Pension Fund ALM with Longevity Hedging

Can Longevity Hedging Help Pension Funds Improve Their Asset Liability Management?

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In this paper we analyze the impact hedging longevity risk can have on a pension fund's funding ratio volatility and asset liability management (ALM) strategy. Our model captures all relevant aspects of the ALM problem and is calibrated to industry statistics; however, we've sacrificed model complexity to make the solution more intuitive and presentable. Our main conclusion is that hedging longevity risk creates an additional risk budget to be put towards more rewarding asset allocation strategies, thereby improving the overall ALM outcome of the modelled pension fund.

We show that at representative parameters for the risk and return on the balance sheet, and a range of realistic hedge prices, executing longevity hedges elevates the Efficient Frontier across all reasonable risk budgets. Therefore, implementing a longevity hedging strategy can improve the fund's Sharpe Ratio and ALM outlook considerably. This is especially true for funds with a low risk budget, e.g. when the funding ratio is close to 105%.

Our results are consistent with earlier work on this topic by Cocco and Gomes (2012), who demonstrate the benefits of financial assets designed to hedge shocks to the survival probabilities in a life cycle model with longevity risk. Our analysis differs since we focus on the pension fund rather than the household balance sheet, use a more extensive model for the financial market, explicitly define the hedge instrument, and use market information on the pricing of longevity hedges.[1]

The remainder of the paper is organized as follows. First, we discuss how we jointly model longevity risk and investment risk stochastic framework. Second, we formulate the ALM problem of a pension fund in the context of our framework. Third, we discuss the impact of hedging longevity risk on the pension fund's funding level volatility and the optimal asset allocation. Finally, we explain our conclusions and provide an outlook for future research.

Joint Stochastic Model for Longevity and Investment Risk

The mortality model chosen for this analysis is the Gaussian Makeham Model defined in Schrager (2006). The force of mortality of an x-year old, $\mu_x(t)$ is modeled as an affine function of a 2-vector of stochastic processes, Y_i

$$\mu_x(t) = Y_1(t) + Y_2(t)c^x$$

Y_i follows an Ornstein-Uhlenbeck process (which is the continuous time equivalent of an AR(1)-process),

$$dY_i(t) = a_i(\theta_i - Y_i(t))dt + \sigma_i dW_i(t) \;,\; Y_i(0) = Y_{i0} \text{ for } i = 1,2$$
$$dW_1(t)dW_2(t) = \rho dt$$

For simplicity we assume,

$$\theta_2 = 0$$

The $(n-t)$ -year survival probability of an $(x+t)$ -year old at time t in this model is given by,

$$p(t,n,x) = \exp\left(C(n,x) - D_1(n,x) \bullet Y_1(t) - D_2(n,x) \bullet Y_2(t)\right)$$

The benefit of this model is that all mortality and survival probabilities are completely tractable. The analytical expressions for $C(n,x)$, $D_1(n,x)$ and $D_2(n,x)$ can be found in (Schrager, 2006).

The best-estimate mortality path is implied by the expected value of the stochastic factor

$$\bar{Y}_i(t) = Y_{i0}e^{-a_i t}$$

Mortality trend risk is quantified by the application of a shock to the factor $Y_t^{99.5\%} = Y_0 - \sigma_Y \bullet \Phi^{-1}(99.5\%)$. We use these expressions to calibrate the model parameters to public mortality data published by the CBS.[2,3] The results are displayed in Exhibits 1 and 2.

Life Expectancy 65 Year Unisex	Current	2027	2037
CBS 2017	19.88	20.92	22.09
CBS 2017 90% quantile (*)	-	22.18	23.87
Affine Makeham Model	19.88	20.94	22.01
Affine Makeham 90%	-	21.83	24.13

Exhibit 1: Life expectancy of 60-year male from different mortality models. Both current and projected life expectancies in the Affine Makeham Model match the output of the two main Dutch mortality models closely. (*) 90% quantile is derived based on Gaussian distribution assumption for the life expectancy.

Parameter	Value
α_1	4.83e-7
α_2	0.0150
θ_1	0
ρ	0
σ_1	1.52e-5
σ_2	2.00e-8
c	1.13003
$Y_1(0)$	3.79e-10
$Y_2(0)$	2.48e-6

Exhibit 2: Parameters of the Affine Makeham Model.

The model gives a very satisfactory fit to the CBS data. We conclude the Affine Makeham Model at the fitted parameters provides a good representation of longevity risk over the long run. We will use these parameters to derive the volatility of an annuity in the next section.

We now turn to modelling the asset side of the balance sheet. Our model of the financial markets assumes the fund invests in three investment categories,

- "Return" investments: an optimized combination of equity, direct and indirect real estate, private equity and, hedge fund investments
- "Spread" investments: a fixed income portfolio optimized for credit risk and duration;
- "Safe" investments: a portfolio of high credit quality fixed income instruments and collateralized derivative positions optimized for interest rate risk management.

A duration gap, D_i , between assets and liabilities is explicitly modeled with the yield of the duration strategy assumed to follow a Geometric Brownian Motion process with zero drift and volatility equal to σ_{IR} . These asset classes follow correlated Geometric Brownian Motion processes, with drift vector $\vec{\mu} = [\,\mu_R \;\; \mu_{Sp} \;\; \mu_{Sf} \;\; \mu_{IR}\,]^T$ and volatility matrix $\Sigma = \mathrm{diag}[\,\sigma_R \;\; \sigma_{Sp} \;\; \sigma_{Sf} \;\; \sigma_{IR}\,]$ for the Return, Spread, Safe investments, and the (long-term) interest rate level respectively. Asset allocation is given by a weight vector $\vec{w} = [\,w_R \;\; w_{Sp} \;\; w_{Sf} \;\; D]^T$.

The market value of the assets can be described by the following equation,

$$A(t) = \exp\left(\left[\vec{w}^T\vec{\mu} - \frac{1}{2}\vec{w}^T\Sigma \mathrm{P}\Sigma\vec{w}\right]dt + \vec{w}^T\Sigma\, dW^M(t)\right)$$

Assumptions for the expected excess-return as well as the volatility of those excess-returns are in line with historical averages (return) and the correlation matrix, P , which is taken from the DNB ("De Nederlandsche Bank", the Dutch regulator) standard model for pension funds.[4] We also calibrate interest rate volatility to be in line with an absolute downward shock of 1.25%, as proposed by Noorman[5] and assume this corresponds to a 1-in-50 scenario under a Gaussian distribution for interest rates: $\frac{1.25\%}{N^{-1}(0.975)} = 0.6378\%$. These assumptions are displayed in Exhibit 3.

Volatility & Correlation	Volatility (*)	Correlation			
"Return" investment	20%	1	50%	0%	40%
"Spread" investment	8.5%	50%	1	0%	40%
"Safe" investment	0%	0%	0%	1	0%
Interest Rate	0.6378%	40%	40%	0%	1
Duration Gap	3 years	-	-	-	-

Exhibit 3a: Financial markets parameters. Volatility and correlation of main ALM drivers. (*) Volatility of "Return", "Spread," and "Safe" investment categories is measured relative to the market value of the liability.

With respect to sensitivity of the liabilities to inflation, we assume 0% indexation during the horizon of the ALM optimization given current indexation policies and low nominal funding levels of pension funds.

	Expected Excess Return
"Return" investment	3.5%
"Spread" investments	2%
"Safe" investments	0%
Interest Rate (*)	0%

Exhibit 3b: Financial markets parameters. Expected excess return of different asset categories. (*) Interest Rate assumption shows the expected change in the value of the interest rate level. The interaction with the duration gap determines the impact on the return. We assume no expected change in the interest rate level.[6]

The ALM Problem of the Pension Fund

For simplicity, we model the pension fund liability using a single model point consisting of a 60-year male with a constant pension benefit of 1 starting at age 60.

The pension liability at time *t*, for an individual that is x -year old at time zero, with a pension age of $x_{pension}$ equals,

$$L(t) = \sum_{i=t+1}^{\varpi} D(t,i)\, p(t, i-t, x+t) \mathbf{I}_{[x+i > x_{pension}]}$$

Where $D(t,T)$ is the discount factor at time *t* for payment at time T. We assume a flat interest rate curve at 1% and the Affine Makeham parameters in Exhibit 2. Liability volatility due to longevity risk, σ_L, then equals 6.1% which is simplified using the "freezing of the weights" approximation.[7,8]

Liability duration at time t, $D_L(t)$, equals,

$$D_L(t) = \sum_{i=t+1}^{\varpi} (i-t)\, D(t,i)\, p(t, i-t, x+t)\, \mathbf{I}_{[x+i > x_{pension}]}$$

The pension fund's funding level measures the degree to which the market value of the assets is expected to cover the liability,

$$F(t) = \frac{A(t)}{L(t)}$$

Optimization without Longevity Hedge

In our joint model of longevity and investment risk the excess-return of the funding level without longevity hedging equals,

$$\mu_F = \vec{w}^T \vec{\mu}$$

and the volatility of the funding level without longevity hedging equals,

$$\sigma_F = \sqrt{\vec{w}^T \Sigma \mathrm{P} \Sigma \vec{w} + \sigma_L^2}$$

the fund's ALM problem can be formulated as the optimization of the return of the funding level subject to a risk-budget constraint,

$$\max_{W} \mu_F \qquad \text{s.t. } \sigma_F \le \frac{VaR - limit}{N^{-1}(VaR - probability)}$$

Longevity Hedge Instruments

Different types of longevity hedge instruments exist in the market for longevity risk. Simple quota-share reinsurance that hedges only longevity risk can be achieved by a longevity swap. Alternative covers exist, either in index or indemnity format, including finite stop-loss cover for the first-loss of the risk distribution, as well as finite or infinite tail-risk covers. Index-based hedges reference general population mortality rates (e.g., those published by the Central Bureau of Statistics, CBS, in The

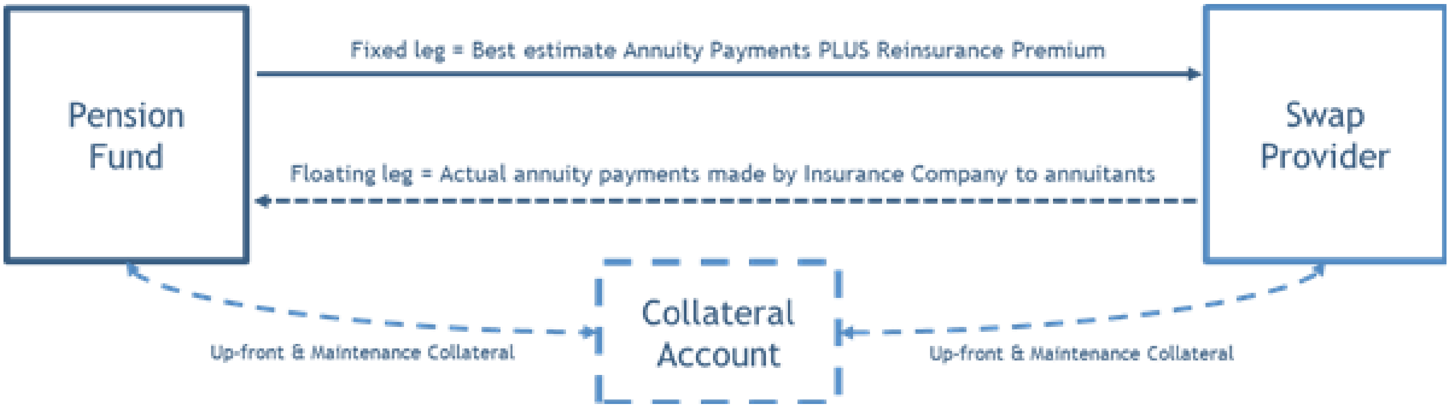

Exhibit 4: Structural diagram of longevity swap. A collateral arrangement is usually part of the contract.

Netherlands) and therefore include some elements of basis risk. Indemnity contracts, such as the longevity swap which we use as a hedge instrument in our analysis below, reference actual portfolio cash-flows, and hence contain no basis risk.[10]

Longevity swaps feature a pre-agreed stream of cash-flows paid by the hedger to the risk taker representing the expected pension payments plus an additional premium charge (usually expressed as a percentage of the expected payments). In return, the risk taker agrees to pay the hedger the actual total pension benefits it ultimately pays to the pensioners. See Exhibit 4.

The payments between the hedger and risk taker are typically netted on a monthly basis so that a smaller amount of cash is exchanged, and only in one direction. If longevity turns out exactly as expected, then only the fixed premium charge would be paid.

In our analysis we use a longevity swap as the hedge instrument for simplicity, and assume a premium range of $p = 3 - 5\%$ which is consistent with observed market prices. In practice, the price depends on the specific pension fund mortality risk experience, market dynamics, and the Terms & Conditions of the swap contract. The optimal choice of hedge instrument will be explored further in a future publication.

Optimization with Longevity Hedge

The longevity swap premium, p, is a percentage of the expected liability cash-flow each year. Given a liability duration, D_L, the average impact on the fund's return from hedging a proportion, H of the longevity exposure is $-H \bullet \frac{p}{D_L}$.

Hedging a portion of the fund's longevity risk using a longevity swap changes the expressions for the funding level return and volatility to,

$$\mu_F = \vec{w}^T \vec{\mu} - H \bullet \frac{p}{D_L}$$

$$\sigma_F = \sqrt{\vec{w}^T \Sigma \mathrm{P} \Sigma \vec{w} + (1-H)^2 \sigma_L^2}$$

The optimization problem now contains an additional decision variable but is the same in principle,

$$\max_{W,H} \mu_F \qquad \text{s.t. } \sigma_F \leq \frac{VaR - limit}{N^{-1}(VaR - probability)}$$

Impact of Hedging Longevity Risk on a Pension Fund's ALM

In this section we present and discuss the results of the optimization problem at representative parameters. Whereas the previous section introduced the optimization problem with a generic hedge ratio H, in this section we simplify by determining the Efficient Frontier assuming no hedging (i.e. $H = 0$) and full hedging (i.e. $H = 1$).

In practice, selective hedging is possible (either in terms of instrument, risk-layer, sub-portfolio, and hedge ratio) which should further extend the potential impact of longevity hedging on the pension fund's ALM. Before explaining the results of the optimization, we display some of the risk-return outcomes from our model under the asset allocations defined in Exhibit 5 on the next page.

The results in the third and the last column of Exhibit 5 show that hedging longevity risk significantly reduces funding ratio volatility at a given asset allocation thereby reducing the worst-case outcome. We propose that the reduction in volatility created by the hedge allows for a higher allocation to Return or Spread assets, thereby achieving higher total returns at the same level of overall risk. Hedging is therefore expected to generally improve the Sharpe Ratio because risk is hedged at a cost that is lower than the available risk-return in the market.

Considering the impact of the hedge's cost on expected return, a 5% swap premium (as a percentage of the liability) should be divided by duration to derive an annual cost of hedging. In the optimization process, this annual cost is measured against extra return that can be achieved, per unit of risk, by investing in risky assets. The resulting cost of hedging is therefore 5% / 20 year duration = 25bps on the liability. It is important to then determine how much the risk budget increases due to the longevity hedge, and what is the resulting improvement in the overall return of the portfolio.

In Exhibit 6a we display the result of the ALM problem (i.e., the Efficient Frontier) in terms of funding level volatility, given a defined risk budget. We compare the Efficient Frontier with and without hedging. In Exhibit 6b we display the same results at a higher duration gap, and witness that introducing more risk in the balance sheet slightly mutes the effect of longevity hedging, however the benefits are still present.

Asset Allocation	Funding Ratio Excess Return	Funding Ratio Volatility	Funding Ratio Sharpe-Ratio	1-Yr 99.5% - VaR Scenario F(0) = 110%
Risk Minimization, 100% Safe Investments	0.00%	6.4%	0.0	93.5%
Return Maximization, 100% Return Investments	3.50%	21.7%	15.6	54.1%
Equal Weights, 33.3% in each Asset Class, No Hedging H=0	1.83%	11.3%	16.3	81.0%
Equal Weights, 33.3% in each Asset Class, Full Hedging H=100%	1.58%	9.5%	16.7	85.6%
Traditional Allocation Weights, 20% Return, 30% Spread, No Hedging H=0	1.30%	9.2%	14.2	86.4%
Traditional Allocation Weights, 20% Return, 30% Spread, Full Hedging H=1	1.05%	6.8%	15.4	92.5%

Exhibit 5: Numerical results for example allocation rules. Hedging reduces return but also risk, eventually increasing Sharpe Ratio for a simple equal weight strategy and a traditional allocation strategy.

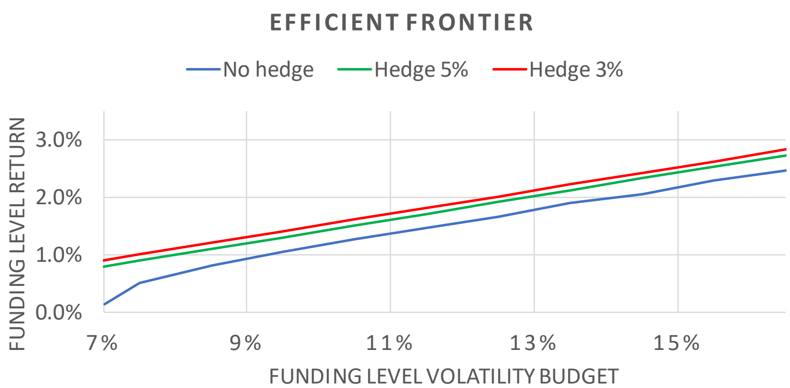

Exhibit 6a: Efficient Frontier (optimal asset allocation given a volatility budget) for a pension fund with and without longevity hedge. Parameters are taken from Exhibits 2 and 3. Introducing a longevity hedge significantly improves the optimal return outcome. This can be further improved by allowing for optimization over the hedge ratio.

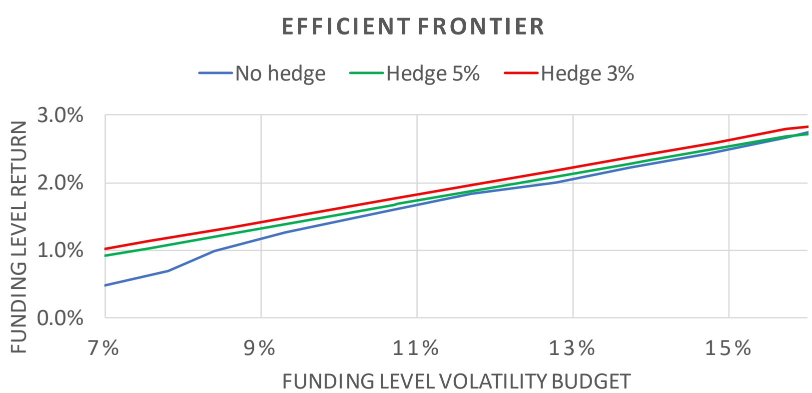

Exhibit 6b: Efficient Frontier with and without longevity hedge. Duration gap is increased to 5 years. Introducing a longevity hedge still improves the optimal return outcome. Although the impact is less pronounced than in exhibit 6a. A lower longevity hedge premium makes the impact of hedging more pronounced.

	Without Hedge			With Hedge		
Allocation / Vol budget	**7.5%**	**10%**	**12%**	**7.5%**	**10%**	**12%**
"Return" Investment	4%	33%	47%	28%	41%	51%
"Spread" Investments	19%	6%	2%	9%	16%	20%
"Safe" Investments	77%	61%	51%	63%	43%	29%

Exhibit 7: Optimal asset allocation given a risk budget, with and without logevity hedge.

It is clear from both graphs the introduction of longevity hedging has a positive impact on the Efficient Frontier as it improves the optimal expected return at a given level of risk. We show the results under a premium charge ranging from 3% to 5%. As would be expected, a change in the longevity swap premium shifts the Efficient Frontier in a parallel way.

In both scenarios, there is very interesting potential for improvement where the risk budget is limited. Without longevity hedge the minimum risk level the fund can achieve equals $\sigma_F = \sqrt{D\sigma_{IR}^2 + \sigma_L^2}$. This implies full allocation to Safe (i.e high-quality duration matching) investments. However, this doesn't make sense as a Strategic Asset Allocation because it destroys the return potential of the fund.

With a longevity hedge in place the fund can achieve the same amount of risk with a much higher expected return. This implies that hedging longevity risk should be an interesting option to include in ALM studies of funds that have limited buffers, since their risk budget should be limited.

Exhibit 7 shows the optimal asset allocation, with and without longevity hedges, at different levels of risk budget.

We see that the shift to more risky assets is reasonably pronounced, especially at lower risk budgets. However, the shift to more risky assets doesn't imply a widening of the duration gap as the interest rate gap strategy is not affected by the shift in optimal asset allocation.

When a pension fund needs to reduce its funding level volatility it now has two options. The traditional choice for most pension funds is to reduce the exposure in risky assets, i.e. reduce the volatility of the numerator of the funding ratio. Although this reduces the funding level volatility, it also reduces the investment return potential of the investment portfolio. The other choice is to reduce the volatility in the denominator of the funding ratio, by reducing the exposure to longevity risk. This second alternative leaves the investment return intact, which enhances the recovery potential for pension funds.

Conclusion

We've analyzed a pension fund's ALM optimization including the introduction of longevity risk hedges in a stylized model of the balance sheet. We conclude that, at representative parameters, hedging longevity risk enables a pension fund to allocate a higher proportion of return seeking assets, thereby improving the Sharpe Ratio for a given risk budget. This outcome is not impacted by the duration gap or the price of hedging.

The following avenues for future analysis are envisioned:

1. We can apply the same approach of creating risk-budget through longevity hedging to life-cycle funds. Currently, life-cycle funds in defined contribution schemes apply allocation rules to traditional asset classes like fixed-income and equity, however, this grossly overlooks the pensioner's needs to manage the risks of living longer than expected, and hence requiring additional income. We plan to analyze the problem of optimal asset allocation on a fixed horizon (with the horizon linked to the desired retirement age), based on an updated liability definition of an individual accumulating assets for retirement, in a pure risk/return framework.
2. The model in this paper is purposefully simplistic because the focus is on concepts. Future work could allow for parameter uncertainty in the asset returns, a more sophisticated measure of downward risk and more sophisticated stochastic modelling of the assets and liabilities.
3. Analyze the inconsistency of the market price of risk implied by index-based hedges and observed indemnity longevity swap quotes.

Endnotes

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David Schrager (Senior Partner at Longitude Solutions) has extensive experience in balance sheet management and structuring (longevity) hedge solutions for insurance and pension products. David held positions in both insurance and banking at ING, ABN Amro and NN Group and worked in culturally diverse teams for most of his career. David, in his previous role as head of pricing & hedging at NN Life, executed the second largest pension buy-out in the Netherlands in 2015 (ENCI) and the first Index-based Longevity Swap by NN Group in 2017. He was responsible for setting up internal models for valuation of complex insurance liabilities and economic capital, (longevity) hedge programs, pricing and structuring of pension buy-outs and has been involved in many strategic de-risking projects including M&A. David holds a Ph. D. in quantitative economics from the University of Amsterdam. Major contributions of the thesis focus on analytical solutions for complex stochastic problems in valuation and capital modelling.