe ratios than hedge funds, they also exhibited lower correlations with equity indices. The lower correlation persisted when focusing either on the bull or bear markets during that time period. The authors concluded that FOHFs are a better diversification tool than hedge funds due to their lower correlation to equity indices.

**5. Factor diversification**

Bender, Briand, Nielsen, and Stefek (2010) looked at diversification in the context of correlations across bull and bear markets. They examined factors constructed to represent a specific risk premium, classified by asset class, style, and strategy characteristics. For example, the MSCI Value Minus Growth index is able to capture the exposure only to the value premium. Style and strategy factors exhibited low correlations with one another, hence offering diversification benefits to investors. Furthermore, the data exhibited very low correlations with various asset classes, particularly the bond premium. The authors compare the Sharpe ratios of a traditional 60/40 equity/bond mix with an equally weighted mix of risk premia. Both portfolios were rebalanced on a monthly basis between May 1995 and September 2009. The risk premia portfolio exhibited similar returns with less than a third of the volatility. During the most recent five financial crises, diversification enabled the risk premia portfolio to avoid extreme losses, in sharp contrast to the traditional portfolio. Similarly, Page and Taborsky (2011) stated that even if a combination of risky and risk-free assets seems to offer diversification benefits in most periods, such combinations perform poorly during periods of financial crises, when correlations between asset classes increase. By following a regime approach, investors can achieve lower correlations across risk factors and hence better diversification.

**6. Time varying correlation**

The issue of correlation asymmetry was more formally established in Ang and Chen (2001). Using weekly equity portfolio returns over the period July 1963 to December 1998, the authors find that correlations are lower in bear markets than in regular markets, while correlations are higher in bull markets than both calm and bear markets. In contrast, the normal distribution predicts that both bull and bear markets exhibit lower correlations than calm periods. This constitutes a contradiction between what the data indicates and the normal distribution predicts.

As a result, any diversification benefit implied by a normal distribution is overstated during bear markets and understated in bull markets. Such correlation measures exhibit higher asymmetry for small, value, past-loser, and lower-beta stocks. They stated that regime-switching models are more capable of capturing such asymmetry. Butler and Joaquin (2011) later reported similar findings in the context of international stock portfolios.

In an updated study, Chua, Kritzman, and Page (2009) reinforced such findings across most asset classes using data for the period 1970 to 2008. They compared portfolios based on downside, upside, and full sample correlations and reported that portfolios constructed based on downside correlations maximize utility. The critical contribution of the paper is what they call full-scale optimization. By assigning a utility function that abruptly penalizes large losses, they implicitly took into account correlation asymmetries. They then reported that portfolios constructed in this way achieved better diversification, defined in terms of lower downside correlation and higher upside correlation, as well as higher utility, than portfolios based on mean-variance optimization.

**7. Tail measures**

The way portfolio risk is measured is the foundation upon which portfolios are optimized and portfolio diversification is measured. While variance has been widely used as such a measure, distortion risk measures provide an alternative. In a portfolio optimization context, they offer a different way to assign greater weight on the tails (Adam, Houkari, Laurent, 2008). Such measures can place greater weight on high losses and deflate the weight put on positive events. Distortion risk measures are equivalent to spectral risk measures; an example of a spectral risk measure is the expected shortfall. Adam et al. examined 16 hedge funds with monthly returns from January 1990 to July 2001. They first minimized the risk of a portfolio invested in those funds by using distortion, i.e., moment-based and spectral risk measures for a given level of return and constraints. They found high rank correlations between the formed portfolios, which showed the robustness of optimal allocations relative to the risk measure chosen. This was confirmed by the fact that the first principal component of the returns of these portfolios accounted for more than 90% of the total risk. Similar robustness was found when minimizing expected shortfall for different thresholds ranging from

-5% to 40%. It is only when examining the worst-case scenario that allocations change compared to the previous thresholds. With the 10% threshold, the Herfindahl diversification index

$$\text{Herfindahl index} = \sum_{i=1}^{N} w_i^2 \qquad (1)$$

started to decrease under a certain level of expected portfolio return, showing that in extremely demanding risk constraints, portfolios are concentrated on fewer funds with less catastrophic risk characteristics.

Brandtner (2013) criticizes spectral risk measures as a portfolio selection tool when used together with spectral utility functions. He begins by noting that current literature lacks an integrated framework that analyzes both the determination of efficient frontiers and the choice of optimal portfolios. He proceeds to define a framework that is based on decision theory and takes into account any dependence structure among the assets.

Assuming an investor maximizes a spectral utility function, then for two co-monotonic risky assets, he shows that the efficient frontier is a straight line between the risky assets and therefore, contrary to using variance, diversification is never optimal. Instead, the investor will prefer an exclusive investment in one of the risky assets. Similarly, if there are only two states of the world, then all or nothing decisions hold, irrespective of the dependence structure. If a risk free asset is added to the portfolio, then the investor obtains either the risk free asset or the tangency portfolio as the optimal solution, hence diversification is still not preferable. If spectral utility functions are used in accordance with spectral risk measures, then maximizing utility is equivalent to maximizing return, and as a result, only corner solutions are obtained.

The latter argument was formally established by Ibragimov (2007) for VaR, where he showed that diversification, defined in terms of VaR subadditivity, does not always work as expected. In a world of extremely heavy tail risks with unbounded distribution support, VaR can become super-additive. From a utility perspective, Samuelson (1967) showed that any investor with a strictly concave utility function will uniformly diversify among independently and identically distributed risks with finite second moments. In that case, the portfolio will have equal weights. However, Ibragimov points out that if there is a point far out in the tails beyond which the utility is not concave but convex, then diversification may not be optimal.

In a similar context, Cholette, Pena, and Lu (2011) defined diversification in terms of several measures related to correlation. First, they examined the level of dependence between financial indices with regard to Pearson or Spearman correlations.

$$\begin{aligned}\rho_{Pearson} &= \frac{Cov(X,Y)}{\sqrt{Var(X)\cdot Var(Y)}}\\ \rho_{Spearman} &= \frac{Cov(F_X(X),F_Y(Y))}{\sqrt{Var(F_X(X))\cdot Var(F_Y(Y))}}\end{aligned} \qquad (2)$$

They showed that lower dependence implies greater diversification. Using weekly returns from international stock market indices over the period January 1990 to May 2006, they first measured asymmetric dependence and found that Pearson and rank correlations do not always provide consistent results, particularly for East Asian and Latin American country indices. They then considered six copulas and used them to fit each group of countries. The shape of the best fit copula described positive or negative tail dependence for each set of countries and its parameter provided an estimate of such dependence. The authors found little evidence of asymmetric dependence in the East Asian countries and larger evidence in the G5 and Latin America. They also found that over time, average tail dependence increased for each region, which was true whether using symmetric or asymmetric copulas. They then measured how left and right tail dependence, as well as Kendall's $\tau$ relate to returns of each country group for each of the six copulas examined.

$$\tau = P\left[(X-\tilde{X})(Y-\tilde{Y})>0\right] - P\left[(X-\tilde{X})(Y-\tilde{Y})<0\right] \qquad (3)$$

where tilde denotes independent copies of the relevant random variable. They found that Latin American indices exhibited the highest returns while having the lowest dependence, whereas the G5 exhibited the opposite behavior. The fact that diversification is not present during extreme tail dependence confirms the theoretical findings of Ibragimov (2007) and Adam, Houkari, and Laurent (2008).

**8. Return**

Showing the impact to return, Booth and Fama (1992) proved that a portfolio's compound return is higher than the weighted average of the compound returns on the assets in the portfolio. This is due to the fact that the contribution of each asset to the portfolio return is greater than its compound return. The justification for this is that the contribution of each asset and portfolio variance is less than its own variance due to less than perfect correlation. Examining seven asset classes for different time periods ranging between 1941 and 1990, the authors found that the incremental returns due to diversification are greater for small-cap stocks than for other assets. This is because small-cap stocks have volatile returns and their risk is easily diversified away, as they have low correlations with other assets. They further demonstrated the implications of active management to diversification. By generating 1,000 portfolios that randomly invested half the time in stocks and the rest in bonds over the period 1986-1990, they found that the average standard deviation of returns corresponded to a constant-mix portfolio invested 53% in stocks and 47% in bonds. The constant-mix portfolio achieved a compound return 14 basis points higher than the average random portfolio return and had a 52 basis point annual diversification return. Its volatility was also much lower than the average random portfolio.

A similar concept related to diversification, called the return gap, was introduced by Statman and Scheid (2007). They defined the return gap as the difference between the returns of two assets. Their justification was that return gaps take into account not just correlations but also standard deviations and are more intuitive than correlation.

$$Return\_gap = 2\sigma\sqrt{\frac{1-\rho}{2}} \qquad (4)$$

Two assets might exhibit a high correlation over a time period, but the realized returns might be very different. Such assets offer increased diversification, as viewed from the definition of return gaps.

Focusing on a group of hedge funds, Kinlaw, Kritzman, and Turkington (2013) show in a recent paper that diversification is not optimal when performance fees are taken into account. They provide an example based on a Monte Carlo simulation of an equally weighted investment across ten funds, each with an expected return of 7%, standard deviation 15%, and benchmark return of 4%. The base fee each fund charges is 2% and the performance fee is 20%. Assuming no correlation among the funds, they find a reduction in the collective expected fund return of about 0.7%. Such reduction is due to the fact that investors always pay a fee when funds outperform the benchmark or risk free rate, but they are not reimbursed for underperformance. As correlation increases and the funds become less diversified, the reduction in the investment decreases. In practice, this effect is less pronounced due to claw back provisions, termination of underperforming funds, or reset of performance fees without loss reimbursement to investors.

**9. International diversification**

Diversification can be beneficial across countries from the perspective of a local investor (Driessen and Laeven, 2007). Using monthly data from 1985-2002 across 52 countries, investors were first allowed to trade in regional equity markets based on the fact that investors prefer familiar investing opportunities (Huberman, 2001; Grinblatt and Keloharju, 2001). Then they were allowed to invest in global equity indices. For the first case, the authors regressed each of possible three global indices or one regional index against a local index in order to measure the statistical significance of diversification possibilities. If the regression alpha is zero and beta equal to one, it means that the global or regional indices do not add to the expected return, but rather only to the variance of the portfolio spanned by the local index. In that case, the optimal mean-variance portfolio consists only of the local index. To measure the economic significance of diversification, Driessen and Laeven first calculated by how much the Sharpe ratio of a mean-variance portfolio based only on local indices changed versus the Sharpe ratio of a mean-variance portfolio that included global indices. In addition, they measured the change in expected return when adding these global indices, given the same variance as for the optimal portfolio of the local indices, and assuming no risk-free asset. Driessen and Laeven found that the benefits of diversification as measured by all of these criteria were greater for developing countries relative to developed ones and this was mainly due to their increased country risk. Over their sample period, diversification benefits have decreased as country risk has decreased.

**10. Risk contribution**

Another way to define diversification is in terms of risk contribution, which is equivalent to the beta of a security to the portfolio. It closely relates to loss contribution and, under certain instances, the two measures are identical (Qian 2005). One such example is a portfolio that is optimal from a mean-variance perspective. In that case, risk contribution is equal to the expected return contribution. To the extent that a portfolio is not mean-variance efficient, loss contribution will dominate risk contribution, which will in turn dominate return contribution. For extreme losses, loss and risk contributions will be equal.

Under this concept, diversification can be defined as the uniformity of risk contributions across a portfolio's components (Maillard, Roncalli, Teiletche, 2009). Equally weighted risk portfolios ensure that all portfolio components contribute the same amount to the total risk. In contrast, the minimum variance portfolio equalizes marginal risk contributions. This means that a small increase in any component will increase the total risk by the same amount as a small increase in any other component. The risk contributions, however, will be unequal and the portfolio will be highly concentrated. Consequently, a portfolio with equal risk contributions may be viewed as a portfolio located between the *1/N* and the minimum variance portfolios, with the latter having the lowest variance and *1/N* having the highest variance.

As the Lee (2011) study indicates, the portfolio weights of the equal risk contribution (ERC) portfolio are inversely proportional to the portfolio's betas with respect to the assets. That means that high volatility or correlation of an asset to the portfolio will result in lower weights. In order for the ERC portfolio to be efficient, all assets must possess identical Sharpe ratios and exhibit the same correlations among all other assets. Using data for the top 10 US industry sectors between January 1973 and December 2008, the authors found that the performance and risk statistics of the ERC portfolio were very close to the *1/N* strategy. The ERC portfolio was more concentrated in terms of weights, but the *1/N* portfolio was concentrated in terms of risk contributions. The MV portfolio had better risk-adjusted performance, but worse diversification. Repeating the process for agricultural commodities over a similar period, the authors found that ERC dominated *1/N* both in terms of return and risk. MV dominated over all, but showed larger drawdowns and tail risk. Finally, looking at global asset classes, the ERC portfolio had superior Sharpe ratios and average returns. The authors noted that the solution obtained for the ERC portfolio is numerically challenging and a global optimum cannot be always guaranteed.

**11. Risk ratio**

Another commonly used measure is formalized by Tasche (2006, definition 4.1). For an arbitrary risk measure ρ, position weight $w_i$ with return $r_i$, Tasche calculates the following ratio:

$$DF_{\rho,i} = \frac{\rho_i}{\sum_j \rho\left(w_j r_j\right)} \tag{5}$$

Based on this, the study considers the diversification ratio defined as the ratio of the weighted average volatilities divided by the portfolio volatility (Choueifaty and Coignard 2008).

$$DR(w) = \frac{\sum_i w_i \sigma_i}{\sigma_w} \tag{6}$$

If the expected returns of portfolio components are proportional to their risks, then maximizing the expected return is equivalent to maximizing risk. In that case, the most diversified portfolio (MDP) is also the mean-variance optimal portfolio. This is also the case in a universe where all portfolio components have the same volatility. Any stock not belonging in the most diversified portfolio is more correlated to that portfolio than any stock that belongs in it. Furthermore, all stocks have the same correlation to the portfolio. Using U.S. and European stock return data between December 1991 and 2008, the authors demonstrated that the maximum diversification portfolio was consistently less risky than market cap-weighted indices and had a higher Sharpe ratio than the market cap benchmarks, minimum variance portfolio, and the equally weighted portfolio.

Following up in 2011, and using standard deviation as the risk measure, Choueifaty and Coignard decomposed the diversification ratio in terms of the volatility-weighted average correlation and the concentration ratio, defined as the sum of variances divided by the sum of weighted volatilities squared.

$$
\begin{aligned}
DR(w) &= \left[\rho(w)(1-CR(w)) + CR(w)\right]^{-1/2} \\
\rho(w) &= \frac{\sum_{i\neq j} w_i\sigma_i w_j\sigma_j\rho_{i,j}}{\sum_{i\neq j} w_i\sigma_i w_j\sigma_j} \\
CR(w) &= \frac{\sum_i (w_i\sigma_i)^2}{\left(\sum_i w_i\sigma_i\right)^2}
\end{aligned}
\tag{7}
$$

The latter is a generalization of the Herfindahl-Hischman index. They also showed that the portfolio diversification ratio can be decomposed into the volatility-weighted average of its components' diversification ratios divided by its volatility. The diversification ratio equals the number of independent factors necessary for a portfolio that allocates risk to these factors in order to achieve the same *DR*. It is therefore equal to the effective number of uncorrelated factors. The authors further showed that any stock not belonging in the most diversified portfolio is more correlated to that portfolio than any stock that belongs in it.

Assuming that *X* and *Y* are two assets with identical Sharpe ratios, a new company *Z* can be created by holding shares of *X* and *Y* in the balance sheet. The Sharpe ratio of *Z* is higher than *X* and *Y*, unless the correlation between *X* and *Y* is 1. The existence of assets with non-identical Sharpe ratio to others makes the most diversified portfolio nonefficient in the mean-variance space. As Meucci (2009) points out, this is a differential and not an absolute diversification measure. Focusing on a portfolio of 10 U.S. sectors in the Russell 1000 universe and using 10 years of monthly returns as of March 2010, Lee (2011) demonstrated that the MDP was more concentrated relative to the market capitalization-weighted portfolio in terms of risk contributions. In terms of cumulative risk contribution, Lee showed that the minimum variance portfolio (MVP) was the most concentrated, followed by the MDP. The market capitalization-weighted portfolio was again found to be more diversified in that context than the MDP.

Frahm and Wiechers (2013) proposed the ratio of the smallest possible variance among the portfolio constituents divided by the actual variance of the portfolio as an alternative diversification measure.

$$\rho(w) = \frac{\sigma_{MVP}}{\sigma_w} \tag{8}$$

It shows how much removable variation is still contained in the portfolio.

Pérignon and Smith (2010) examined VaR results reported from major banks in the U.S. on a quarterly basis between the end of 2001 and beginning of 2007 and tried to calculate the diversification benefit of individual VaR across broad risk categories (equity, interest rate, commodity, credit spread, foreign exchange) to the aggregate VaR. Having access only to individual risk VaRs, they proxied each category to major market indices and used the correlation between these indices to aggregate the individual VaRs. Defining the diversification measure as:

$$\delta = \frac{\sum_i^N VaR_i - VaR}{\sum_i^N VaR_i} \tag{9}$$

Pérignon and Smith reported that their proxies closely approximated the aggregate VaR reported.

**12. Information theory**

Using the number of portfolio constituents as a measure of diversification has been criticized, since it only provides an adequate picture if the portfolio is equally weighted. Information theory provides diversification measures that focus on the quantification of the disorder of a random variable. Using monthly returns between 1965 and 1985 from 483 U.S. companies, Woerheide and Persson (1993) repeated the experiment of Evans and Archer (1968) to determine which diversification index related to information theory or economic concentration is mostly related to volatility reduction in the case of unequally and positively weighted portfolios. They found that the complement of the Herfindahl index was the best performer with an $R_2$ of 0.548.

$$\text{Compliment of Herfindahl index} = 1 - \sum_{i=1}^{N} w_i^2 \tag{10}$$

Thus they recommended that specific index as a measure of diversification. The study was repeated later by Frahm and Wiechers (2013) with updated data providing with similar results.

Another popular measure from information theory is the Shannon entropy. It was used by Bouchaud et al. (1997), Bera and Park (2008), and Meucci (2009) and is revisited below.

**13. Principal portfolios**

Rudin and Morgan (2006) examined equally weighted portfolios and constructed the principal portfolios, namely the components of a portfolio that are uncorrelated linear combinations of the original portfolio constituents. To see how this is done, consider a set of $N$ securities in a portfolio. The portfolio variance is given by:

$$Variance = W'\Sigma W \tag{11}$$

where Σ is the covariance among the securities and can be further decomposed as:

$$\Sigma = \mathrm{E}\Lambda\mathrm{E}' \tag{12}$$

where $E^{NxN}$ contains the eigenvectors of Σ and $\Lambda^{NxN}$ is a diagonal matrix that contains the eigenvalues of Σ. The portfolio variance can then be written as:

$$Variance = W'\mathrm{E}\Lambda\mathrm{E}'W \tag{13}$$

Instead of working with the original security weights $\tilde{W} = E^{-1}W$ , we can instead choose weights $W$.

These form the principal portfolios. Note that while the original securities had returns $R$, the principal portfolios have returns $\tilde{R} = E^{-1}R$ .

The portfolio variance is finally written as:

$$Variance = \tilde{W}\,\Lambda\tilde{W} \tag{14}$$

Denoting $\lambda$ as the eigenvalue of each principal portfolio, and hence its variance, Rudin and Morgan formed the diversification index:

$$DI = 2\sum_{k=1}^{N} kw_i - 1 \quad where\ w_i = \frac{\lambda_i}{\sum_i \lambda_i} \tag{15}$$

This index measures the relative importance of principal components in a portfolio. If the original constituents have a high correlation with each other, the first few principal portfolios will account for most of the variance; hence the index will be small. If all assets are uncorrelated, then the index will equal $N$, since in that case each $w_i$ will equal $1/N$.

Meucci (2009) followed the same approach of constructing principal portfolios, but refined the diversification measure. In the spirit of Tasche, he first defined the diversification distribution, with $\rho$ being the variance of a principal portfolio.

$$p_i = \frac{\tilde{w}_i^2 \lambda_i^2}{\sum_i \tilde{w}_i^2 \lambda_i^2} \tag{16}$$

He then applied the exponential of the Shannon entropy on that diversification distribution to form the below diversification measure:

$$N_{Ent} = \exp\left(-\sum_i p_i \ln(p_i)\right) \tag{17}$$

A low number means that the effective number of uncorrelated risk factors is low and hence the portfolio is not diversified. The defined entropy of the principal portfolios can achieve its maximum value equal to the number of portfolio constituents. This means that the portfolio is fully diversified. Portfolios can be then constructed on the mean-diversification frontier.

Meucci's approach, also called diversified risk parity, was compared against the equal risk contribution portfolio, the minimum variance portfolio, and the equally weighted portfolio in a paper by Lohre, Opfer, and Orszag (2011). Using global indices that represented broad asset classes between December 1987 and September 2011 and long-only constraints, they found all strategies yielding similar returns. The *1/N* strategy showed a slightly higher return, with a much higher volatility and drawdown. The minimum variance strategy exhibited the lowest return, with a much lower volatility and hence the highest Sharpe ratio. It also had the lowest drawdown. The equal risk contribution strategy was in between the *1/N* and minimum variance.

The diversified risk parity approach displayed a low Sharpe ratio, while its drawdown was between the *1/N* and risk parity strategy. In terms of tracking error, the diversified risk parity was similar to the risk parity strategy. The diversified risk parity strategy was the most resilient to the 2008 crisis, when using a rolling window. However, that came at the expense of higher turnover. In terms of diversification, the risk parity strategy was not evenly distributed across the principal portfolios. The diversified risk parity was evenly distributed across three out of five asset classes and was found to react more timely in terms of allocation shifts, when calculating with a rolling window. Relaxing the long-only constraints allows the diversified risk parity to be more homogeneous across all assets.

In a recent paper, Meucci, Santangelo, and Deguest (2014) explain that using principal components to measure diversification presents various drawbacks. The principal components are statistically unstable, they are not invariant under transformations, they are not unique, they are not easy to interpret, and they can give rise to counter-intuitive results. The authors propose instead to look for the zero-correlation transformation of the original factors that disrupts these factors as little as possible. Such transformation is called minimum torsion linear transformation and is formally achieved by minimizing the tracking error between the torsion and the original factors. They then derive the effective number of minimum torsion bets, similar to the effective number of uncorrelated risk factors from Meucci's previous paper (2009). This approach overcomes these limitations, based on principal components.

## 14. Conclusion

The quest for diversification is never ending; its definition is not unique and diversification measures are continuously evolving. It is important to understand the advantages and limitations of diversification and the context in which it is applied. While there is significant research behind this concept, going back many decades, more studies are needed in order for investors to better understand the potential impact of diversification on their portfolios.

## Endnotes

1. In contrast, geometric returns are expected to increase for a given level of arithmetic returns as diversification increases.

2. See Markowitz (1952), Sharpe (1964), Lintner (1965), Mossin (1966) and Samuelson (1967).

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## Author Bio

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