CAIA® Level I Workbook
September 2020

Chartered Alternative Investment Analyst Association®
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PREFACE

Welcome to the workbook to “Alternative Investments” Level 1, 4th edition 2020. The CAIA® program, organized by the CAIA Association® and co-founded by the Alternative Investment Management Association (AIMA) and the Center for International Securities and Derivatives Markets (CISDM), is the only globally recognized professional designation in the area of alternative investments, the fastest growing segment of the investment industry.

The following is a set of materials designed to help you prepare for the CAIA Level I exam.

Workbook

The exercises are provided to help candidates enhance their understanding of the reading materials. The questions that will appear on the actual Level I exam will not be of the same format as these exercises. In addition, the exercises presented here have various levels of difficulty and therefore, they should not be used to assess a candidate’s level of preparedness for the actual examination.

September 2020 Level I Study Guide

It is critical that each candidate should carefully review the study guide. It contains information about topics to be studied as well as a list of equations that the candidate MAY see on the exam. The study guide can be found on the CAIA website, on the Curriculum page.

Errata Sheet

Correction notes appear in the study guide to address known errors existing in the assigned readings that are viewed as being substantive. Occasionally, additional errors in the readings and learning objectives are brought to our attention after publication. At those points we will then post the errata directly to a separate errata sheet on the Curriculum and Study Materials section of the CAIA website.

It is the responsibility of the candidate to review these errata prior to taking the examination. Please report suspected errata to curriculum@caia.org.

The Level II Examination and Completion of the Program

All CAIA candidates must pass the Level I examination before sitting for the Level II examination. A separate study guide is available for the Level II curriculum. As with the Level I examination, the CAIA Association administers the Level II examination twice annually. Upon successful completion of the Level II examination, and assuming that the candidate has met all the Association’s membership requirements, the CAIA Association will confer the CAIA Charter upon the candidate. Candidates should refer to the CAIA website, www.caia.org, for information about examination dates and membership requirements.
REVIEW QUESTIONS & ANSWERS

Chapter I  What is an Alternative Investment?

1. Define investment.
   - Investment is deferred consumption.

2. List four major types of real assets other than land and other types of real estate.
   - Natural resources, commodities, infrastructure and intellectual property

3. List the three major types of alternative investments other than real assets in the CAIA curriculum.
   - Hedge Funds, Private Equity, Structured Products

4. Name the assets that are often characterized as traditional by some and as alternatives by others for each of the following categories: hedge funds, private equity and real assets.
   - Hedge Funds – liquid alternative mutual funds
   - Private Equity – closed-end funds with illiquid holdings
   - Real Assets – public real estate and public equities of corporations with performance dominated by stable positions in real assets

5. Approximately when did average-quality corporate bonds and international equities become commonly viewed as institutional-quality investments in the United States?
   - Between 1950 to 1980

6. Name the four return characteristics that differentiate traditional and alternative investments.
   - Diversification, Illiquidity, Inefficiency, Nonnormality

7. Name four major methods of analysis that distinguish the analysis of alternative investments from the analysis of traditional investments.
   - Return Computation Methods, Statistical Methods, Valuation Methods, Portfolio Management Methods

8. Describe an incomplete market.
   - An incomplete market refers to the lack of investment opportunities that causes market participants to be unable to implement an investment strategy that satisfies their exact preferences such as risk preferences.

9. Define active management.
   - Active management refers to efforts of buying and selling securities in pursuit of superior combinations of risk and return.
10. What distinguishes use of the term *pure arbitrage* from the more general usage of the term *arbitrage*?

- Pure arbitrage is risk-free, while arbitrage, as a more general term, is not risk-free. Pure arbitrage is an attempt to earn risk-free profits through the simultaneous purchase and sale of identical positions trading at different prices in different markets. Whereas, arbitrage is used to represent efforts to earn superior returns even when risk is present because the long and short positions are not in identical assets or are not held over the same time period.
1. What is the term for a private management advisory firm that serves a group of related and ultra-high net worth investors?
   - Family office
2. In a large financial services organization, what is the name used to denote the people and processes that play a supportive role in the maintenance of accounts and information systems as well as in the clearance and settlement of trades?
   - Back office operations
3. Are dealer banks described as buy-side or sell-side market participants?
   - Sell-side market participants
4. List several advantages of Separately Managed Accounts (SMAs) relative to funds.
   - A fund investor owns shares of a company (the fund) that in turn owns other investments, whereas an SMA investor actually owns the invested assets as the owner on record.
   - A fund invests for the common purposes of multiple investors, while an SMA may have objectives tailored to suit the specific needs of the investor, such as tax efficiency.
   - A fund is often opaque to its investors to promote confidentiality; an SMA offers transparency to its investor.
   - Fund investors may suffer adverse consequences from redemptions (withdrawals) and subscriptions (deposits) by other investors, but an SMA provides protection from these liquidity issues for its only investor.
5. Which of the following participants is LEAST LIKELY to be classified as an outside service provider to a fund: Arbitrageurs, accountants, auditors or attorneys?
   - Arbitrageurs
6. List four major legal documents necessary for establishing and managing a hedge fund.
   - Private-placement memoranda, partnership agreement, subscription agreement, management company operating agreement
7. What is a qualified majority?
   - More than 75% of LPs voting to make a decision (e.g., the decision to extend the investment period or the fund’s duration).

8. Is the New York Stock Exchange a secondary or third market?
   - Secondary market

9. What are the three constraints against achieving alternative investment benefits through liquid products?
   - Leverage: there is a 300% asset coverage rule that requires a mutual fund to have assets totaling at least three times the total borrowings of the fund, thus limiting borrowing to 33% of assets. UCITS restrictions are even tighter.
   - Regulatory constraints on concentration
   - Illiquidity constraints

10. What is systemic risk?
    - Systemic risk is the potential for economy-wide losses attributable to failures or concerns over potential failures in financial markets, financial institutions, or major participants.
Chapter 3  Quantitative Foundations

1. What is the general term denoting compound interest when the interest is not continuously compounded?
   - Discrete compounding

2. What is the primary challenge that causes difficulty in calculating the return performance of a forward contract or other position that requires no net investment? How is that challenge addressed?
   - If the forward contract has a starting value of zero, it would cause division by zero. One solution to the problem of computing return for derivatives is to base the return on notional principal. Another is to include collateral.

3. An IRR is estimated for a fund based on an initial investment when the fund was created, several annual distributions and an estimate of the fund’s value prior to its termination. What type of IRR is this?
   - Since Inception IRR

4. An investment has two solutions for its IRR. What can be said about the investment and the usefulness of the two solutions?
   - There are two sign changes in the cash flow stream of the investment. None of the IRRs should be used.

5. Two investments are being compared to ascertain which investment would add the most value to a portfolio. Both investments have simplified cash flow patterns of an initial cost followed by positive cash flows. Why might the IRRs of the investment provide an unreliable indication of which investment adds more value?
   - The major challenge with comparing IRRs across investments is when investments have scale differences. Scale differences are when investments have unequal sizes and/or timing of their cash flows.
6. An analyst computes the IRR of one alternative to be 20% and another to be 30%. When the analyst combines the cash flows of the two alternatives into a single investment, must the IRR of the combination be greater than 20% and less than 30%?

   - No. The answer is not immediately apparent because the IRR of a portfolio of two investments is not generally equal to a value-weighted average of the IRRs of the constituent investments. If the cash flows from two investments are combined to form a portfolio, the IRR of the portfolio can vary substantially from an average of the IRRs of the two investments.

7. Is an IRR a dollar-weighted return or a time-weighted return? Why?

   - The IRR is the primary method of computing a dollar-weighted return.

8. What is the primary cause of the shape of the J-curve of interim private equity fund returns?

   - It is caused by a combination of early expense recognition, early loss recognition, and deferred gain recognition.

9. In which scenario will a clawback clause lead to payments?

   - A clawback clause, clawback provision or clawback option is designed to return incentive fees to LPs when early profits are followed by subsequent losses.

10. What is the difference between a hard hurdle rate and a soft hurdle rate?

    - A hard hurdle rate limits incentive fees to profits in excess of the hurdle rate. A soft hurdle rate allows fund managers to earn an incentive fee on all profits, given that the hurdle rate has been achieved.
1. Describe the difference between an ex ante return and an ex post return in the case of a financial asset.

- Ex post returns are realized outcomes rather than anticipated outcomes. Future possible returns and their probabilities are referred to as expectational or ex ante returns.

2. Contrast the kurtosis and the excess kurtosis of the normal distribution.

- Kurtosis serves as an indicator of the peaks and tails of a distribution. In the case of a normally-distributed variable the kurtosis is 3. Excess kurtosis is equal to kurtosis minus 3. Thus a normally distributed variable has an excess kurtosis of 0. Excess kurtosis provides a more intuitive measure of kurtosis relative to the normal distribution since it varies around zero to indicate kurtosis that is larger (positive) or smaller (negative) than the case of the normal distribution.

3. How would a large increase in the kurtosis of a return distribution affect its shape?

- Kurtosis is typically viewed as capturing the fatness of the tails of a distribution, with high values of kurtosis, or positive values of excess kurtosis, indicating fatter tails (i.e., higher probabilities of extreme outcomes) than is found in the case of a normally distributed variable. Kurtosis can also be viewed as indicating the peakedness of a distribution, with a sharp narrow peak in the center being associated with high values of kurtosis, or positive values of excess kurtosis.

4. Using statistical terminology, what does the volatility of a return mean?

- Volatility is often used synonymously with standard deviation in investments.

5. The covariance between the returns of two financial assets is equal to the product of the standard deviations of the returns of the two assets. What is the primary statistical terminology for this relationship?

- The covariance will equal the product of the standard deviations when the
6. What is the formula for the beta of an asset using common statistical measures?

\[ \beta = \frac{\text{Cov}(R_m, R_i)}{\text{Var}(R_m)} = \frac{\sigma_{im}}{\sigma_m^2} = \frac{\rho_{im} \sigma_i}{\sigma_m} \]

7. What is the value of the beta of the following three investments: a fund that tracks the overall market index, a riskless asset, and a bet at a casino table?

- +1, 0, 0 (assuming the casino bet is a traditional bet not based on market outcomes).

8. In the case of a financial asset with returns that have zero autocorrelation, what is the relationship between the variance of the asset's daily returns and the variance of the asset's monthly return?

- The variance of the monthly returns are \( T \) times the variance of the daily returns where \( T \) is the number of trading days in the month.

9. In the case of a financial asset with returns that have autocorrelation approaching +1, what is the relationship between the standard deviation of the asset's monthly returns and the standard deviation of the asset's annual return?

- In the perfectly correlated case the standard deviation of a multiperiod return is proportional to \( T \). In this case the annual vol is 12 times the monthly vol.

10. What is the general statistical issue addressed when the GARCH method is used in a time series analysis of returns?

- The tendency of an asset's variance to change through time.
1. Jane studies past prices and volume of trading in major public equities and establishes equity market neutral positions based on her forecasts of prices. Jane consistently outperforms market indices of comparable risk. Does the performance indicate that the equity market is informationally inefficient at the semistrong level?
   
   - The underlying equity market is informationally inefficient at both the weak level and the semi-strong level since any inefficiency at a “lower” level indicates inefficiency at a “higher” level because the underlying information sets are cumulative moving from weak to strong.

2. List two major factors that drive informational market efficiency through facilitating better investment analysis.
   
   - Assets will also tend to trade at prices closer to their informationally efficient values when there is easier access to better information.
   - Assets will also tend to trade at prices closer to their informationally-efficient values when there is less uncertainty about their valuation. In other words, when there are better valuation methods.

3. What does the modified Fisher equation express regarding minimal interest rate determinants?
   
   - It expresses the nominal interest rate as the combination of the after-tax real interest rate, \( r \), and the anticipated rate of inflation (\( \pi \)), with an adjustment for the income tax rate.

4. What does it mean to bootstrap a yield curve?
   
   - It is the process of recursively estimating spot rates using one or more zero-coupon bonds on the short end and coupon bonds on the medium- and long-term regions of the term structure.

5. In which theory of the term structure of interest rates do all bonds have the same expected return?
   
   - Unbiased expectations theory
6. What differentiates a relative pricing model from an absolute pricing model?
   - A relative pricing model prescribes the relationship between two prices. An absolute pricing model attempts to describe a value, or a price level, based on its underlying economic factors.

7. What makes a binomial tree a recombining tree?
   - A binomial tree with an upward movement followed by a downward movement that recombines with a pathway with a downward movement and an upward movement. A recombining binomial tree has $n+1$ possible final outcomes for an $n$ period tree, rather than $2^n$ outcomes.

8. What is the term used to describe a framework for specifying the return or price of an asset based on its risk, as well as future cash flows and payoffs?
   - Asset pricing model

9. What is the market portfolio and what is a market-weight?
   - The market portfolio is a hypothetical portfolio containing all tradable assets in the world.
   - The market weight of an asset is the proportion of the total value of that asset to the total value of all assets in the market portfolio.

10. What is an ex post excess return?
    - A realized return (an observed historical return) expressed as an excess return by subtracting the riskless return from the asset’s total return.
Chapter 6  Derivatives and Risk-Neutral Valuation

1. What two spot interest rates imply the value of a six-month forward contract from a six-month Treasury bill?
   - The current six-month and 12-month spot rates are needed to find the six-month forward contract for a six-month Treasury bill.

2. What is the relationship between a forward interest rate and its expected value at settlement under the unbiased expectations hypothesis and the liquidity premium hypothesis?
   - Under the unbiased expectations hypothesis, forward bond prices (whether implied by spot rates or observed in the forward price of forward contracts) are unbiased predictors of subsequent spot or cash market prices.

3. What are the carrying costs (and benefits) of physical inventory such as a commodity?
   - The carrying costs of physical inventory include interest (r) and storage (c), the benefit of physical inventory is the convenient yield.

4. Which is more likely to be more liquid, a forward contract or a futures contract?
   - A futures contract is more likely to be more liquid.

5. What does it mean when a future is marked-to-market?
   - When a future is marked-to-market, it means that the side of a futures contract that benefits from a price change receives cash from the other side of the contract (and vice versa) throughout the contract’s life.

6. What is maintenance margin?
   - A maintenance margin is collateral put up by the investor on an ongoing basis until the position is closed out.

7. What two assets form a long straddle?
   - A long call and long put with the same strike price.

8. What is the equation for put-call parity?
   - $Call + bond – Put = Underlying Asset$

9. What are the five variables that determine the price of an option on a non-dividend stock according to the Black-Scholes option pricing model?
   - The price of the underlying asset
   - The strike price
   - The return volatility of the underlying asset
   - The time to the option’s expiration
   - The riskless rate

10. What are the names of the first and second derivatives of an option price with respect to the price of the option’s underlying asset?
    - Delta and gamma
Chapter 7  Measures of Risk and Performance

1. What are the two main differences between the formula for variance and the formula for semivariance?
   - The semivariance uses a formula otherwise identical to the variance formula except that it only includes the negative deviations in the numerator and a smaller number of observations in the denominator.

2. What are the main differences between the formula for semistandard deviation and target semistandard deviation?
   - Target semivariance is similar to semivariance except that target semivariance substitutes the investor's target rate of return in place of the asset's mean return.

3. Define tracking error and average tracking error
   - Tracking error indicates the dispersion of the returns of an investment relative to a benchmark return, where a benchmark return is the contemporaneous realized return on an index or peer group of comparable risk.
   - Average tracking error simply refers to the average difference between an investment's return relative to its benchmark. In other words, it is the numerator of the information ratio.

4. What is the difference between value at risk and conditional value-at-risk?
   - Value at risk (VaR or VAR) is the loss figure associated with a particular percentile of a cumulative loss function. In other words, VaR is the maximum loss over a specified time period within a specified probability.
   - Conditional value-at-risk (CVaR), also known as expected tail loss, is the expected loss of the investor given that the VaR has been equaled or exceeded. CVaR will exceed VaR (if the overall maximum potential loss exceeds the VaR).

5. Name the two primary approaches for estimating the volatility used in computing value-at-risk.
   - Estimate the standard deviation (volatility) as being equal to the asset's historical standard deviation of returns
   - Estimate volatility based on the implied volatilities from option prices
6. What are the steps involved in directly estimating VaR from historical data rather than through a parametric technique?
   - Collect the percentage price changes
   - Rank the gains/losses from the highest to the lowest
   - Select the outcome (loss) reflecting the quantile specified by the VaR (e.g., for a VaR based on 95% confidence pick the observation with a loss larger than 95% of the other outcomes).

7. When is Monte Carlo analysis most appropriate as an estimation technique?
   - It is best used in difficult problems where it is not practical to find expected values and standard deviations using mathematical solutions.

8. What is the difference between the formulas for the Sharpe and Treynor ratios?
   - The Treynor ratio differs from the Sharpe ratio by the use of systematic risk rather than total risk in the denominator.

9. Define return on VaR.
   - Return on VaR (RoVaR) is simply the expected or average return of an asset divided by a specified VaR (expressing VaR as a positive number):

10. Describe the intuition of Jensen’s alpha.
    - Jensen’s alpha is a direct measure of the absolute amount by which an asset is estimated to outperform, if positive, the return on efficiently priced assets of equal systematic risk in a single-factor market model.
1. Provide two common interpretations of the investment term alpha.
   - Alpha refers to any excess or deficient investment return after the return has been adjusted for the time value of money (the risk-free rate) and for the effects of bearing systematic risk (beta).
   - Alpha can also refer to the extent to which the skill, information, and knowledge of an investment manager generates superior risk-adjusted returns (or inferior risk-adjusted return in the case of negative alpha).
   - Note that the first interpretation can include high returns from luck.

2. Provide two common interpretations of the investment term beta.
   - Beta is the proportion by which an asset's excess return moves in response to the market portfolio's excess return (the return of the asset minus the return of the riskless asset).
   - Beta refers to any of a number of measures of risk or the bearing of risk, wherein the underlying risk is systematic (shared by at least some other investments and usually unable to be diversified or fully hedged without cost) and is potentially rewarded with expected return without necessarily specifying that the systematic risk is the risk of the market portfolio.

3. Does ex ante alpha lead to ex post alpha?
   - Not necessarily. While ex ante alpha may be viewed as expected idiosyncratic return, ex post alpha is realized idiosyncratic return. Simply put, ex post alpha is the extent to which an asset outperformed or underperformed its benchmark in a specified time period. Ex post alpha can be the result of luck and/or skill. To the extent that an investor suffers bad luck, ex ante will not guarantee ex post alpha.

4. What are the two steps to an analysis of ex ante alpha using historical data?
   - An asset pricing model or benchmark must be used to divide the historical returns into the portions attributable to systematic risks (and the risk-free rate) and those attributable to idiosyncratic effects.
   - The remaining returns, meaning the idiosyncratic returns (i.e., ex post alpha), should be statistically analyzed to estimate the extent, if any, to which the superior returns may be attributable to skill rather than luck.
5. List the three major types of model misspecification in the context of estimating systematic risk.
   - Omitted (or misidentified) systematic return factors
   - Misestimated betas
   - Nonlinear risk-return relationships

6. What is the goal of an empirical investigation of abnormal return persistence?
   - To identify ex ante alpha

7. What is the term for investment products trying to deliver systematic risk exposure with an emphasis on doing so in a highly cost-effective manner?
   - Beta drivers (or passive indexers)

8. Does an analyst select a p-value or a significance level in preparation for a test?
   - The significance level. The p-value is the output of the statistical computations.

9. What is the relationship between selection bias and self-selection bias in hedge fund datasets?
   - Selection bias is a distortion in relevant sample characteristics from the characteristics of the population, caused by the sampling method of selection or inclusion used by the data manager. If the selection bias originates from the decision of fund managers to report or not to report their returns, then the bias is referred to as a self-selection bias.

10. What are two methods of detecting outliers in a statistical analysis?
    - Detection through visual inspection of plots
    - Ordered listings of the variables and regression residuals
Chapter 9  Natural Resources and Land

1. What is the difference between row cropland and permanent cropland?
   - Row cropland is annual cropland that produces row crops, such as corn, cotton, carrots, or potatoes from annual seeds. Permanent cropland refers to land with long-term vines or trees that produce crops, such as grapes, cocoa, nuts, or fruit.

2. What is the name of an option with no expiration date? Would that option typically be a European option or an American option?
   - A perpetual option. A perpetual option is an American option because if it were European it would never be able to be exercised and would be worthless.

3. What is the name of a lot of land that is vacant, approved for development but for which infrastructure construction has not commenced?
   - Paper lots

4. When and why are risk-neutral probabilities used?
   - Risk-neutral probabilities are often used in derivative pricing models. A risk-neutral probability may be useful to price derivatives even when investors are risk neutral because the derivative pricing model is being calculated relative to the price of the underlying asset and because the price of the underlying asset can often be viewed as already reflecting risk aversion.

5. What is the role of a Timberland investment management organization (TIMO)?
   - Timberland investment management organizations (TIMOs) provide management services to timberland owned by institutional investors such as pension plans, endowments, foundations, and insurance companies.

6. How do agency risks and political risks relate to institutional ownership of farmland?
   - An investor in farmland does not necessarily actively manage the crops. As such, the investor relies on payment from the lessee (the agent) that
operates the property. The risk that the lessor (the principal or investor) does not get paid by the agent is agency risk. Political risk is the economic uncertainty due to changes in government policy that may affect returns. The investor in farmland can be hurt by such political issues such as decreases in support payments or changes in land ownership rights.

7. Other than moneyness of the best available use, what are three factors regarding the uses that would cause a multiple use option to have a low value?

- High similarity among the profitability of each alternative
- Low volatility of the profitability of each alternative
- High correlation between the profitability of each alternative.

8. What is the effect of smoothed or stale asset values on the estimation of long-term average returns?
- For large samples there would typically be only a small difference between the mean based on stale returns and the mean based on true returns, so the use of stale valuation tends to have little effect on estimations of long-run returns.

9. What is the effect of smoothed or stale asset values on the estimation of historic return volatility?
- Smoothed asset values understate risk, therefore historical return volatility will be lower than unsmoothed risk.

10. What is contagion in a financial market?
- Contagion is the general term used in finance to indicate any tendency of major market movements, especially declines, to be transmitted from one financial market to other financial markets.
1. Explain the implications of Hotelling’s Theory on long-term commodity prices.
   - Hotelling’s theory states that prices of exhaustible commodities, such as various forms of energy and metals, should increase by the prevailing nominal interest rate—perhaps with a risk premium. Therefore, ignoring storage costs, expected spot prices of a commodity should be equal to the future value of the current spot price compounded at the nominal riskless rate plus a risk premium.

2. What are the three costs of carry that determine the price of a forward contract on a physical asset?
   - Storage costs, convenience yield (when viewed as a negative cost), and interest (financing) charges.

3. What is the name of the condition in which the expected spot price of a commodity in one year exceeds the one-year forward price of the commodity?
   - Normal backwardation

4. What is the primary reason that causes a commodity futures market to be in contango or backwardation?
   - The term structure takes on a positive or negative slope (contango or backwardated) based on carrying costs so that the risk-adjusted returns of spot positions and fully-collateralized futures positions will tend to be equal.

5. What is the name of the following quantity: the spot price of a commodity minus a forward price on the commodity?
   - The basis of the commodity contract

6. What is the primary reason that the forward price of an asset could be substantially smaller than the price generated by the cost-of-carry model?
   - The cost of carry model indicates a maximum forward price. When arbitrageurs cannot borrow a commodity without incurring expenses (other than the time value of money), it is possible that forward prices will be less...
than those implied by the cost of carry model.

7. In the context of analyzing the returns of futures contracts, what is excess return?

- The excess return of a futures contract is the return generated exclusively from changes in futures prices. The term “excess return” is used elsewhere in investment with a different meaning: the total return minus the riskless rate.

8. What is the definition of roll return that is earned through holding futures contracts?

- Roll return or roll yield from holding futures contracts is defined as the portion of the return of a futures position attributable only from the change in the contract’s basis through time.

9. List three important propositions regarding the accrual of roll return through holding futures contracts through time.

- Proposition 1: Roll return is not generated at the time that one position is closed and a new position is opened.
- Proposition 2: Realized roll return is not necessarily positive when markets are backwardated.
- Proposition 3: A position that generated a positive roll return does not indicate that the position’s total returns were superior (i.e., that there was alpha).

10. List four explanations that commodities should help diversify a portfolio of traditional assets.

- Unlike financial securities, commodities have prices that are not directly determined by the discounted value of future cash flows.
- More so than traditional asset prices, nominal commodity prices should be positively correlated with inflation largely because commodity prices form part of the definition and computation of inflation.
- Commodity price changes may be negatively correlated with the returns to stocks and bonds is that they may react very differently at different parts of the business cycle.
- Low or negative correlation between commodity prices and financial assets is based on commodities being a major cost of corporate production and thus consistent with lowered corporate profits.
Chapter 11 Other Real Assets

1. Name three factors that theory suggests should drive the extent to which natural resource price changes drive the performance of firms that process those natural resources.
   - The price elasticity of the demand for the good
   - The price elasticity of the supply for the good
   - The extent to which an operating firm is exposed to or has hedged its expenses and revenues (i.e., its profits).

2. To what extent have gold prices driven the equity values of gold mining firms based on data from the financial crisis in late 2008?
   - In the short run, it appeared that the operationally-intensive firms related to gold production were driven more by the volatility of the equity markets than by the volatility of gold prices.

3. Why are most listed MLPs in the U.S. involved in producing, processing and distributing energy products?
   - MLPs receive tax treatment predicated on adhering to regulations, including that at least 90% of the entities’ revenues come from specified businesses, such as energy (in the U.S.)

4. Do infrastructure assets need to have all seven of the elements that identify investable infrastructure? Why or why not?
   - No. There are no clear, hard lines separating infrastructure from other assets. Gray areas exist. Most infrastructure assets lack one or more of the seven elements, but they must contain many or most.

5. What is the difference between economic and social infrastructure? Provide an example of each.
   - Economic infrastructure assets are assets with economic value that is driven by the revenue they generate, typically with end users paying for the services provided by these assets. Examples include toll roads and bridges, railways, airports, and maritime terminals.
   - Social infrastructure assets are assets that have end users who are unable to pay for the services or that are used in such a way that it is difficult to determine how many services were used by each person. Examples include
schools, public roads, prisons, administrative offices, and other government buildings.

6. What is the primary defining difference between greenfield projects and brownfield projects?
   - A greenfield project is new, whereas a brownfield project is existing.

7. What is the term used to describe when a governmental entity sells a public asset to a private operator?
   - Privatization

8. Is investable intellectual property a public good or a private good?
   - Private good because the cash generated can be privately received and owned.

9. What are the four inputs to the simplified model of intellectual property values?
   - \( p \) = the probability of generating large positive cash flows
   - \( CF1 \) = Denote the first-year cash flows of the project
   - \( g \) = perpetual growth rate
   - \( r \) = discount rate

10. What is the empirical evidence on the very long-term annual financial returns of works of art?
    - The median real return to holding art over extended periods of time is 2.2%
1. List three potential disadvantages of real estate as an investment

- Heterogeneity
- Lumpiness
- Illiquidity

2. Name the three styles of real estate investing.

- Core
- Value Added
- Opportunistic

3. Provide an example of a common real estate investment for each of the three styles of real estate investing.

- Core real estate: A large office building or apartment complex
- Value-added real estate: Hotels, resorts, assisted-care living facilities, low-income housing, outlet malls, hospitals
- Opportunistic real estate: Development of raw property, redevelopment of property that is in disrepair, or acquisition of property that experiences substantial improvement in prospects through major changes, such as urban renewal.

4. Define mortgage

- A mortgage loan is a loan secured by property.

5. How do unscheduled principal payments affect the lender of a fixed-rate mortgage at different levels of market interest rates?

- Unscheduled principal payments cause a wealth transfer between the borrower and the lender, depending on the relationship between the mortgage’s interest rate and current market interest rates. When market rates are lower than the mortgage rate, unscheduled principal payments generally benefit the borrower and harm the lender. The lender receives additional cash flows that if reinvested at prevailing interest rates will earn less return than the mortgage offers. Borrowers can make unscheduled prepayments to reduce the total interest costs of their mortgage by an amount greater than the amount that they could earn from interest income
in the market. Thus, borrowers have an incentive to make prepayments on mortgages when interest rates decline below the mortgage's rate.

6. How does increased interest rate volatility affect the borrower of a fixed-rate mortgage in which the borrower can make unscheduled principal payments?
   - The option to prepay is a call option in which the mortgage borrower, much like a corporation with a callable bond, can repurchase its debt at a fixed strike price. Mortgage borrowers, like all call option holders on fixed income securities, benefit from increased interest rate volatility.

7. How does the interest rate risk of a variable-rate mortgage compare to that of a fixed-rate mortgage from the perspective of the lender?
   - A variable-rate type of mortgage to a lender protects the lender from the valuation fluctuations due to interest rate changes experienced with fixed-rate mortgages. To the extent that rates adjust quickly and completely, the variable-rate loan tends towards having little or no interest rate risk.

8. What is the “option” in an option adjustable-rate mortgage loan?
   - An option adjustable-rate mortgage loans (option ARM) is an adjustable-rate mortgage that provides borrowers with the flexibility to make one of several possible payments on their mortgage every month. The payment alternatives from which borrowers may select each month typically include an interest-only payment, one or more payments based on given amortization periods, or a prespecified minimum payment amount.

9. Are investors in commercial mortgages typically more or less concerned than investors in residential mortgages about: (a) rental income, (b) default risk, and (c) prepayment risk?
   - More concerned (residences are owner-occupied)
   - More concerned (residential mortgages are usually insured)
   - Less concerned (commercial loans are less subject to prepayment without penalty).
10. Describe the three major advantages of REIT ownership relative to direct real estate ownership.

- REITs provide management services in the selection and operation of properties.
- REITS provide liquid access to an illiquid asset class. Investors can add to or trim their exposure to real estate quickly and easily through purchase and sale of shares in REITs.
- REITs avoid taxation of income at the corporate level. This would be an advantage to an investor otherwise holding the real estate in a taxable corporation.
Chapter 13  Real Estate Equity

1. What is the complementary option type to financial options?
   - A real option. A real option is an option on a real asset rather than a financial security. The real option may be a call option to purchase a real asset, a put option to sell a real asset, or an exchange option involving exchange of nonfinancial assets.

2. What is the name of the point in a decision tree at which new information arrives?
   - An information node denotes a point in a decision tree at which new information arrives.

3. List the two major approaches to valuing private commercial real estate equity.
   - The income approach and valuations based on comparable sale prices

4. Define net operating income.
   - Net operating income (NOI) is a measure of periodic earnings that is calculated as the property’s rental income minus all expenses associated with maintaining and operating the property.

5. How does the numerator of a pretax discounting approach differ from the numerator of an after-tax discounting approach?
   - In the pretax discounting approach, pre-tax cash flows are used in the numerator. In the after-tax discounting approach the estimated after-tax cash flows are used in the numerator.

6. How does the equity residual approach to real estate valuation differ from a discounted cash flow approach applied to the assets of a real estate project?
   - The equity residual approach focuses on the perspective of the equity investor by subtracting the interest expense and other financing outflows due to mortgage holders (in the numerators) and by discounting the remaining cash flows using an interest rate reflective of the required rate of return on the equity of a leveraged real estate investment (in the denominator). The discounted cash flow approach discounts all cash flows from assets using a rate commensurate with the risk of the assets, not the equity.
7. What are the characteristics that distinguish syndications from other real estate investment vehicles?
   - Syndications are formed by a group of investors
   - Syndications are usually formed to undertake a particular real estate project.

8. A real estate project is estimated to offer a 10% after-tax rate of return when the depreciation allowed for tax purposes is equal to the true economic depreciation. In what direction would the expected rate of return change if the depreciation allowed for tax purposes were accelerated relative to the true economic depreciation, and why?
   - When depreciation for tax accounting purposes is accelerated in time relative to true economic depreciation, the after-tax return generally increases and exceeds the pretax return reduced by the stated income tax rate.

9. Summarize two major unresolved risk-related issues of equity REIT returns.
   - There is no consensus on whether the high correlation of REIT returns with equity markets returns is unique to public REITs or simply masked in private REIT and appraisal-based returns.
   - Another unresolved issue is whether owners of a publicly traded REIT receive an expected return that includes a risk premium for illiquidity.

10. Contrast the NAREIT and NCREIF real estate indices as measures of private commercial real estate performance.
    - The NAREIT US Real Estate Index Series is a family of REIT-based performance that covers different sectors of the U.S. commercial real estate space. The returns of the NAREIT indices are based on equity REIT transaction data.
    - The NCREIF Property Index (NPI) is a large popular, value-weighted index published quarterly and is based on unleveraged commercial-property appraisals.
1. List the three primary elements that differentiate a hedge fund from other investment pools.
   - A hedge fund is privately organized in most jurisdictions
   - A hedge fund usually offers performance-based fees to its managers
   - A hedge fund usually can apply leverage, use derivatives or utilize other investment flexibility

2. Describe consolidation in the hedge fund industry in recent years.
   - Consolidation in the hedge fund industry has been manifested in much higher percentages of assets being invested with the biggest funds. Institutional investors are showing a clear preference for the largest funds.

3. Define high-water-mark in the context of hedge fund fee computation.
   - The high-water mark (HWM) is the highest NAV of the fund on which an incentive fee has been paid.

4. How can managerial co-investing contribute to optimal contracting?
   - The idea is that by having their own money in the fund, through co-investing, managers will work hard to generate high returns and control risk, i.e. helping to align the interests of the hedge fund managers with that of the investors. Specifically, managerial co-investing may mitigate “gaming” emanating from large incentive fees.

5. What is an example of a perverse incentive caused by incentive fees?
   - If a fund experiences negative returns within a reporting period, the fund's managers may view the fund as likely to close, in which case the managers may have a strong incentive to take excessive risks in an attempt to recoup losses and stay in business. Even if the managers do not fear that the fund will close, if the fund's NAV falls substantially below its HWM, the managers may foresee no realistic chance of earning incentive fees in the near term unless the fund's risk is increased. Thus, an incentive fee structure may encourage enormous and inappropriate risk taking by the managers.
6. How does the annuity view of hedge fund fees differ from the option view of hedge fund fees?
   • The annuity view of hedge fund fees represents the prospective stream of cash flows from fees available to a hedge fund manager through the long term.
   • The option view of incentive fees uses option theory to demonstrate the ability of managers to increase the present value of their fees by increasing the volatility of the fund’s assets.

7. What is the primary difference between a fund of finds and a multistrategy fund?
   • In a multistrategy fund, there is a single layer of fees, and the submanagers are part of the same organization. The underlying components of a fund of funds are themselves hedge funds with independently organized managers and with a second layer of hedge fund fees to compensate the manager for activities relating to portfolio construction, monitoring, and oversight.

   • Short volatility exposure is any risk exposure that causes losses when underlying asset return volatilities increase.

9. When do convergent strategies generate profit?
   • Convergent strategies profit when relative value spreads move tighter, meaning that two securities move toward relative values that are perceived to be more appropriate. This tends to be associate with calm markets rather than turbulent markets.

10. What is fee bias?
    • Fee bias is when index returns overstate what a new investor can obtain in the hedge fund marketplace because the fees used to estimate index returns are lower than the typical fees that a new investor would pay.
Chapter 15  Macro and Managed Futures Funds

1. Distinguish discretionary fund trading from systematic fund trading.
   - Discretionary fund trading is where the decisions of the investment process are made directly by the judgment of human traders.
   - Systematic fund trading, often referred to as black-box model trading because the details are hidden in complex software, is where the ongoing trading decisions of the investment process are automatically generated by computer programs.

2. Describe the strategy of a global macro fund.
   - Global macro funds have the broadest investment universe: They are not limited by market segment, industry sector, geographic region, financial market, or currency and therefore tend to offer high diversification. They search diverse markets for perceived opportunities to achieve attractive returns.

3. What does market risk mean in the context of macro investing?
   - Market risk refers to exposure to directional moves in general market price levels. The definition is this context is not restricted to equity market risk.

4. Describe the strategy of a managed futures.
   - Managed futures refers to the active trading of futures and forward contracts on physical commodities, financial assets, and exchange rates. The purpose of the managed futures industry is to enable investors to receive the risk and return of active management within the futures market, while enhancing returns and diversification.

5. What is a commodity trading advisor (CTA)?
   - Commodity trading advisers (CTAs) are professional money managers who specialize in the futures markets.

6. List three questions in evaluating a systematic trading system.
   - What is the trading system, and how was it developed?
   - Why and when does the trading system work, and why and when might it not work?
   - How is the trading system implemented?
7. In a market trending upward, explain how the value of a simple moving average compares to the value of an exponential moving average?
   - Exponential moving averages place higher weights on more recent observations for typical values of the exponential weighting parameter. If prices are trending upward the exponential moving average will tend to recognize an upward trend more quickly and more profoundly due to the higher weight on the more recent (and higher) prices.

8. Does whipsawing tend to occur in a trending market or a sideways market?
   - Sideways markets where upward and downward movements tend to alternate.

9. What is a breakout strategy?
   - Breakout strategies focus on identifying the changes from a sideways market to the commencement of a new trend by observing the range of recent market prices (e.g., looking back at the range of prices over a specific time period).

10. List the six major potential risks of managed futures funds.
    - Transparency risk
    - Capacity risk
    - Liquidity risk
    - Model risk
    - Regulatory risk
    - Lack of trends risk
Chapter 16  Event-Driven Hedge Funds

1. List the three primary categories of single-strategy event-driven hedge funds.
   - Activist Hedge Funds
   - Merger Arbitrage Funds
   - Distressed Securities Funds

2. Why are event-driven hedge funds often characterized as selling insurance?
   - Event-driven hedge funds are often characterized as selling insurance because they purchase shares during the period near an event (such as a proposed merger announcement) and the eventual resolution of uncertainty regarding the event. This act may be viewed as providing event risk insurance to the equity market.

3. Why would activist hedge fund managers need to understand corporate governance?
   - Corporate governance is central to the activist hedge fund’s investment strategy as it is the means to assert change or threat of change into the management of the target corporation.

4. List the five dimensions of shareholder activists.
   - Financial versus social activists
   - Activists versus pacifists
   - Imitators versus followers
   - Friendly versus hostile activists
   - Active activists versus passive activists

5. What is the economic term for a person or entity who allows others to pay initial costs and then benefits from those expenditures?
   - A free rider

6. Is Form 13F a U.S.-required form that is targeted towards activist hedge funds?
   - No, Form 13F is a required quarterly filing of all long positions by all U.S. asset managers with over $100 million in assets under management, including hedge funds and mutual funds, among other investors.
7. What is the difference between a spin-off and a split-off?
   • A spin-off occurs when a publicly traded firm splits into two publicly traded firms, with shareholders in the original firm automatically becoming shareholders in the new firm.
   • A split-off occurs when Company A splits off (divests) Company B and the investors in Company A have the choice to retain their ownership in Company A or exchange their shares in A for shares in the newly created firm (Company B).

8. What are the positions utilized in a traditional merger arbitrage strategy?
   • Traditional merger arbitrage generally uses leverage to buy the stock of the firm that is to be acquired and sell short the stock of the firm that is to be the acquirer in a stock-for-stock merger.

9. What is financing risk in the context of an event-driven investment strategy?
   • Financing risk is the economic dispersion caused by failure or potential failure of an entity, such as an acquiring firm, to secure the funding necessary to consummate a plan such as an acquisition.

10. How is short selling of equity in a distressed firm similar to an option position?
    • Shares in highly leveraged firms resemble call options, therefore short-selling distressed equities is analogous to writing naked call options on the firm’s assets and generates a negatively skewed return distribution. An investor has a naked option position when the investor is short an option position for which the investor does not also have a hedged position.
Chapter 17  Relative Value Hedge Funds

1. Describe the positions utilized in a classic convertible bond arbitrage trade.
   - The classic convertible bond arbitrage trade is to purchase a convertible bond that is believed to be undervalued and to hedge its risk using a short position in the underlying equity.

2. What are the three terms used to describe convertibles bonds differentiated by whether the implicit option in the bond is in-the-money, at-the-money or out-of-the-money?
   - Equity-like convertible, hybrid convertibles, busted convertibles, (respectively)

3. What is the difference between delta and theta in measuring the price sensitivity of an option?
   - Delta is the change in the value of the option with respect to a change in the value of the underlying, whereas theta is the change in the value of the option with respect to the time to expiration of the option (i.e., passage of time).

4. What is the term that describes when additional equity is issued at below market values causing the per share value of the holdings of existing shareholders to be diminished?
   - Dilution

5. List the components of the returns of a traditional convertible arbitrage strategy.
   - Convertible Bond Arbitrage Income:
     i. (Bond Interest − Stock Dividends + Short Stock Rebate − Financing Expenses)

   - Convertible Bond and Stock Net Capital Gains and Losses:
     i. (Capital Gains on Stock and Bond − Capital Losses on Stock and Bond)
6. What is the key difference between a volatility swap and a variance swap?
- Variance swaps are forward contracts wherein one party agrees to make a cash payment to the other party linearly based on the realized variance of a price or rate in exchange for receiving a predetermined cash flow. A volatility swap mirrors a variance swap except that the payoff of the contract is linearly based on the standard deviation of a return series rather than the variance.

7. What is the primary term for financial arrangements that protect an investor's portfolio from tail risk?
- Portfolio insurance

8. What are the differences between duration, modified duration and effective duration?
- Duration is a measure of the sensitivity of a fixed-income security to a change in the general level of interest rates. Traditional duration may also be viewed as a weighted average of the longevity of the cash flows of a fixed-income security.
- Modified duration is equal to traditional duration divided by the quantity \[1 + \left(\frac{y}{m}\right)\], where \(y\) is the stated annual yield, \(m\) is the number of compounding periods per year, and \(y/m\) is the periodic yield. With continuous compounding, \(m\) is infinity, and traditional duration equals modified duration. Modified duration scales traditional duration to adjust for the compounding assumption used in the interest rate computations.
- Effective duration is a measure of the interest rate sensitivity of a position that includes the effects of embedded option characteristics. As such it is not generally equal to the weighted average longevity of the cash flows.

9. What is the difference between a yield curve and a term structure of interest rates?
- The yield curve plots yields to maturity of coupon bonds, while the term structure of interest rates generally is used to denote actual or hypothetical yields of zero-coupon bonds.
10. For what type of interest rate shift is a duration-neutral position best protected?

- A duration-neutral position is protected from value changes due to shifts in the yield curve that are small (infinitesimal), immediate (instantaneous), and parallel (additive).
1. Name the three major types of equity hedge funds and describe their typical systematic risk exposures.
   - Equity long/short hedge funds
   - Equity market-neutral fund
   - Short-bias funds

2. Describe the role of a market maker in the context of taking and/or providing liquidity in a market with anxious traders.
   - A market maker is a market participant that offers liquidity, typically both on the buy side by placing bid orders and on the sell side by placing offer orders. A market maker meets imbalances in supply and demand for shares caused by idiosyncratic trade orders from anxious traders. Typically, the market maker’s purpose for providing liquidity is to earn the spread between the bid and offer prices by buying at the bid price and selling at the offer price.

3. Why is an empirical test of informational market efficiency a test of joint hypotheses?
   - The test is a joint hypothesis of the appropriateness of the particular model of returns (in determining what constitutes an abnormal return) and a test of whether a particular investment has generated statistically significant abnormal returns.

4. Define standardized unexpected earnings, and describe how the measure is used.
   - Standardized unexpected earnings (SUE) is a measure of earnings surprise, with some measure of unexpected earnings in the numerator and some measure of earnings volatility in the denominator.

5. What have empirical studies generally concluded about the relationship between the net stock issuance of a firm and the subsequent returns of the firm’s shareholders?
   - There is evidence that positive or negative net stock issuance is one of the most profitable anomalies. Companies that issue large amounts of new shares, such as more than 20% of the shares currently outstanding, frequently see their stock price substantially underperform the market.
6. What is the name of the measure that describes managerial skill as the correlation between managerial return predictions and realized returns?
   - The information coefficient, which measures managerial skill as the correlation between managerial return predictions and realized returns.

7. Explain how limits to arbitrage prevent markets from being perfectly efficient.
   - There is a limit to the risk that an arbitrager can tolerate and/or is allowed to take, which provides a limit on the level of arbitrage activity by a single manager.
     i. Managers who want to be successful in the long run must limit the risk of their fund, especially in periods where their strategy might be out of favor (e.g., value managers in the late 1990s/early 2000). This might prevent them from taking aggressive bets.
     ii. Market structures may prevent successful arbitrage in some cases. For example, institutional investors may be too large to participate in micro-cap stocks.
     iii. Limits on short selling could be a limit to arbitrage. Some countries do not allow short selling altogether, recent IPOs or spin-offs may not have shares available to be shorted, or some shares may be temporarily unavailable for borrowing when the demand to sell the shares short exceeds the supply of borrowable shares.

8. What is the name of the modeling approach that combines the factor scores of a number of independent anomaly signals into a single trading signal?
   - Multiple-factor scoring models, which combine the factor scores of a number of independent anomaly signals into a single trading signal.

9. Consider a skilled manager implementing a pairs trading strategy. What is the concern that tends to limit the size of the positions that the manager might take in attempting to increase expected alpha?
   - The limits to arbitrage, which refers to the potential inability or unwillingness of speculators, such as pairs traders, to hold their positions without time constraints or to increase their positions without size constraints. Very large positions with high degrees of leverage increase the probability of financial ruin and the inability to survive short-term displacements.
10. What distinguishes mean neutrality from variance neutrality in equity market-neutral strategies?
   - Mean neutrality is when a portfolio is shown to have zero beta exposure or correlation to the underlying market index. In other words, when the market experiences a move in one direction, mean-neutral portfolios are no more likely to move in the same direction as in the opposite direction. Variance neutrality is when portfolio returns are uncorrelated to changes in market risk.
Chapter 19  Funds of Hedge Funds

1. List the four functions of fund of funds management.
   - Strategic and Manager Selection
   - Portfolio Construction
   - Risk Management and Monitoring
   - Due Diligence

2. Name four benefits to investing in funds of funds that may lead to higher net returns to limited partners without causing higher risk.
   - Economies of Scale
   - Informational Advantage
   - Access to Certain Managers
   - Negotiated fees

3. Name five benefits to investing in funds of funds that may lead to lower investment risk to limited partners without sacrificing expected return.
   - Diversification
   - Liquidity
   - Regulation
   - Currency Hedging
   - Educational Role

4. Describe the double layer of fees in funds of funds.
   - FoF managers effectively pass on to their investors all fees charged by the underlying hedge funds in their portfolios, while also charging an extra set of fees for their own work as well as for an additional layer of service providers.
   - Many FoFs charge a 1% management fee and a 10% performance fee on top of the average underlying hedge fund management fee of 2% and incentive fee of 20% for the hedge funds.

5. In theory, how would the volatility of an equally-weighted portfolio of 16 uncorrelated, zero-beta and equally-risky funds compare to the volatility of a single such fund?
   - The volatility of an equally-weighted portfolio of sixteen uncorrelated, zero-beta and equally-risky funds would be reduced (as shown in Equation
21.1) through division by the square root of the number of funds (16). Division by four generates a 75% risk reduction.

6. Why might the incentive fees of a multistrategy fund differ substantially from the incentive fees of an otherwise similar fund of funds even if the stated fees are equal?
   - Multistrategy funds net the profits and losses of all underlying investment to determine any profit on which an incentive fee is paid. Funds of funds structures pay out incentive fees to each underlying manager separately, meaning that the profitable fund managers receive full incentive fees but there is no offset to aggregate incentive fees due to underlying managers with losses.

7. Why might the operational risks of a multistrategy fund differ substantially from the operational risks of a fund of funds?
   - While funds of funds diversify operational risk across 10 to 20 independent managers and organizations, a multistrategy fund has a single operational infrastructure. Market risk may also be a concern, as a catastrophic loss in even one of the multistrategy fund's underlying strategies may sink the entire fund. Conversely, the failure of one of a fund of funds' 20 managers may subject investors to only a 5% loss and not affect the fund's other investments.

8. What is a seeding fund?
   - Seeding funds, or seeders, are funds of funds that invest in newly created individual hedge funds, often taking an equity stake in the management companies of the newly-minted hedge funds.

9. What investment pools in the United States and Europe provide liquid access of investors to alternative investment strategies?
   - UCITS in the EU and some '40 Act funds in the U.S.

10. List the four major categories of funds of funds.
    - Market-Defensive Fund of Funds
    - Conservative Fund of Funds
    - Strategic Fund of Funds
    - Diversified Fund of Funds
Chapter 20  Private Equity Assets

1. What are the three major forms of private debt introduced in the chapter?
   - Mezzanine debt
   - Distressed debt
   - Leveraged loans

2. List major contrasts between venture capital and buyouts.
   - Whereas VC funds target nascent, start-up companies, buyouts target more established and mature companies.
   - VC is necessary to get a prototype product or service out the door. In a buyout, the capital is necessary not for product development but to take the company private so that it can concentrate on maximizing operating efficiencies. Venture capital relies on new technology or innovation; buyouts look to see where they can add operating efficiencies or expand product distribution
   - A VC firm typically acquires a substantial but minority position in the company. Control is not absolute. Conversely, in a buyout, all of the equity is typically acquired, and control is absolute.
   - Venture capital and buyout firms target different internal rates of return. While both are quite high, not surprisingly, VC targets are higher. The reason is simple: There is more risk funding a nascent company with brand-new technology than an established company with regular and predictable cash flows.

3. Identify three major methods of executing an exit from VC.
   - Conducting an initial public offering of the company’s securities
   - Sale to acquiring firms
   - Leveraged recapitalization (where the proceeds from the debt are paid to the venture capitalist

4. What differentiates the angel investing stage of VC from the seed stage of venture capital financing?
   - Angel investing is the earliest stage of venture capital. Angel investors are often friends and family, and they fund an entrepreneur’s ideas. At this stage, the angel investor is funding an idea. There is no formal business plan, no management team, no product, no market analysis, just an idea. This
phase usually raises $50,000 to $500,000.

- The seed stage is the first stage where institutional investors commit their capital into a venture and is typically prior to having established the viability of the product. At this stage, the business plan is completed, some members of management have been assembled, and the entrepreneur and management team have performed market analysis and addressed parts of the business plan. This phase usually raises $1 million to $5 million.

5. What is a compound option and how do compound options relate to VC?
   - A compound option is an option on an option. It allows its owner the right but not the obligation to pay additional money at some point in the future to obtain an option. As this relates to VC financing, the owner of this option is able to delay further capital until new information has arrived or the entrepreneur has reached a new milestone.

6. What is a springing board remedy?
   - A springing board remedy occurs when the investor designates a majority of the defaulting issuer’s board of directors.

7. What is the primary difference between a management buy-in LBO and a management buyout LBO?
   - A management buyout is led by the target firm’s current management, whereas a management buy-in is led by an outside management team. Thus the buy-out retains all or most top management while the buy-in replaced all or most top management.

8. Describe the evolution of the buyout market.
   - 1970s – KKR was founded with $3 million of its own funds to invest.
   - 1980s – the rise of a key element of the growth in buyouts: financing the buyouts using bonds with low credit ratings (i.e., junk bonds). Buyouts reached a peak in 1989, when KKR bought the giant food conglomerate RJR Nabisco for $31 billion.
   - 1990s – buyout activity declined for two reasons: (1) the recession of 1990-91 pushed credit spreads to high levels which lowered the attractiveness of junk bond financing for buyouts, and (2) in 1998, the Russian government defaulted on its sovereign bonds, which also sent credit spreads upward.
   - 2000s – started quietly for the buyout market before availability of credit created a boom from 2003 to 2007 before falling off after the liquidity bubble burst in late 2007.
• 2010s – buyout activity resumed growth, but have yet to reach pre-crisis levels of 2006 and 2007.

9. What are the two primary conflicts of interest that emanate from the potentially lucrative compensation schemes offered to exiting management teams in a management buy-in?
   • Incumbent management has a strong incentive to resist any buyout attempt that displaces them as managers if the buyout does not provide them with generous compensation even if the buyout benefits shareholders.
   • Incumbent management has a strong incentive to encourage buyouts that offer them generous compensation even if the buyout benefits shareholders.

10. List the five general categories of LBOs designed to create value.
   • Efficiency buyouts
   • Entrepreneurship stimulators
   • The overstuffed corporation
   • Buy-and build strategies T
   • Turnaround strategies
Chapter 21  Private Equity Funds

1. Fill in the blanks of the following sentence using the terms PE fund, PE firm, and underlying business enterprises: A _______ serves as the general partner to a _______ that invests its money in ___________.
   - PE firm
   - PE fund
   - underlying business enterprises

2. What two roles do PE firms play in a partnership and how do carried interest and management fees line up with those two roles?
   - General partner (GP) to Partnership
   - Investment Adviser

3. What are the five stages of the lifecycle of a VC fund?
   - Fundraising
   - Sourcing Investments
   - Investing
   - Operations and Management
   - Windup and Liquidation

4. What are the three phases in the relationship between LPs and GPs of PE funds?
   - Entry and establish
   - Build and harvest
   - Decline, exit, or transition to new managers

5. Describe bad-leaver and good-leaver clauses in PE partnerships.
   - Bad-leaver clause: a for-cause removal of the GP that, if exercised, causes investments to be suspended until a new fund manager is elected or, in the extreme, the fund is liquidated.
   - Good-leaver clause: enables investors to cease additional funding of the partnership with a vote requiring a qualified majority. This “without-cause” clause provides a clear framework for shutting down a partnership that is not working, or when confidence is lost.

6. What is a club deal?
   - A club deal is when two or more LBO firms work together to share costs, present a business plan, and contribute capital to the deal.
7. Discuss the following statement: empirical evidence indicates that investors in listed BDCs are subject to greater return volatility and enjoy less diversification benefits than investors in private equity that is not publicly traded.

- Since private equity lacks liquid market price data, such empirical evidence is likely formed by comparing liquid market prices with illiquid pricing data. Analysis of prices for private equity based on illiquid trading data or professional judgment can be argued to understate risk. Returns based on illiquid trading data or professional judgment are likely smoothed and therefore analysis based on those data should be expected to underestimate true return volatilities and correlations.

8. Why might a PE fund of funds be especially appropriate for an investor new to PE?

- They can provide the necessary resources and address the information gap for inexperienced PE investors through the expertise in due diligence, monitoring, and restructuring.

9. What is the primary difference between a traditional PIPE and a toxic PIPE?

- Traditional PIPEs allow investors to buy common stock at a fixed price. A toxic PIPE is a PIPE with adjustable conversion terms that can generate high levels of shareholder dilution in the event of deteriorating prices in the firm's common stock.

10. Describe six differences between typical PE and hedge fund fees.

- Hedge fund incentive fees are front loaded. PE fund fees tend to be collected at the termination of deals.
- Hedge fund incentive fees are based on changes in net asset value whether the gains are realized or unrealized. PE fund fees are based on realized values of exited positions.
- Hedge fund incentive fees are collected on a regular basis, either quarterly or semiannually. PE fund incentive fees tend to be collected at the time of an event, such as exit.
- Investor capital does not need to be returned first to collect incentive fees in a hedge fund. PE funds typically do not distribute incentive fees until the original investor capital has been repaid.
- Hedge funds often have no provisions for the clawback of management or incentive fees. PE funds typically have clawback provisions requiring the return of fees on prior profits when subsequent losses are experienced.
- Hedge funds rarely have a preferred rate (hurdle rate) of return (e.g., 6%)
that must be exceeded before the hedge fund manager can collect an incentive fee. Most PE funds have a hurdle rate.
1. What is a fulcrum security and how might it facilitate a private equity fund strategy?
   • A fulcrum security is the senior-most debt security in a reorganization process that is not paid in full with cash but rather is the security that is most likely to be repaid with equity in the reorganized firm.

2. Explain how and why an increase in the portion of loans that are covenant-lite will affect default rates and the magnitude of losses given default.
   • An increase in covenant-lite loans will likely lead to higher default rates and lower recovery rates. Covenants protect the creditor’s interests by requiring the borrower to comply with certain requirements, such as maintaining a debt service coverage ratio, or prohibit them from certain actions, such as increasing their debt load or issuing higher seniority debt. Not having these covenants could cause the company to over-lever and, in the event of a default, could increase risk to the creditor in the bankruptcy process.

3. Briefly describe mezzanine financing.
   • Mezzanine debt derives its name from its position in the capital structure of a firm: between the ceiling of senior secured debt and the floor of equity. Mezzanine finance defies generalization. Mezzanine financing is highly customized, often focused on equity appreciation while still maintaining the characteristics of (usually, high yield) debt, i.e. principal and interest payments.

4. Does mezzanine debt with an equity kicker exhibit the J-curve return pattern of private equity? Why or why not?
   • With a mezzanine fund, the J-curve effect is not a factor. One of the distinct advantages of mezzanine financing is its immediate cash-on-cash return. Mezzanine debt bears a coupon that requires twice-yearly interest payments to investors. As a result, mezzanine financing funds can avoid the early negative accounting returns associated with venture capital or leveraged buyout funds.
5. What would be the primary justification for believing that the use of mezzanine financing can lower a firm’s weighted average cost of capital?
   - The justifications for advantages to mezzanine debt are based on inefficiencies and imperfections in the capital markets for the size companies that tend to utilize mezzanine financing.

6. How does mezzanine debt tend to differ from high-yield bonds and leveraged loans in seniority, term, and liquidity?
   - Seniority: Mezzanine debt is usually lower in seniority than high-yield bonds and, especially, leveraged loans
   - Term: Mezzanine debt is usually 4-6 years, similar to leveraged loans but shorter than high-yield bonds
   - Liquidity: Mezzanine debt has minimal liquidity, especially compared the relatively high liquidity of leveraged loans.

7. By what standards or measures is distressed debt usually distinguished from non-distressed debt?
   - Distressed debt is often defined as debt that has deteriorated in quality since issued and that: has a market price less than half its principal value, yields 1,000 or more basis points over the riskless rate, or has a credit rating of CCC (Caa) or lower.

8. Provide two major sources of distressed debt.
   - A company with a deteriorated financial condition
   - Debt issued by a private equity firms or leveraged buyout firm

9. What is the primary distinction between Chapter 7 bankruptcy and Chapter 11 bankruptcy in the U.S.?
   - Chapter 7 bankruptcy is where the company is no longer viewed as a viable business and the assets of the firm are liquidated. Chapter 11 bankruptcy attempts to maintain operations of a distressed corporation that may be viable as a going concern.

10. Who is the initial investor in debtor-in-possession financing?
    - Senior and subordinated creditors
Chapter 23  
Introduction to Structuring

1. What is the similarity between a structured product and the capital structure of an operating firm?
   - Both are used to structure risk (and longevity). The capital structure of an operating firm is used to structure risk in the business enterprise, whereas the structure product is used to structure risk of a financial portfolio.

2. What is the primary role of structuring in an economy?
   - The primary economic role of structured products is usually market completion – making available a broader spectrum of investment opportunities.

3. How could a financial market become less complete?
   - By having a reduction in the number of unique investment opportunities or an increase in the number of uncertainties facing investors.

4. From an investor’s viewpoint, what is the difference between owning a tranche in a sequential-pay CMO and a tranche in a targeted amortization class CMO in a rising interest rate environment?
   - In a sequential-pay CMO the order or prepayment does not change: the senior most tranche is paid first, the next senior tranche second, and so on. In the targeted amortization class CMO the tranches receive payments in accordance with a more complex priority that changes with major changes in prepayment speeds such that various tranches may experience substantial increases or decreases in seniority in receiving cash distributions.

5. What is the extension risk and contraction risk of the PO tranche to a CMO?
   - Typically, principal-only tranches are positively exposed to extension risk in that their values decline when their payments extend in longevity (i.e., prepayments slow) since PO holders receive no coupons. Conversely, principal-only tranches are negatively exposed to contract risk; typically, as interest rates decline, the speed of prepayments accelerates and the values of PO’s rise.
6. What are the two major types of investor motivations to investing in a tranche of a CMO rather than investing directly in mortgages similar to the mortgages of the CMO’s collateral pool?
   - Risk management: Investors may be better able to manage risk through structured products (e.g., by selecting tranches with specific longevities)
   - Return enhancement: Investors may be better able to establish positions that will enhance returns if the investor’s market view is superior

7. Name two prominent time periods when structured mortgage products are believed to have increased systemic risk and led to a financial crisis? What is the major difference between the underlying economic events that led to the losses in these two crises?
   - 1994 and 2007
   - In 1994, the combination of extended maturities and higher interests caused market values of many CMO tranches to collapse based more on interest rate risk than default risk. In 2007, the creditworthiness of the CMOs caused the market values of many tranches to fall substantially. In both situations, the dramatic fall in market values caused investors and institutions to liquidate their positions which exacerbated the crisis by lowering CMO valuations even further.

8. In Merton’s structural model, how is debt with default risk viewed as having exposure to a put option?
   - The risky debt of a levered firm can be viewed as being equivalent to owning a riskless bond and writing a put option that allows the stockholders to put the assets of the firm to the debt holders without further liability (i.e., in exchange for the debt).
   - Debt of Levered Firm = + Riskless Bond – Put Option on Firm’s Assets

9. In Merton’s structural model, what is the conflict of interest between stockholders and debt holders with regard to asset risk and how does this conflict relate to structured products?
   - There is an inherent conflict between the stockholders and the bondholders with regard to the optimal level of risk for the firm’s assets. The equity holders, with their long position in a call option prefer higher levels of risk, especially when the value of the firm’s assets is near or below the face value of the debt. Conversely, bond holders prefer safer projects and reduced asset volatility as seen through their short position in a put option. The conflict of interest may be viewed as a zero sum game in which
managers can transfer wealth from bondholders to stockholders by increasing the risk of the firm’s assets (or vice versa).

- The conflict of interest between stockholders and bondholders in the capital structure of a firm is analogous to the case of structured products with multiples tranches. The manager of the collateral pool can cause wealth transfers between tranches by altering the risk of the assets. In most structures, high levels of asset risk benefit junior tranche holders at the expense of senior tranche holders.

10. What are three major option strategies that resemble the ownership of a mezzanine tranche?
    - A collar position, a bull call spread and a bull put spread
Chapter 24  Credit Risk and Credit Derivatives

1. Why is the market for fixed income securities other than riskless bonds often termed the spread product market?
   • Because other U.S. dollar-denominated fixed income products, such as bank loans, high-yield bonds, investment-grade corporate bonds, or emerging markets debt, trade at yields containing a credit spread relative to U.S. Treasury securities.

2. What are the three factors that determine the expected credit loss of a credit exposure?
   • Probability of default (PD), which specifies the probability that the counterparty may fail to meet its obligations
   • Exposure at default (EAD), which specifies the nominal value of the position that is exposed to default at the time of default
   • Loss given default (LGD), which specifies the economic loss in case of default

3. What is the relationship between the recovery rate and the loss given default?
   • Loss given default (LGD) = (1-R), where R = the recovery rate

4. List the two key characteristics that can make risk-neutral modeling a powerful tool for pricing financial derivatives.
   • The risk-neutral modeling approach provides highly simplified and easily tractable modeling, and
   • Often derivative prices generated by risk-neutral modeling must be the same as the prices in an economy where investors are risk-averse.

5. List the four stages in the evolution of credit derivative activity.
   • The first, or defensive, stage, which started in the late 1980s was characterized by ad hoc attempts by banks to lay off some of their credit exposures. The second stage, which began about 1991 was the emergence of an intermediated market in which dealers applied derivatives technology to the transfer of credit risk and investors entered the market to seek exposure to credit risk. The third stage was maturing into resembling other forms of derivatives with major regulatory
• guidance. Dealers began warehousing risks and running hedged and diversified portfolios of credit derivatives. The fourth stage centered on the development of a liquid market.

6. What is the primary difference between a total return swap on an asset with credit risk and a credit default swap on that same asset?
  • In the case of a credit default swap, the credit protection buyer makes fixed payments, known as the swap premium, to the credit protection seller. In the case of a total return swap, the credit protection buyer makes payments to the credit protection seller based on the total market return of the underlying asset. The total market return is comprised of any coupon payments and any change in the underlying bond’s market price.

7. List the seven kinds of potential trigger events in the standard ISDA agreement.
  • Bankruptcy
  • Failure to pay
  • Restructuring
  • Obligation acceleration
  • Obligation default
  • Repudiation/moratorium
  • Government intervention.

8. How can one party to a CDS terminate credit exposure (other than counterparty risk) to a CDS without the consent of the counterparty to the CDS?
  • By entering an offsetting position, by assigning the contract to a dealer or other approved counterparty (with permission of the original counterparty, or by reaching an agreement with the original counterparty to mutually terminate the contract.

9. If a speculator believes that the financial condition of XYZ Corporation will substantially deteriorate relative to expectations reflected in market prices, should the speculator purchase a credit call option on a spread or a price?
  • Spread. Deterioration in credit increases credit spreads and lowers risky bond price.
10. What CDS product should an investor consider when attempting to hedge the credit risk of a very large portfolio of credit risks rather than the hedge a few issues?
   • CDS index products
Chapter 25  CDO Structuring of Credit Risk

1. How would the exposure to credit risk of the most senior and most junior tranches of a CDO tend to compare to the average credit risk of the collateral pool?
   • In one sense the tranches should have credit risks that are dispersed above and below the credit risk of the underlying assets. The amount of credit risk in the senior tranches would be lower than in the collateral pool. The credit risk of the most junior tranche would be higher than the credit risk of the collateral pool. Diversification and credit enhancements make the relationship complex.

2. List two major economic motivations to the CDO structuring of non-investment grade debt.
   • Risk management: Investors may be better able to manage risk through structured products by selecting tranches that match their preferences.
   • Return enhancement: Investors may be better able to establish positions that will enhance returns if the investor's market view is superior.

3. What is the WARF of a portfolio?
   • The weighted average rating factor (WARF) as described by Moody's is a numerical scale from 1 (for AAA-rated credit risks) to 10,000 (the worst credit risks) that reflects the estimated probability of default.

4. What is the primary difference between the motivations of creating a balance sheet CDO and creating an arbitrage CDO?
   • Balance sheet CDOs are created to assist a financial institution in divesting assets from its balance sheet. Arbitrage CDOs are created to attempt to exploit perceived opportunities to earn superior profits through money management.

5. What is the primary difference between a cash-funded CDO and a synthetic CDO?
   • The distinction focuses on whether the SPV obtains the risk of the portfolio using actual (cash) holdings of assets or through derivative positions. A cash-funded CDO holds the portfolio of risky securities as collateral for the trust, whereas the synthetic CDO obtains the risk exposure through the use of a credit derivative.
6. Is subordination an internal or external credit enhancement?
   - It is an internal credit enhancement involving the structure of the product.

7. How many tranches can be in a single-tranche CDO?
   - In a single-tranche CDO, the CDO may have multiple tranches but the sponsor issues (sells) only one tranche from the capital structure to an outside investor.

8. Suppose that the total value of the collateral pool of a CDO remains constant but the riskiness of the pool increases. If the value of the senior-most tranches decreases, what should happen to the combined value of the other tranches?
   - The combined value of the other tranches will increase.

9. What is the explanation, based on option theory, as to why the junior-most tranche of a CDO would fall in value when the collateral pool of assets becomes more diversified?
   - The junior-most tranche of a CDO is similar to a long call position on the collateralized asset. As the collateral pool of assets becomes more diversified, thus less risky, the value of the long call position declines due to its negative vega.

10. What is the primary purpose of using a copula approach to analyze a CDO?
    - To ascertain the risks of tranches due to potential default risk in the CDO portfolio.
1. List the six primary types of structured product wrappers.
   • Over-the-counter contracts (OTC), medium term notes/certificates/warrants, funds, life insurance policies, structured deposits, Islamic wrappers

2. What can cause the after-tax rate of return of a product with tax deferral and tax deduction to be higher than the after-tax return of an otherwise identical product with tax deferral only?
   • When the income tax rate at withdrawal (e.g., retirement) is lower than the income tax rate of the investor when the contribution was made.

3. What does a participation rate indicate in a structured product?
   • The participation rate indicates the ratio of the product’s payoff to the value of the underlying reference, asset or index.

4. How does a long position in an up-and-in call differ from a short position in a down-and-out put?
   • Both option positions are bullish, but the up-and-in call has a payoff that is positively correlated with the underlying asset only in the region above the strike price and barrier, while bullish payoff region of the short put position is below its strike price. The up-and-in call has unlimited profit potential while the profit potential of the short position in the down-and-out put is limited to the premium received.

5. What is the name of an option that offers a payout in a currency based on the numerical value of an underlying asset with a price that is expressed in another currency?
   • A quanto option is an option with a payoff based on one currency using the numerical value of the underlying asset expressed in a different currency.

6. What simple option portfolio mimics the payout to an absolute returns structured product?
   • A long position in an at-the-money straddle, which generate profits via large movements in either direction (of the price of the underlier).
7. List the three major approaches to estimating the value of a highly complex structured product.
   - Partial differential equation approach (PDE approach)
   - Simulation, such as Monte Carlo simulation
   - The building blocks approach (i.e., portfolio approach)

8. Describe the difference between an analytical solution and a solution estimated with numerical methods.
   - Analytical solutions such as the Black-Scholes model are exact, because the model can be solved using a finite set of common mathematical operations. A solution estimated with numerical methods is not exact. It utilizes a potentially complex set of procedures to form an estimate.

9. In an informationally efficient market, can a structured product be engineered to offer both any payoff diagram shape and any payoff diagram level?
   - No. The level drives whether the opportunity has a positive, negative or zero net present value.

10. Briefly summarize the evidence on whether the offering prices of structured products are over-priced or underpriced relative to the values of similar exposures composed of market-traded products.
    - Based on evidence, the offering prices of some structured products are over-priced to the values of similar exposures composed of market-traded products.
    - Deng and others (2011) find that the fair price of ARBNs “is approximately 4.5% below the actual issue price on average”.
    - McCann and Luo estimate that “between 15% and 20% of the premium paid by investors is a transfer of wealth from unsophisticated investors to insurance companies and their sales forces”.

APPLICATIONS

INTRODUCTION

This workbook discusses the solutions to the applications in the 4th Edition CAIA Level I: Alternative Investments using the Texas Instruments BA II Plus Professional calculator.

1. The calculator keystroke instructions are very detailed. Users should note that concepts such as using the financial functions and using the calculator memories are detailed in solutions to the applications but are not repeated each time the concepts are used. Therefore, readers with limited knowledge of the TI BA II Plus calculator might benefit from working through the applications in the order in which they appear in the workbook.

2. Before we begin solving these applications, we need to set the calculator to the decimal free format as well as P/Y = 1. Follow these steps:

   Press 2nd → Format → 9 → Enter → CE/C
   Press 2nd →
   P/Y = 1 → Enter → CE/C

3. Clearing values from previous calculations is critical. Turning the calculator off and on, or hitting CE/C clears the screen but it does not clear the financial registers. There are two special ways to clear the financial registers of the calculator.

   First, to clear the values in the second row of keys:

   Press CF → 2nd → CE/C [clear work]

   Second, to clear the values in the third row of keys:

   Press 2nd → FV

   Failure to clear the financial registers is likely to lead to errors when the financial functions of the calculator are used in subsequent problems.
APPLICATION 2.8.2A (page 51)
Suppose that a short seller establishes a short position in one share of XYZ Corporation at $100 per share and that XYZ pays a dividend of $1.00 per share each year. The current rebate on XYZ shares is 1% per year and lenders require loans to be collateralized 102%. What would be the dollar profit or loss to the short seller if XYZ rose to $103 at the end of one year when the position was liquidated, and what is the effect of the dividend on that profit or loss? Assume that commission and other transaction fees totaled 5 cents per share per trade.

The short position loses $3 (a capital loss) from the stock rising from $100 to $103. The dividends require the short-seller (borrower) to pay $1 to the lender in lieu of dividends (the substitute dividend). The short-seller incurs round trip transaction costs of $0.05 x 2 = $0.10. Finally, the short seller receives a short stock rebate on the collateral which is below a market return by $102 x 1% = $1.02. The total loss is ($3.00 + $1.00 + $0.10 - $1.02) or $3.08. Note that if dividends are indeed irrelevant the payment of the dividend did not increase the short-seller’s loss. The $1 loss on the dividend would, in theory, be offset by a lower stock price. Presumably, the stock would have rise to $104 rather than $103 had it not been for the dividend.

EXPLANATION
There are four components of profit/loss in this short position, capital gain/loss, dividend gain/loss, rebate gain/loss, and commissions paid. When the stock rises from $103 from $100, the short position loses $3 of capital loss. The stock pays an annual dividend, which the short seller pays the securities lender, so $1.00 of dividends the short sellers owes the securities lender. The short seller also earns a rebate equal to 0.01 multiplied by the collateralized loan, which is $100 x 102%. Therefore, 0.01 is multiplied by $100 which is multiplied by 1.02, or $1.02. Lastly, the short seller must pay a commission for every share traded, which is $0.05 per share x two transactions, or $0.10. Putting it all together, -$3 (capital loss as share price rises from $100 to $103) - $1.00 (annual dividend owed to securities lender from short seller) + $1.02 (short stock rebate) – $0.10 (round trip transaction commission).

CALCULATIONS
Step One: Press 100 → - → 103
Step Two: Press = “-3”
Step Three: Press = “-1.00”
Step Four: Press 0.01 → x → 102
Step Five: Press = “+1.02”
Step Six: 0.05 → x → 2
Step Seven: 0.10 → +|-.
Step Eight: - → 0.10
Step Nine: =
Answer: -3.08
WORKOUT AREA: Here are sample problems – cover one of the values and see if you can solve it using the others

<table>
<thead>
<tr>
<th>Dollar Return to Short Seller</th>
<th>Short Position Shares</th>
<th>Price of XYZ Corp. per Share</th>
<th>Rebate on XYZ Corp</th>
<th>Annual Dividend</th>
<th>Collateral Requirement</th>
<th>Transaction Costs Per Share Traded</th>
<th>Price of XYZ Corp. in One Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>($3.08)</td>
<td>1</td>
<td>$100.00</td>
<td>1.00%</td>
<td>$1.00</td>
<td>102%</td>
<td>$0.05</td>
<td>$103.00</td>
</tr>
<tr>
<td>($6.00)</td>
<td>10</td>
<td>$95.00</td>
<td>1.00%</td>
<td>$1.00</td>
<td>105%</td>
<td>$0.05</td>
<td>$100.00</td>
</tr>
<tr>
<td>$5.32</td>
<td>25</td>
<td>$80.00</td>
<td>1.50%</td>
<td>$1.00</td>
<td>110%</td>
<td>$0.10</td>
<td>$70.00</td>
</tr>
<tr>
<td>$15.00</td>
<td>50</td>
<td>$50.00</td>
<td>2.00%</td>
<td>$1.00</td>
<td>100%</td>
<td>$0.05</td>
<td>$30.00</td>
</tr>
<tr>
<td>($6.00)</td>
<td>100</td>
<td>$200.00</td>
<td>1.00%</td>
<td>$3.00</td>
<td>100%</td>
<td>$0.20</td>
<td>$165.00</td>
</tr>
<tr>
<td>($9.64)</td>
<td>200</td>
<td>$150.00</td>
<td>1.50%</td>
<td>$2.00</td>
<td>105%</td>
<td>$0.05</td>
<td>$140.00</td>
</tr>
<tr>
<td>$3.90</td>
<td>300</td>
<td>$20.00</td>
<td>2.00%</td>
<td>$0.50</td>
<td>101%</td>
<td>$0.01</td>
<td>$10.00</td>
</tr>
</tbody>
</table>
APPLICATION 3.2.3 (page 60)
The notional value of a derivative contract is \( l \) time the amount of collateral required to fund a position in the derivative. If \( R \) is the non-annualized return of the derivative based on its notional value and \( R_f \) is the non-annualized return on the riskless asset, what is the non-annualized log return of the collateralized position, assuming that the position is fully levered?

\[
l = 3, \ R = 6\%, \ \text{and} \ R_f = 3\%
\]

EXPLANATION
To solve this, we must use Equation 3.7:

\[
R_{pcoll} = \left[ l \times \ln(1 + R) \right] + R_f
\]

\[
R_{pcoll} = [3 \times \ln(1 + 0.06)] + 0.03
\]

\[
R_{pcoll} = 0.2048
\]

The non-annualized log return of the collateralized position is 20.48%

CALCULATIONS
Step One: Press 1 \( \rightarrow + \rightarrow 0.06 \)
Step Two: Press LN
Step Three: Press \( x \rightarrow 3 \)
Step Four: Press = “0.1748”
Step Five: Press + \( \rightarrow 0.03 \)
Step Six: Press =

Answer = 0.2048

WORKOUT AREA: Here are sample problems – cover one of the values and see if you can solve it using the others

<table>
<thead>
<tr>
<th>( l )</th>
<th>( R )</th>
<th>( R_f )</th>
<th>( R_{pcoll} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>6%</td>
<td>3%</td>
<td>20.48%</td>
</tr>
<tr>
<td>2</td>
<td>4%</td>
<td>4%</td>
<td>11.84%</td>
</tr>
<tr>
<td>3</td>
<td>7%</td>
<td>5%</td>
<td>25.30%</td>
</tr>
<tr>
<td>3</td>
<td>2%</td>
<td>1%</td>
<td>6.94%</td>
</tr>
<tr>
<td>5</td>
<td>1%</td>
<td>0%</td>
<td>4.98%</td>
</tr>
</tbody>
</table>
APPLICATION 3.3.3A (page 63)

Investment A is expected to cost $100 and to be followed by cash inflows of $10 after one year and then $120 after the second year, when the project terminates. The IRR is based on anticipated cash flows and is an anticipated lifetime IRR. The IRR of the investment is 14.7%.

EXPLANATION

To solve this problem, we will use Equation 3.9:

\[ \frac{CF_0}{(1 + IRR)} + \frac{CF_1}{(1 + IRR)^2} + \frac{CF_2}{(1 + IRR)^3} + \cdots + \frac{CF_T}{(1 + IRR)^T} = 0 \]

It is much easier to solve this problem with your financial calculator doing the search. However, to perform the search by hand, note that \( CF_0 = -\$100 \), \( CF_1 = \$10 \), and \( CF_2 = \$120 \). Putting this into the equation:

\[ 100 + \frac{10}{(1 + IRR)^1} + \frac{120}{(1 + IRR)^2} = 0 \]

A method to solve for IRR that does not use the advanced features of the calculator is the “trial and error” method. That is put in an interest rate (as a decimal) for IRR and solve. If the NPV is higher than zero, increase the interest rate. If the NPV is lower than zero, decrease the interest rate. Continue guessing the rate until the NPV converges “close enough” to zero. In this situation, let’s guess 10%.

\[ 100 + \frac{10}{(1 + 0.1)^1} + \frac{120}{(1 + 0.1)^2} = 0 \]

This works out to have a NPV of $8.26. Therefore, we need to increase the interest rate. Let’s try an interest rate of 15%.

\[ 100 + \frac{10}{(1 + 0.15)^1} + \frac{120}{(1 + 0.15)^2} = 0 \]

With an interest rate of 15%, the NPV is ($0.56). The NPV is less than zero, so the IRR is lower than 15%. Using an interest rate of 14.66% (a contrived “guess” that is the answer), as shown below, the NPV is zero. Therefore, the IRR is about 14.66%.

\[ 100 + \frac{10}{(1 + 0.1466)^1} + \frac{120}{(1 + 0.1466)^2} = 0 \]
CALCULATIONS

Using third row of keys…

Step One: Press 2nd → CLR TVM
Step Two: Press 2 → N
Step Three: Press 100 → +/- → PV
Step Four: Press 10 → PMT
Step Five: Press 110 → FV

(Note: Application 3.3.3a mentions the future value is 120. When doing the TVM calculation of the calculator with a payment (PMT) we need to remember that we get the PMT of 10 for two periods. That is, 10 in period 1 and 10 in period 2. Period 2 also includes the FV of 110. Therefore, in Period 2 the total payment is 120, which reflects the information and verbiage in the Application.)

The key point is this: the value placed into FV must equal the final cash flow minus the value already included in PMT!

Step Six: Press CPT → I/Y
Answer: 14.66

WORKOUT AREA: Here are sample problems – cover one of the values and see if you can solve it using the others. Thus, the following table can be used to create 28 sample problems!

<table>
<thead>
<tr>
<th>IRR</th>
<th>CF&lt;sub&gt;0&lt;/sub&gt;</th>
<th>CF&lt;sub&gt;1&lt;/sub&gt;</th>
<th>CF&lt;sub&gt;2&lt;/sub&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>14.66%</td>
<td>($100.00)</td>
<td>$10.00</td>
<td>$120.00</td>
</tr>
<tr>
<td>73.98%</td>
<td>($130.00)</td>
<td>$25.00</td>
<td>$350.00</td>
</tr>
<tr>
<td>29.53%</td>
<td>($250.00)</td>
<td>$15.00</td>
<td>$400.00</td>
</tr>
<tr>
<td>26.88%</td>
<td>($350.00)</td>
<td>$50.00</td>
<td>$500.00</td>
</tr>
<tr>
<td>16.14%</td>
<td>($75.00)</td>
<td>$1.00</td>
<td>$100.00</td>
</tr>
<tr>
<td>31.27%</td>
<td>($500.00)</td>
<td>$85.00</td>
<td>$750.00</td>
</tr>
<tr>
<td>83.15%</td>
<td>($350.00)</td>
<td>$95.00</td>
<td>$1,000.00</td>
</tr>
</tbody>
</table>
APPLICATION 3.3.3B (page 63)

Fund B expended $200 million to purchase investments and distributed $30 million after one year. At the end of the second year, it is being appraised at $180 million. The IRR of the fund is a since-inception IRR (or interim IRR) of 2.7%.

EXPLANATION

Please see the explanation in this workbook for Application 3.3.3A.

CALCULATIONS

Step One: Press 2nd → CLR TVM
Step Two: Press 2 → N
Step Three: Press 200 → +/- → PV
Step Four: Press 30 → PMT
Step Five: Press 150 → FV
Step Six: Press CPT → I/Y
Answer: 2.66

WORKOUT AREA: Here are sample problems – cover one of the values and see if you can solve it using the others

<table>
<thead>
<tr>
<th>IRR</th>
<th>CF₀</th>
<th>CF₁</th>
<th>CF₂</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.66%</td>
<td>($200.00)</td>
<td>30.00</td>
<td>$180.00</td>
</tr>
<tr>
<td>12.58%</td>
<td>($180.00)</td>
<td>25.00</td>
<td>$200.00</td>
</tr>
<tr>
<td>8.70%</td>
<td>($200.00)</td>
<td>15.00</td>
<td>$220.00</td>
</tr>
<tr>
<td>6.96%</td>
<td>($350.00)</td>
<td>150.00</td>
<td>$240.00</td>
</tr>
<tr>
<td>15.80%</td>
<td>($150.00)</td>
<td>1.00</td>
<td>$200.00</td>
</tr>
<tr>
<td>8.19%</td>
<td>($200.00)</td>
<td>50.00</td>
<td>$180.00</td>
</tr>
<tr>
<td>13.61%</td>
<td>($300.00)</td>
<td>200.00</td>
<td>$160.00</td>
</tr>
</tbody>
</table>
APPLICATION 3.3.3C (page 64)

Investment C was purchased three years ago by BK Fund for $500. In the three years following the purchase, the investment distributed cash flows to the investor of $110, $120, and $130. Now in the fourth year, the investment has been appraised as being worth $400. The IRR of the investment is based on realized previous cash flows and a current appraised value. The IRR may be described as an interim IRR and is 15.0%.

EXPLANATION

Please see the explanation in this workbook for **Application 3.3.3A**.

CALCULATIONS

Since there are at least three unique cash inflows, the second row of calculator keys will not handle the problem. While the problem can be solved using a trial and error search by the user, this problem should be solved using the third row of keys. Using the third row of keys involves entering cash flows one-by-one into the financial registers.

Step One: Press 2nd → CLR WORK
Step Two: Press CF → “CF0 =” 500 → +/- → ENTER
Step Three: Press ↓ → “C01” 110 → ENTER
Step Four: Press ↓ → “F01” 1 → ENTER
Step Five: Press ↓ → “C02” 120 → ENTER
Step Six: Press ↓ → “F02” 1 → ENTER
Step Seven: Press ↓ → “C03” 130 → ENTER
Step Eight: Press ↓ → “F03” 1 → ENTER
Step Nine: Press ↓ → “C04” 400 → ENTER
Step Ten: Press ↓ → “F04” 1 → ENTER
Step Eleven: Press IRR → CPT
Answer: 15.05

NOTE: The F01, F02 and so forth are shortcut keys that allow the user to enter the cash flow once and select F0n>1 when the flow is repeated.
WORKOUT AREA: Here are sample problems – cover the IRR values and see if you can solve them using the others

<table>
<thead>
<tr>
<th>IRR</th>
<th>CF₀</th>
<th>CF₁</th>
<th>CF₂</th>
<th>CF₃</th>
<th>CF₄</th>
</tr>
</thead>
<tbody>
<tr>
<td>15.05%</td>
<td>($500.00)</td>
<td>$110.00</td>
<td>$120.00</td>
<td>$130.00</td>
<td>$400.00</td>
</tr>
<tr>
<td>26.10%</td>
<td>($200.00)</td>
<td>$25.00</td>
<td>$50.00</td>
<td>$100.00</td>
<td>$250.00</td>
</tr>
<tr>
<td>34.48%</td>
<td>($200.00)</td>
<td>$50.00</td>
<td>$100.00</td>
<td>$150.00</td>
<td>$150.00</td>
</tr>
<tr>
<td>-6.68%</td>
<td>($350.00)</td>
<td>$25.00</td>
<td>$25.00</td>
<td>$25.00</td>
<td>$200.00</td>
</tr>
<tr>
<td>65.29%</td>
<td>($150.00)</td>
<td>$0.00</td>
<td>$300.00</td>
<td>$0.00</td>
<td>$300.00</td>
</tr>
<tr>
<td>36.13%</td>
<td>($200.00)</td>
<td>$50.00</td>
<td>$50.00</td>
<td>$50.00</td>
<td>$400.00</td>
</tr>
<tr>
<td>15.86%</td>
<td>($300.00)</td>
<td>$10.00</td>
<td>$10.00</td>
<td>$10.00</td>
<td>$500.00</td>
</tr>
</tbody>
</table>

Note that in the fourth example that the $25 cash flow is repeated three times. The keystrokes entering the cash flows can be simplified by entering $25 as C01 and then F01 as 3 (i.e., frequency of that cash flow is three consecutive, equally-spaced occurrences). Then the final $200 would be entered as C02 (the second distinct cash flow) and its frequency (F02) could be left at the default value of 1.
APPLICATION 3.4.5 (page 70)

Assume that a private equity project has the following cash flows:

\[ C_0 = -200, \ C_1 = 50, \ C_2 = -50, \ C_3 = 100, \ C_4 = 0, \ C_5 = 250 \]

Compute the IRR and MIRR given RR = 10% and CC = 8%.

Solution: The IRR is 14.38% (which can be found iteratively or on an advanced calculator). To find the MIRR, first find the \( PV \) of the outflows and the \( FV \) of the inflows. The sum of the present values (at time 0) of \( C_0 = -200 \) and \( C_2 = -50 \) at 8% is -242.867. The sum of the future values (time \( T = 5 \)) of \( C_1 = 50, \ C_3 = 100, \) and \( C_5 = 250 \) at 10% is 444.205. Given \( PV = -242.867, \ FV = 444.205, \) and \( T = 5 \), the MIRR is 12.83%, found by applying the lump sum time value of money formula or using a financial calculator.

EXPLANATION

Regarding the IRR calculation, please see the explanation for Application 3.3.3A.

To find the Modified Internal Rate of Return (MIRR), we must use equation 3.11:

\[
MIRR_T = \left( \frac{FV \text{ of Positive Cash Flows Using the Reinvestment Rate}}{PV \text{ of Negative Cash Flows Using the Cost of Capital}} \right)^{\frac{1}{T}} - 1
\]

To find the future value of positive cash flows using the reinvestment rate, we use a similar equation to finding the IRR, except we use the reinvestment rate (RR) instead of the IRR:

\[
\frac{50}{(1 + 10\%)^1} + \frac{100}{(1 + 10\%)^3} + \frac{250}{(1 + 10\%)^5} = 444.205
\]

Next, we must find the present value of the negative cash flows using the cost of capital (CC). Again, we use a similar equation to the IRR, except we substitute the CC for IRR:

\[
-200 + \frac{-50}{(1 + 10\%)^2} = -242.867
\]

The MIRR is calculated by plugging these two numbers into the numerator and denominator, respectively:

\[
MIRR_T = \left( \frac{444.205}{-(-242.867)} \right)^{\frac{1}{5}} - 1 = 12.83\%
\]
To find the IRR:

Step One: Press CF → Press 2nd → CLR WORK
Step Two: Press → “CF0” = 200 → +/- → ENTER
Step Three: Press ↓ → “C01” 50 → ENTER
Step Four: Press ↓ → “F01” 1 → ENTER
Step Five: Press ↓ → “C02” -50 → ENTER
Step Six: Press ↓ → “F02” 1 → ENTER
Step Seven: Press ↓ → “C03” 100 → ENTER
Step Eight: Press ↓ → “F03” 1 → ENTER
Step Nine: Press ↓ → “C04” 0 → ENTER
Step Ten: Press ↓ → “F04” 1 → ENTER
Step Eleven: Press ↓ → “C05” 250 → ENTER
Step Twelve: Press ↓ → “F05” 1 → ENTER
Step Thirteen: Press IRR → CPT
Answer: 14.38

To find the MIRR, we must perform three separate steps:

Step One: Press CF → Press 2nd → CLR WORK
Step Two: Press → “CF0” = 0 → ENTER
Step Three: Press ↓ → “C01” 50 → ENTER
Step Four: Press ↓ → “F01” 1 → ENTER
Step Five: Press ↓ → “C02” 0 → ENTER
Step Six: Press ↓ → “F02” 1 → ENTER
Step Seven: Press ↓ → “C03” 100 → ENTER
Step Eight: Press ↓ → “F03” 1 → ENTER
Step Nine: Press ↓ → “C04” 0 → ENTER
Step Ten: Press ↓ → “F04” 1 → ENTER
Step Eleven: Press ↓ → “C05” 250 → ENTER
Step Twelve: Press ↓ → “F05” 1 → ENTER
Step Thirteen: Press NPV → “I” → 10
Step Fourteen: Press ↓ → “NPV” → CPT
Answer: 444.205
Step Eight: Press ↓ → “F03” 1 → ENTER
Step Nine: Press ↓ → “C04” 0 → ENTER
Step Ten: Press ↓ → “F04” 1 → ENTER
Step Eleven: Press ↓ → “C05” 0 → ENTER
Step Twelve: Press ↓ → “F05” 1 → ENTER
Step Thirteen: Press NPV → “l” → 8
Step Fourteen: Press ↓ → “NPV” → CPT
Answer: -242.867

Step One: Press 444.205 → + → -242.867 → +|-
Step Two: Press = “1.829”
Step Three: Press → y^x → ( → 1 → + → 5 → )
Step Four: Press = “1.1283”
Step Five: Press → - → 1
Step Six: Press =
Answer = 0.1283

WORKOUT AREA: Here are sample problems – cover the IRR values and see if you can solve them using the others. Cover one variable from the second table and see if you can solve for it using the others.

Use the following table to practice calculating IRR.

<table>
<thead>
<tr>
<th>IRR</th>
<th>CF0</th>
<th>CF1</th>
<th>CF2</th>
<th>CF3</th>
<th>CF4</th>
<th>CF5</th>
</tr>
</thead>
<tbody>
<tr>
<td>14.38%</td>
<td>($200.00)</td>
<td>$50.00</td>
<td>($50.00)</td>
<td>$100.00</td>
<td>$0.00</td>
<td>$250.00</td>
</tr>
<tr>
<td>29.72%</td>
<td>($100.00)</td>
<td>$50.00</td>
<td>($50.00)</td>
<td>$150.00</td>
<td>$25.00</td>
<td>$50.00</td>
</tr>
<tr>
<td>-12.71%</td>
<td>($500.00)</td>
<td>$200.00</td>
<td>$0.00</td>
<td>$0.00</td>
<td>$100.00</td>
<td>$50.00</td>
</tr>
<tr>
<td>5.55%</td>
<td>($300.00)</td>
<td>$50.00</td>
<td>$75.00</td>
<td>$75.00</td>
<td>$75.00</td>
<td>$80.00</td>
</tr>
<tr>
<td>281.19%</td>
<td>($100.00)</td>
<td>$300.00</td>
<td>$300.00</td>
<td>$50.00</td>
<td>$0.00</td>
<td>($200.00)</td>
</tr>
<tr>
<td>30.00%</td>
<td>($250.00)</td>
<td>$100.00</td>
<td>$20.00</td>
<td>$20.00</td>
<td>$50.00</td>
<td>$500.00</td>
</tr>
</tbody>
</table>

Use the following table to practice calculating MIRR.

<table>
<thead>
<tr>
<th>PV of Positive Cash Flows</th>
<th>PV of Negative Cash Flows</th>
<th>t</th>
<th>MIRR</th>
</tr>
</thead>
<tbody>
<tr>
<td>444.205</td>
<td>-242.867</td>
<td>5</td>
<td>12.83%</td>
</tr>
<tr>
<td>323.506</td>
<td>-150.453</td>
<td>5</td>
<td>16.55%</td>
</tr>
<tr>
<td>501.459</td>
<td>-451.756</td>
<td>3</td>
<td>3.54%</td>
</tr>
<tr>
<td>720.459</td>
<td>-100.369</td>
<td>10</td>
<td>21.79%</td>
</tr>
<tr>
<td>100.101</td>
<td>-235.984</td>
<td>4</td>
<td>-19.30%</td>
</tr>
</tbody>
</table>
APPLICATION 3.7.3A (page 80)

Fund A at the end of its term has risen to a total net asset value (NAV) of $300 million from its initial size of $200 million. Assuming no hurdle rate and an 80%/20% carried-interest split, the general partner is entitled to receive carried interest equal to how much? The answer is $20 million. The answer is found by multiplying the GP’s share (20%) by the total profit ($100 million). The total profit is found as the difference in the NAVs. The NAVs are calculated after adding revenues and deducting expenses.

EXPLANATION

Total profit (ending NAV minus Initial NAV), no hurdle rate, and the carried interest split are needed to determine how much the general partner is entitled to receive. In this case, $300 million minus $200 million equals $100 million. $100 million in profit multiplied by 20%, which is the percentage of carried interest the general partner is entitled to, equals $20 million.

CALCULATIONS

Step One: Press 300 → - → 200 → x → 0.2
(Note: the 300 and 200 figures are in millions, as seen in the application.)

Step Two: Press = Answer:
$20 million

WORKOUT AREA: Here are sample problems – cover one of the values and see if you can solve it using the others

<table>
<thead>
<tr>
<th>General Partners’ NAV (millions)</th>
<th>Beginning NAV (millions)</th>
<th>Ending NAV (millions)</th>
<th>Carried-Interest Split</th>
</tr>
</thead>
<tbody>
<tr>
<td>$20.00</td>
<td>$200.00</td>
<td>$300.00</td>
<td>80.0% 20.0%</td>
</tr>
<tr>
<td>$120.00</td>
<td>$100.00</td>
<td>$500.00</td>
<td>70.0% 30.0%</td>
</tr>
<tr>
<td>$20.00</td>
<td>$450.00</td>
<td>$500.00</td>
<td>60.0% 40.0%</td>
</tr>
<tr>
<td>$25.00</td>
<td>$150.00</td>
<td>$200.00</td>
<td>50.0% 50.0%</td>
</tr>
<tr>
<td>$120.00</td>
<td>$200.00</td>
<td>$400.00</td>
<td>40.0% 60.0%</td>
</tr>
<tr>
<td>$105.00</td>
<td>$850.00</td>
<td>$1,000.00</td>
<td>30.0% 70.0%</td>
</tr>
<tr>
<td>$480.00</td>
<td>$250.00</td>
<td>$850.00</td>
<td>20.0% 80.0%</td>
</tr>
</tbody>
</table>
APPLICATION 3.7.3B (page 80)

Fund B terminates and ultimately returns $132 million to its limited partners, and the total initial size of the fund was $100 million. Assuming a carried interest rate of 20%, the general partner is entitled to receive carried interest equal to how much? The answer is $8 million. Note that if $32 million is the profit only to the LP, the total profit of the fund was higher. The answer is found by solving the following equations: LP profit = 0.8 × total profit; so $32 million = 0.8 × total profit; therefore, total profit = $40 million. The second equation is GP carried interest = 0.2 × total profit; therefore, carried interest = $8 million.

EXPLANATION

We are given only the net profit to the limited partners, $32 million (i.e. $132 million minus $100 million). To determine the carried interest that the general partner is entitled to, the total profit is needed. Therefore, we divide $32 million by 80%, which is the carried interest percentage that the limited partners are entitled to. The quotient is $40 million which indicates the total profit. Multiplying the total profit by the 20%, which is the carried interest percentage that the general partners are entitled to, equals $8 million in carried interest to the general partners. Again, the annual management fee is set to zero in this problem for simplicity.

CALCULATIONS

Step One: Press 32 → ÷ → 0.8 (Note: the 32 and 100 figures are in millions, as seen in the application.)

Step Two: Press = “40”

Step Three: Press 40 → × → 0.2 Step

Four: Press =

Answer: $8 million
WORKOUT AREA: Here are sample problems – cover one of the values and see if you can solve it using the others

<table>
<thead>
<tr>
<th>General Partners' Share (millions)</th>
<th>Initial Fund Size (millions)</th>
<th>Return to Limited Partners (millions)</th>
<th>Total Profit</th>
<th>Carried-Interest Split</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Limited Partner</td>
</tr>
<tr>
<td>$8.00</td>
<td>$100.00</td>
<td>$132.00</td>
<td>$40.00</td>
<td>80.0%</td>
</tr>
<tr>
<td>$10.71</td>
<td>$100.00</td>
<td>$125.00</td>
<td>$35.71</td>
<td>70.0%</td>
</tr>
<tr>
<td>$5.56</td>
<td>$450.00</td>
<td>$500.00</td>
<td>$55.56</td>
<td>90.0%</td>
</tr>
<tr>
<td>$16.67</td>
<td>$150.00</td>
<td>$200.00</td>
<td>$66.67</td>
<td>75.0%</td>
</tr>
<tr>
<td>$12.50</td>
<td>$350.00</td>
<td>$400.00</td>
<td>$62.50</td>
<td>80.0%</td>
</tr>
<tr>
<td>$37.50</td>
<td>$850.00</td>
<td>$1,000.00</td>
<td>$187.50</td>
<td>80.0%</td>
</tr>
<tr>
<td>$17.65</td>
<td>$250.00</td>
<td>$350.00</td>
<td>$117.65</td>
<td>85.0%</td>
</tr>
</tbody>
</table>
APPLICATION 3.7.4A (page 81)

Consider a fund that makes two investments, A and B, of $10 million each. Investment A is successful and generates a $10 million profit, whereas Investment B is a complete write-off (a total loss). Assume that the fund managers are allowed to take 20% of profits as carried interest. How much carried interest will they receive if profits are calculated on a fund-as-a-whole (aggregated) basis, and how much will they receive if profits are calculated on a deal-by-deal (individual transaction) basis?

On the fund-as-a-whole basis, the fund broke even, so no incentive fees will be distributed. On the deal-by-deal basis, Investment A earned $10 million, so $2 million in carried interest will be distributed to the managers.

EXPLANATION

On a fund-as-a-whole basis, Investment A generated a profit of $10 million, while Investment B lost $10 million. Therefore, the net of the two investments in the whole fund is $0. Therefore, there is no carried interest to distribute to either the limited partners or general partners.

On a deal-by-deal basis, Investment A generated a profit of $10 million and the general partners are entitled to their proportion of the carried interest, which is 20%. Therefore, $10 million multiplied by 20% for a product of $2 million.

Investment B does not generate a profit; in fact it loses $10 million. Thus, there is no carried interest and limited partners and general partners do not receive a distribution.

CALCULATIONS

On a fund-as-a-whole basis

Step One: Press 10 → - → 10 → x → 0.2  
(Note: the 10 and 100 figures are in millions, as seen in the application.)

Step Two: Press =  
Answer: $0

On a deal-by-deal basis

Step One: Press 10 → x → 0.2  
(Note: the 10 figure is in millions, as seen in the application.)

Step Two: Press =  
Answer: $2 million
WORKOUT AREA: Here are sample problems – cover one of the values and see if you can solve it using the others

<table>
<thead>
<tr>
<th>Mgr Fees on Deal-by-Deal Basis</th>
<th>Mgr Fees on Fund-as-a-Whole Basis</th>
<th>Investment A Profit (millions)</th>
<th>Investment B Profit (millions)</th>
<th>Limited Partner</th>
<th>General Partner</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2.00</td>
<td>$0.00</td>
<td>$10.00</td>
<td>-$10.00</td>
<td>80.0%</td>
<td>20.0%</td>
</tr>
<tr>
<td>$10.50</td>
<td>$10.50</td>
<td>$10.00</td>
<td>$25.00</td>
<td>70.0%</td>
<td>30.0%</td>
</tr>
<tr>
<td>$1.50</td>
<td>$0.00</td>
<td>-$25.00</td>
<td>$6.00</td>
<td>75.0%</td>
<td>25.0%</td>
</tr>
<tr>
<td>$3.00</td>
<td>$0.00</td>
<td>$15.00</td>
<td>-$35.00</td>
<td>80.0%</td>
<td>20.0%</td>
</tr>
<tr>
<td>$5.00</td>
<td>$4.00</td>
<td>$25.00</td>
<td>-$5.00</td>
<td>80.0%</td>
<td>20.0%</td>
</tr>
<tr>
<td>$3.90</td>
<td>$3.90</td>
<td>$13.00</td>
<td>$13.00</td>
<td>85.0%</td>
<td>15.0%</td>
</tr>
<tr>
<td>$5.00</td>
<td>$4.00</td>
<td>-$5.00</td>
<td>$25.00</td>
<td>80.0%</td>
<td>20.0%</td>
</tr>
</tbody>
</table>
APPLICATION 3.7.5A (page 82)

Consider a fund that calculates incentive fees on a fund-as-a-whole basis and makes two investments, A and B, of $10 million each. Investment A is successful and generates a $10 million profit after three years. Investment B is not revalued until it is completely written off after five years. Assume that the fund managers are allowed to take 20% of profits as carried interest calculated on an aggregated basis. How much carried interest will they receive if there is no clawback provision, and how much will they receive if there is a clawback provision?

Without a clawback provision, the fund earned $10 million after three years and distributed a $2 million carried interest to the managers. When the second investment failed, the incentive fee is not returned. In the case of a clawback provision, the fund distributed a $2 million incentive fee to the managers after three years, but when the second investment failed, the incentive fee is returned to the limited partners, since there is no combined profit.

EXPLANATION

With no clawback provision, Investment A generates $10 million of profit after three years. Therefore, the fund managers can take profits after three years when Investment A generated the $10 million profit. Fund managers are entitled to $10 million multiplied by 20%, the carried interest percentage for general partners, for a product of $2 million. Now, 2 years later when Investment B fails to generate a profit the fund managers do not receive any more carried interest and do not have to give back the prior $2 million distributed in carried interest from Investment A. This is the result of the fund not having a clawback provision.

With a clawback provision, the scenario begins similarly. Investment A generates $10 million of profit after three years. Therefore, the fund managers can take profits after three years when Investment A generated the $10 million profit. Fund managers are entitled to $10 million multiplied by 20%, the carried interest percentage for general partners, for a product of $2 million. However, when Investment B fails the $2 million distributed to general partners is returned to the limited partners (in theory!) because Investment A’s profit plus Investment B’s profit equals zero (i.e. there is no combined profit). If there was a combined profit, then the general partners would be entitled to 20% of that combined profit, which may require either an additional distribution to the general partners or a clawback (i.e. if the combined profit was below $2 million).
**CALCULATIONS**

Without a clawback provision

Step One: Press 10 → x → 0.2

(Note: the 10 figure is in millions, as seen in the application.)

Step Two: Press =

Answer: $2 million

With a clawback provision

Step One: Press 10 → x → 0.2

(Note: the 10 figure is in millions, as seen in the application.)

Step Two: Press “2”

Step Three: Press 10 → x → 0.2

(Note: the 10 figure is in millions, as seen in the application.)

Step Four: Press “2”

Step Five: Press 2 → - → 2 Step

Six: Press =

Answer: $0

**WORKOUT AREA:** Here are sample problems – cover one of the values and see if you can solve it using the others

<table>
<thead>
<tr>
<th></th>
<th>Carried - Interest Split</th>
<th>Without Clawback</th>
<th>With a Clawback</th>
<th>Investment A Profit (millions)</th>
<th>Investment B Profit (millions)</th>
<th>Limited Partner</th>
<th>General Partner</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2.00</td>
<td></td>
<td>$0.00</td>
<td>$0.00</td>
<td>$10.00</td>
<td>-$10.00</td>
<td>80.0%</td>
<td>20.0%</td>
</tr>
<tr>
<td>$10.50</td>
<td></td>
<td>$10.50</td>
<td>$10.00</td>
<td>$10.00</td>
<td>$25.00</td>
<td>70.0%</td>
<td>30.0%</td>
</tr>
<tr>
<td>$1.20</td>
<td></td>
<td>$0.00</td>
<td>-$25.00</td>
<td>$6.00</td>
<td>$35.00</td>
<td>85.0%</td>
<td>15.0%</td>
</tr>
<tr>
<td>$2.25</td>
<td></td>
<td>$0.00</td>
<td>$15.00</td>
<td>-$35.00</td>
<td>$25.00</td>
<td>80.0%</td>
<td>20.0%</td>
</tr>
<tr>
<td>$5.00</td>
<td></td>
<td>$4.00</td>
<td>$25.00</td>
<td>-$5.00</td>
<td>$25.00</td>
<td>80.0%</td>
<td>20.0%</td>
</tr>
<tr>
<td>$3.90</td>
<td></td>
<td>$3.90</td>
<td>$13.00</td>
<td>$13.00</td>
<td>$13.00</td>
<td>85.0%</td>
<td>15.0%</td>
</tr>
<tr>
<td>$2.50</td>
<td></td>
<td>$2.00</td>
<td>-$5.00</td>
<td>$25.00</td>
<td>$25.00</td>
<td>90.0%</td>
<td>10.0%</td>
</tr>
</tbody>
</table>
APPLICATION 3.7.6A (page 83)

Consider a $10 million fund with 20% incentive fees that lasts a single year and earns a $2 million profit. Ignoring a hurdle rate, the fund manager would receive $400,000, which is 20% of $2 million. But with a hard hurdle rate of 10%, the fund manager receives the 20% incentive fees only on profits in excess of the 10% return, meaning $200,000. The first $1 million of profit goes directly to the limited partners. The fund manager collects an incentive fee only on profits in excess of the $1 million, which is the profit necessary to bring the limited partners’ return up to the hurdle rate. Thus, the manager receives an incentive fee of $200,000.

EXPLANATION

This application describes a $10 million fund that generated a $2 million profit, with a 10% hurdle rate, where the general partners receive 20% incentive fees (also called carried interest or performance fees). Ignoring the 10% hurdle rate, the fund managers received $2 million multiplied by 20% or $400,000. With the 10% hurdle rate in place, the fund managers only receive the 20% incentive fee on profits in excess of a 10%, that is in excess of 10% multiplied by $10 million or $1 million. With the hurdle rate the general partners receive 20% multiplied by $1 million for an incentive fee of $200,000. Keep in mind that if the fund had returned $1 million or less, the general partners would not be entitled to any incentive fee.
CALCULATIONS – INCLUDING MEMORY INSTRUCTIONS

Ignoring the hurdle rate

Step One: Press 2 → x → 0.2

(Note: the 2 figure is in millions, as seen in the application.)

Step Two: Press =

Answer: $400,000

With a 15% hurdle rate ($2 million in profits and 20% incentive fee)

Step One: Press 2 → x → 0.15 → = (Note: the 2 figure is in millions)

Next store the above total fees (0.3 million) that are waived due to the hard hurdle rate in memory…

Step Two: Press “STO” → 0→CE/C to store this value in memory #0

Note that hitting “STO” begins the process of storing a number in memory, but that process is not complete until the user has selected a memory bank numbered from 0 to 9. The above example selected memory location zero by typing 0 immediately after STO

Step Three: Press 2 → x → 0.20 = to calculate total fees without a hurdle rate

Step Four: Press - →RCL→0 → = to subtract the memory from the previous result

Answer: 0.1 indicating $100,000

WORKOUT AREA: Here are sample problems – cover one of the values and see if you can solve it using the others

<table>
<thead>
<tr>
<th>Hard Hurdle Rate (millions)</th>
<th>No Hurdle Rate (millions)</th>
<th>Initial Investment (millions)</th>
<th>Ending Fund Value (millions)</th>
<th>Hurdle Rate</th>
<th>Incentive Fee</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0.20</td>
<td>$0.40</td>
<td>$10.00</td>
<td>$12.00</td>
<td>10.0%</td>
<td>20.0%</td>
</tr>
<tr>
<td>$0.00</td>
<td>$0.00</td>
<td>$25.00</td>
<td>$20.00</td>
<td>15.0%</td>
<td>20.0%</td>
</tr>
<tr>
<td>$5.10</td>
<td>$6.00</td>
<td>$30.00</td>
<td>$50.00</td>
<td>10.0%</td>
<td>30.0%</td>
</tr>
<tr>
<td>$1.00</td>
<td>$2.50</td>
<td>$15.00</td>
<td>$20.00</td>
<td>20.0%</td>
<td>50.0%</td>
</tr>
<tr>
<td>$0.89</td>
<td>$0.90</td>
<td>$0.50</td>
<td>$5.00</td>
<td>10.0%</td>
<td>20.0%</td>
</tr>
<tr>
<td>$0.70</td>
<td>$1.00</td>
<td>$10.00</td>
<td>$15.00</td>
<td>15.0%</td>
<td>20.0%</td>
</tr>
<tr>
<td></td>
<td>$10.00</td>
<td>$12.50</td>
<td>$100.00</td>
<td>$150.00</td>
<td>10.0%</td>
</tr>
<tr>
<td>-------</td>
<td>--------</td>
<td>--------</td>
<td>---------</td>
<td>---------</td>
<td>-------</td>
</tr>
<tr>
<td>$1.23</td>
<td>$1.25</td>
<td>$50.00</td>
<td>$75.00</td>
<td>1.0%</td>
<td>5.0%</td>
</tr>
</tbody>
</table>
APPLICATION 3.7.7A (page 84)

Fund A with an initial investment of $20 million liquidates with $24 million cash after one year. The hurdle rate is 15%, and the incentive fee is 20%. What is the distribution to the fund manager if the fund uses a hard hurdle? What is the distribution to the fund manager if the fund has a soft hurdle and a 50% catch-up rate?

The first $20 million is returned to the limited partners in both cases. With a hard hurdle, the limited partners receive the first $3 million of profit, which is 15% of the $20 million investment. The limited partners receive 80% of the remaining $1 million, which is $800,000, for a total profit of $3.8 million. With a soft hurdle, the limited partners receive the first $3 million of profit, which is 15% of the $20 million investment. To fulfill the catch-up provision, the fund manager receives 50% of the remaining profit up to the point of being paid 20% of all profit. In this case, 50% of all of the remaining profit, or $1 million, is $500,000. Since $500,000 is less than 20% of the entire $4 million profit, the fund manager is unable to fully catch up. Had the total profits exceeded $5 million, the catch-up of the fund manager would have been completed. With $5 million of profit, the GP would receive 50% of the profits above $3 million, or $1 million (50% of the $2 million profit in excess of the profit necessary to meet the hurdle rate for the LPs). The $1 million of catch-up equals 20% of $5 million. Profits in excess of $5 million would then be split 20% to the fund manager and 80% to the limited partners.

EXPLANATION

Since the fund is liquidating this is not simply a distribution of profits. Thus the first $20 million (i.e. the initial investment) is distributed to investors or limited partners. The additional $4 million ($24 million minus $20 million) is the profit that is subject to the hurdle rate and incentive fees, if applicable.

With the hard hurdle rate in effect we need to apply the 15% hurdle rate to the $20 million initial investment, 15% multiplied by $20 million for a product of $3 million. Therefore, limited partners are entitled to $3 million before general managers can receive the incentive fee on the remainder. In this case, the fund generated a profit of $4 million the fund managers will collect an incentive fee on the difference, $4 million minus $3 million or $1 million. $1 million multiplied by 20% (the fund managers’ incentive fee) equals $200,000. The limited partners receive the other $800,000 or $1 million multiplied by .8 as well as the other $3 million in profit, for a total profit of $3.8 million. At the end of liquidation, fund managers will receive $200,000 and limited partners will receive $23.8 million (i.e. $20 million of initial investment and $3.8 million in profit).

Let’s consider this same application with a soft hurdle rate of 15% and a 50% catch up rate. This scenario begins similarly. We need to apply the 15% hurdle
rate to the $20 million initial investment, 15% multiplied by $20 million for a product of $3 million. Therefore, limited partners are entitled to $3 million before general managers can receive the incentive fee on the remainder. The fund generated a profit of $4 million the fund managers will collect fees on the difference, $4 million minus $3 million or $1 million. In this scenario, the 50% catch up provision is utilized until the fund managers receive 20% of total profits. After the 20% of total profits is received using the catch up provision, the fund managers would collect 20%. Therefore, the fund managers will collect 50% of the $1 million left over after the hurdle rate is satisfied, or $500,000. $500,000 divided by $4 million is 12.5%, which means that the fund managers were unable to fully catch up. If the fund has returned $5 million, the fund managers would have been about to fully catch up (i.e. the catch-up provision would be satisfied). If the fund had earned over $5 million in profit the catch up provision would have been satisfied and the 20% incentive fee would apply for fund managers.

CALCULATIONS

With a 15% hard hurdle rate

Step One: Press 20 → x → .15 (Note: the 10 figure is in millions, as seen in the application.)

Step Two: Press = “3”
Step Three: Press 24 → - → 20
Step Four: Press = “4”
Step Five: Press 4 → - → 3 Step
Six: Press = “1”
Step Seven: Press 1 → x → .2 Step
Eight: Press =
Answer: $200,000

With a 15% soft hurdle rate and a 50% catch up provision

Step One: Press 20 → x → .15 (Note: the 10 figure is in millions, as seen in the application.)

Step Two: Press = “3”
Step Three: Press 24 → - → 20
Step Four: Press = “4”
Step Five: Press 4 → - → 3 Step
Six: Press = “1”
Step Seven: Press $1 \rightarrow x \rightarrow .5$ Step

Eight: Press = “.5”

Step Nine: Press $500,000 \rightarrow \div \rightarrow 4,000,000$ Step

Ten: Press = “.125”

Answer: $500,000$

**WORKOUT AREA:** Here are sample problems – cover one of the values and see if you can solve it using the others

<table>
<thead>
<tr>
<th>Hard Hurdle Rate (millions)</th>
<th>Soft Hurdle Rate (millions)</th>
<th>Initial Investment (millions)</th>
<th>Ending Fund Value (millions)</th>
<th>Hurdle Rate</th>
<th>Incentive Fee</th>
<th>Catch-up Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0.20$</td>
<td>$0.50$</td>
<td>$20.00$</td>
<td>$24.00$</td>
<td>15.0%</td>
<td>20.0%</td>
<td>50.0%</td>
</tr>
<tr>
<td>$2.63$</td>
<td>$3.75$</td>
<td>$50.00$</td>
<td>$75.00$</td>
<td>15.0%</td>
<td>15.0%</td>
<td>30.0%</td>
</tr>
<tr>
<td>$0.70$</td>
<td>$1.00$</td>
<td>$20.00$</td>
<td>$30.00$</td>
<td>15.0%</td>
<td>10.0%</td>
<td>25.0%</td>
</tr>
<tr>
<td>$1.13$</td>
<td>$1.25$</td>
<td>$10.00$</td>
<td>$15.00$</td>
<td>5.0%</td>
<td>25.0%</td>
<td>50.0%</td>
</tr>
<tr>
<td>$2.45$</td>
<td>$2.50$</td>
<td>$5.00$</td>
<td>$30.00$</td>
<td>10.0%</td>
<td>10.0%</td>
<td>25.0%</td>
</tr>
<tr>
<td>$0.87$</td>
<td>$0.90$</td>
<td>$2.00$</td>
<td>$8.00$</td>
<td>10.0%</td>
<td>15.0%</td>
<td>30.0%</td>
</tr>
<tr>
<td>$0.80$</td>
<td>$1.20$</td>
<td>$20.00$</td>
<td>$26.00$</td>
<td>10.0%</td>
<td>20.0%</td>
<td>50.0%</td>
</tr>
</tbody>
</table>
APPLICATION 4.3.7A (page 103)

A returns series indicates first-order autocorrelation of 0.6 and second-order autocorrelation of 0.2. What is the partial second-order autocorrelation coefficient?

Inserting 0.6 for \( \rho_1 \) and 0.2 for \( \rho_2 \) generates: \( \frac{0.2 - (0.6 \times 0.6)}{1 - (0.6 \times 0.6)} = -0.25 \). Note that even though the returns in periods \( k \) and \( k - 2 \) are positively correlated, that correlation is primarily driven by first-order correlation. The marginal second-order effect is captured in the partial autocorrelation coefficient and indicates a mean-reverting effect (once the first-order effects have been removed).

EXPLANATION

In order to determine the second-order autocorrelation coefficient, we must use Equation 4.22.

Second-Order Partial Autocorrelation Coefficient = \( \frac{\rho_2 - \rho_1^2}{1 - \rho_1^2} \)

Inserting 0.6 for \( \rho_1 \) and 0.2 for \( \rho_2 \) generates: \( \frac{0.2 - (0.6 \times 0.6)}{1 - (0.6 \times 0.6)} = -0.25 \)

CALCULATIONS

Step One: Press \( \rightarrow 0.2 \)
Step Two: Press \( \rightarrow - \rightarrow ( \rightarrow 0.6 \rightarrow x \rightarrow 0.6\rightarrow ) \)
Step Three: Press \( \rightarrow = \) “-0.16”
Step Four: Press \( \rightarrow ÷ \)
Step Five: Press \( \rightarrow ( \rightarrow ( \)
Step Six: Press \( \rightarrow 1 \rightarrow - \)
Step Seven: Press \( \rightarrow (\rightarrow 0.6 \rightarrow x \rightarrow 0.6 \rightarrow ) \rightarrow ) \)
Step Eight: Press = “-0.25”
WORKOUT AREA: Here are sample problems – cover one of the values and see if you can solve it using the others

<table>
<thead>
<tr>
<th>Partial Second-Order Autocorrelation</th>
<th>First-Order Autocorrelation</th>
<th>Second-Order Autocorrelation</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.25</td>
<td>0.6</td>
<td>0.2</td>
</tr>
<tr>
<td>0.07</td>
<td>0.5</td>
<td>0.3</td>
</tr>
<tr>
<td>-0.18</td>
<td>0.7</td>
<td>0.4</td>
</tr>
<tr>
<td>0.48</td>
<td>0.2</td>
<td>0.5</td>
</tr>
<tr>
<td>0.29</td>
<td>0.4</td>
<td>0.4</td>
</tr>
</tbody>
</table>
The daily returns of Fund A have a variance of 0.0001. What is the variance of the weekly returns of Fund A assuming that the returns are uncorrelated through time?

Using Equation 4.28 and five days in a week, the variance is 0.0005.

EXPLANATION

The daily returns have a variance of 0.0001 and are uncorrelated through time. The uncorrelated through time allows us to use equation 4.28:

\[ V(R_t) = T \times V(R_1) \quad \text{when, } \rho_{t,t-k} = 0 \]

Therefore, the variance of weekly returns of Fund A is equal to 0.0001 multiplied by 5 (number of trading days in a week) for a product of 0.0005. Simply put, variance grows linearly with time horizon when returns are uncorrelated.

CALCULATIONS

Step One: Press 0.0001 → x → 5 Step Two: Press = Answer: 0.0005

WORKOUT AREA: Here are sample problems – cover one of the values and see if you can solve it using the others

<table>
<thead>
<tr>
<th>Short-term Variance</th>
<th># of ST Periods in LT</th>
<th>Longer-term Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0001</td>
<td>5</td>
<td>0.0005</td>
</tr>
<tr>
<td>0.0002</td>
<td>5</td>
<td>0.0010</td>
</tr>
<tr>
<td>0.0005</td>
<td>4</td>
<td>0.0020</td>
</tr>
<tr>
<td>0.0100</td>
<td>12</td>
<td>0.1200</td>
</tr>
</tbody>
</table>
The daily returns of Fund A have a standard deviation of 1.4%. What is the standard deviation of a position that contains only Fund A and is leveraged with $3 of assets for each $1 of equity (net worth)?

Using Equation 4.31, the standard deviation of the levered returns is 4.2%.

**EXPLANATION**

Utilizing Equation 4.31,

\[ \sigma_L = L \times \sigma_u \]

we multiply 1.4% (the standard deviation) by 3/1 or 3 for a product of 4.2%, which is the standard deviation of levered returns. Simply put, being levered with $3 of assets to $1 of equity causes the volatility of the equity to be 3 times the volatility.

**CALCULATIONS**

Step One: Press 3 → / → 1
Step Two: Press x → 0.014
Step Three: Press = Answer:
0.042 or 4.2%

**WORKOUT AREA:** Here are sample problems – cover one of the values and see if you can solve it using the others

<table>
<thead>
<tr>
<th>Standard Deviations of Levered Returns</th>
<th>Daily Standard Deviation of Returns</th>
<th>Assets</th>
<th>Equity</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.20%</td>
<td>1.40%</td>
<td>$3</td>
<td>$1</td>
</tr>
<tr>
<td>1.53%</td>
<td>2.30%</td>
<td>$2</td>
<td>$3</td>
</tr>
<tr>
<td>4.17%</td>
<td>5.00%</td>
<td>$5</td>
<td>$6</td>
</tr>
<tr>
<td>20.00%</td>
<td>10.00%</td>
<td>$2</td>
<td>$1</td>
</tr>
<tr>
<td>16.50%</td>
<td>3.30%</td>
<td>$5</td>
<td>$1</td>
</tr>
<tr>
<td>43.00%</td>
<td>4.30%</td>
<td>$10</td>
<td>$1</td>
</tr>
<tr>
<td>0.93%</td>
<td>1.20%</td>
<td>$7</td>
<td>$9</td>
</tr>
</tbody>
</table>
APPLICATION 4.4.4B (page 109)

The daily returns of Fund A have a standard deviation of 1.4%. What is the standard deviation of a position that contains 40% Fund A and 60% cash?

Using Equation 4.33, the standard deviation of the unlevered returns is 0.56%.

EXPLANATION

To solve this application, it is important to understand that cash has a standard deviation of 0. Therefore, we can utilize equation 4.33,

\[ \sigma_p = w \times \sigma_m \]

and multiply 40% (the proportion of the fund with a standard deviation of 1.4%) by 1.4% for a product of 0.56%, which is the standard deviation of the unlevered returns.

CALCULATIONS

Step One: Press 0.4 → x → 0.014 Step

Two: Press =

Answer: 0.0056 or 0.56%

WORKOUT AREA: Here are sample problems – cover one of the values and see if you can solve it using the others

<table>
<thead>
<tr>
<th>Standard Deviation of Unlevered Returns</th>
<th>Daily Standard Deviation of Returns</th>
<th>Fund Allocation</th>
<th>Cash Allocation</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.56%</td>
<td>1.40%</td>
<td>40%</td>
<td>60%</td>
</tr>
<tr>
<td>1.15%</td>
<td>2.30%</td>
<td>50%</td>
<td>50%</td>
</tr>
<tr>
<td>3.00%</td>
<td>5.00%</td>
<td>60%</td>
<td>40%</td>
</tr>
<tr>
<td>7.00%</td>
<td>10.00%</td>
<td>70%</td>
<td>30%</td>
</tr>
<tr>
<td>2.64%</td>
<td>3.30%</td>
<td>80%</td>
<td>20%</td>
</tr>
<tr>
<td>3.87%</td>
<td>4.30%</td>
<td>90%</td>
<td>10%</td>
</tr>
<tr>
<td>1.20%</td>
<td>1.20%</td>
<td>100%</td>
<td>0%</td>
</tr>
</tbody>
</table>
APPLICATION 4.4.4C (page 110)

The daily returns of Fund A have a standard deviation of 1.2%. What is the standard deviation of the returns of Fund A over a four-day period if the returns are uncorrelated through time? What is the maximum standard deviation for other correlation assumptions?

With zero autocorrelation, the standard deviation of four-day returns is 2.4% (based on the square root of the number of time periods). As the correlation approaches +1, the upper bound would be 4.8%.

EXPLANATION

The key to understanding these problems is that this is when the returns are uncorrelated through time volatility grows with the square root of the time horizon. In this case, we can use the equations:

\[ \sigma_T = \sigma_1 \times \sqrt{T} \text{ when } \rho_{t,t-k} = 0 \]

Therefore, we multiply the standard deviation of daily returns of 1.2% by the square root of 4, the number of days in the period

(Note: the unit number is days. If the standard deviation were of annual returns, we would multiply the standard deviation by the square root of the number of years in the period).

This will give us a product of 2.4%, which is the standard deviation of four-day returns.

To find the maximum standard deviation we need to assume that the correlation of returns approaches 1, or perfect correlation. When that happens, we can use the equation:

\[ \sigma_T = \sigma_1 \times T \text{ when } \rho_{t,t-k} = 1 \]

Applying this equation, we multiply 1.2% by 4 for a product of 4.8%.  

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CALCULATIONS

Assuming uncorrelated returns through time Step

One: Press 0.012 → x → 4
Step Two: Press \(\sqrt{x} \rightarrow =\)
Answer: 0.024 or 2.4% Assuming

perfectly correlated returns

Step One: Press 0.012 → x → 4 Step
Two: Press =
Answer: 0.048 or 4.8%

WORKOUT AREA: Here are sample problems – cover one of the values and see if you can solve it using the others

<table>
<thead>
<tr>
<th>Standard Deviation of Returns for Specified Timeframe</th>
<th>Daily Standard Deviation of Returns</th>
<th>Time Frame</th>
<th>Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.40%</td>
<td>1.20%</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>4.80%</td>
<td>1.20%</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>10.84%</td>
<td>2.80%</td>
<td>15</td>
<td>0</td>
</tr>
<tr>
<td>42.00%</td>
<td>2.80%</td>
<td>15</td>
<td>1</td>
</tr>
<tr>
<td>8.73%</td>
<td>3.30%</td>
<td>7</td>
<td>0</td>
</tr>
<tr>
<td>23.10%</td>
<td>3.30%</td>
<td>7</td>
<td>1</td>
</tr>
<tr>
<td>1.70%</td>
<td>1.20%</td>
<td>2</td>
<td>0</td>
</tr>
</tbody>
</table>
APPLICATION 5.2.2A (page 123)

Find the value of a $50 five-year zero-coupon bond for $m = 1, 2, 4, 12, 365$, and $\infty$, given an annual interest rate of $9\%$.

Simply insert $50$ in place of $1$, $9\%$ in place of $r$, $5$ for $t$, and the given value for $m < \infty$ in Equation 5.2 (and analogously in Equation 5.4) to produce: $32.497$, $32.196$, $32.041$, $31.935$, and $31.883$ (and $31.881$ for continuous compounding).

EXPLANATION

In order to find the value of the zero-coupon bonds, when $m = 1, 2, 4, 12, 365$ we will need to use Equations 5.2:

$$B(t) = FV \left[ 1 + \left( \frac{r^m}{t} \right) \right]^{-tm}$$

To find the value of the zero-coupon bond when $m = 1$, we calculate the following:

$$B(t) = 50 \left[ 1 + \left( \frac{0.09}{1} \right) \right]^{-5x1}$$
$$32.497 = 50 \left[ 1 + \left( \frac{0.09}{1} \right) \right]^{-5x1}$$

To find the value of the zero-coupon bond when $m = 2$, we calculate the following:

$$B(t) = 50 \left[ 1 + \left( \frac{0.09}{2} \right) \right]^{-5x2}$$
$$32.196 = 50 \left[ 1 + \left( \frac{0.09}{2} \right) \right]^{-5x2}$$

To find the value of the zero-coupon bond when $m = 4$, we calculate the following:

$$B(t) = 50 \left[ 1 + \left( \frac{0.09}{4} \right) \right]^{-5x4}$$
$$32.041 = 50 \left[ 1 + \left( \frac{0.09}{4} \right) \right]^{-5x4}$$

To find the value of the zero-coupon bond when $m = 12$, we calculate the following:

$$B(t) = 50 \left[ 1 + \left( \frac{0.09}{12} \right) \right]^{-5x12}$$
\[
\$31.935 = 50 \left[ 1 + \left( \frac{0.09}{12} \right) \right]^{-5 \times 12}
\]

To find the value of the zero-coupon bond when \( m = 365 \), we calculate the following:

\[
B(t) = 50 \left[ 1 + \left( \frac{0.09}{365} \right) \right]^{-5 \times 365}
\]

\[
\$31.883 = 50 \left[ 1 + \left( \frac{0.09}{365} \right) \right]^{-5 \times 365}
\]

To find the value of the zero-coupon bond for \( m = \infty \), we must use Equation 5.4:

\[
B(t) = FV e^{-\frac{m \times \infty}{t}}
\]

\[
\$31.881 = 50 e^{-0.09 \times 5}
\]

**CALCULATIONS**

Note: all of these calculations are done using Equation 5.2, except when \( m = \infty \), when we use 5.4.

For the \$50 five-year zero-coupon bond for when \( m = 1 \)

- Step One: Press 0.09
- Step Two: Press \( \to + \to 1 \)
- Step Three: Press \( = \) “1.09”
- Step Four: Press \( \to \times \)
- Step Five: Press \( \to ( \to 5 \to +\to \times \to 1 \to ) \)
- Step Six: Press \( = \) “0.6499”
- Step Seven: Press \( \to \times \to 50 \)
- Step Eight: Press \( = \)
- Answer: 32.497

When \( m = 2 \),

- Step One: Press 0.09 \( \to + \to 2 \)
- Step Two: Press \( = \) “0.045”
- Step Three: Press \( \to + \to 1 \)
- Step Four: Press \( = \) “1.045”
- Step Five: Press \( \to \times \)
- Step Six: Press \( \to ( \to 5 \to +\to \times \to 2 \to ) \)
- Step Seven: Press \( = \) “0.6439”
- Step Eight: Press \( \to \times \to 50 \)
- Step Nine: Press \( = \)
- Answer: 32.196

When \( m = 4 \),
Step One: Press 0.09 → ÷ → 4
Step Two: Press = “0.0225”
Step Three: Press → + → 1
Step Four: Press = “1.0225”
Step Five: Press → y^x
Step Six: Press → ( → 5 → +|→ → x → 4 → )
Step Seven: Press = “0.6408”
Step Eight: Press → x → 50
Step Nine: Press =
Answer: 32.041

When m = 12,

Step One: Press 0.09 → ÷ → 12
Step Two: Press = “0.0075”
Step Three: Press → + → 1
Step Four: Press = “1.0075”
Step Five: Press → y^x
Step Six: Press → ( → 5 → +|→ → x → 12 → )
Step Seven: Press = “0.6387”
Step Eight: Press → x → 50
Step Nine: Press =
Answer: 31.935

When m = 365,

Step One: Press 0.09 → ÷ → 365
Step Two: Press = “0.000247”
Step Three: Press → + → 1
Step Four: Press = “1.000247”
Step Five: Press → y^x
Step Six: Press → ( → 5 → +|→ → x → 365 → )
Step Seven: Press = “0.6377”
Step Eight: Press → x → 50
Step Nine: Press =
Answer: 31.883

When m = ∞,

Step One: Press 0.09 → x → 5
Step Two: Press = “0.45”
Step Three: Press → +|-
Step Four: Press = “-0.45”
Step Five: Press → 2^ND → e^x
Step Six: Press = “0.6376”
Step Seven: Press → x → 50
Step Eight: Press =
Answer: 31.881
WORKOUT AREA: Here are sample problems – cover one of the values and see if you can solve it using the others

<table>
<thead>
<tr>
<th>Par Value of Zero-Coupon Bond</th>
<th>Years to Maturity (t)</th>
<th>Annual Interest Rate</th>
<th>Market Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$50</td>
<td>2</td>
<td>9%</td>
<td>$ 32.196</td>
</tr>
<tr>
<td>$100</td>
<td>50</td>
<td>8%</td>
<td>$ 67.053</td>
</tr>
<tr>
<td>$50</td>
<td>1</td>
<td>10%</td>
<td>$ 31.046</td>
</tr>
<tr>
<td>$75</td>
<td>25</td>
<td>17%</td>
<td>$ 13.780</td>
</tr>
<tr>
<td>$30</td>
<td>4</td>
<td>3%</td>
<td>$ 25.836</td>
</tr>
<tr>
<td>$150</td>
<td>365</td>
<td>4%</td>
<td>$ 122.811</td>
</tr>
<tr>
<td>$200</td>
<td>∞</td>
<td>9.0%</td>
<td>$ 127.526</td>
</tr>
</tbody>
</table>
A six-month zero-coupon bond has a price of $97, while a 12-month 7.00% annual coupon bond (paid semiannually) has a price of $100.50. Both bonds have a face value of $100. Find the 12-month spot rate based on annual compounding, semiannual compounding, and continuous compounding.

Solution: all six-month cash flows are worth 97% of their face value, so the coupon bond’s first coupon is worth 0.97 × $3.50 = $3.395, leaving the 12-month cash flow to the coupon bond worth $100.50 − $3.395 = $97.105, which, based on a face value of $103.50 (including the semiannual coupon) means that 12-month cash flows are worth $97.105 / $103.50 = 93.821% of face value. A 12-month discount factor of 0.93821 implies a 6.59% annually compounded yield, a 6.48% semiannually compounded yield, and a 6.38% continuously compounded yield.
CALCULATIONS

To find the annually compounded yield,

Step One: Press → 0.07 → + → 2
Step Two: Press = “0.035”
Step Three: Press x → 100 → x → 0.97
Step Four: Press = “3.395”
Step Five: Press → 100.50 → - → 3.395
Step Six: Press = “97.105”
Step Five: Press → 97.105 → + → 103.50
Step Six: Press = “0.93821”
Step Seven: Press → “1/x”
Step Eight: Press = “1.0659”
Step Nine: Press → - → 1
Step Ten: Press =
Answer: 0.0659

To find the semiannually compounded yield,

Step One: Press → 0.07 → + → 2
Step Two: Press = “0.035”
Step Three: Press x → 100 → x → 0.97
Step Four: Press = “3.395”
Step Five: Press → 100.50 → - → 3.395
Step Six: Press = “97.105”
Step Five: Press → 97.105 → + → 103.50
Step Six: Press = “0.93821”
Step Seven: Press → “1/x”
Step Eight: Press = “1.0659”
Step Nine: Press → √x
Step Ten: Press = “1.0324”
Step Eleven → - → 1
Step Twelve: Press = “0.0324”
Step Thirteen: Press → x → 2
Step Fourteen: Press =
Answer: 0.0648

To find continuously compounding yield,

Step One: Press → 0.07 → + → 2
Step Two: Press = “0.035”
Step Three: Press x → 100 → x → 0.97
Step Four: Press = “3.395”
Step Five: Press → 100.50 → - → 3.395
Step Six: Press = “97.105”
Step Five: Press → 97.105 → + → 103.50
Step Six: Press = “0.93821”
Step Seven: Press → 1/x
Step Eight: Press = “1.0659”
Step Nine: Press → LN

99
Step Ten: Press =
Answer: 0.0638
As it is explained in the textbook, there should be no difference between investing for $T$ years (the longer dated bond) or combining an investment for $t$ years (shorter dated bond) and a forward for $T-t$ years. In this case, the choice is to invest in a 5-year bond or a 3-year bond + 2-year forward contract in year 3 (which gets you to five years). We are given the 5-year ($T$) and 3-year ($t$) spot rates, so we must solve for the 2-year ($T-t$) forward contract. The spot rates are continuously compounded, so we must use equation 5.13:

$$F(t, T) = \frac{r_T T - r_t t}{T - t}$$

$$F(t, T) = \frac{(7\%) (5) - (6\%) (3)}{5 - 3} = 8.5\%$$

**CALCULATIONS**

Step One: Press $\rightarrow$ 5 $\rightarrow$ $x$ $\rightarrow$ 0.07
Step Two: Press = “0.35”
Step Three: Press $\rightarrow$ 3 $\rightarrow$ $x$ $\rightarrow$ 0.06
Step Four: Press = “0.18”
Step Five: 0.35 $\rightarrow$ $-\rightarrow$ 0.18
Step Six: Press = “0.17”
Step Seven: Press $\rightarrow$ 5 $\rightarrow$ $-\rightarrow$ 3
Step Eight: Press = “2”
Step Nine: 0.17 $\rightarrow$ $+\rightarrow$ 2
Step Ten: Press =
Answer = 0.085
WORKOUT AREA: *Here are sample problems – cover one of the values and see if you can solve it using the others*

<table>
<thead>
<tr>
<th>T</th>
<th>Spot Rate (T)</th>
<th>t</th>
<th>Spot Rate (t)</th>
<th>Implied Continuously Compounded Interest Rate (T-t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>7%</td>
<td>3</td>
<td>6%</td>
<td>8.50%</td>
</tr>
<tr>
<td>10</td>
<td>8%</td>
<td>5</td>
<td>6%</td>
<td>10.00%</td>
</tr>
<tr>
<td>8</td>
<td>5%</td>
<td>3</td>
<td>4%</td>
<td>5.60%</td>
</tr>
<tr>
<td>9</td>
<td>10%</td>
<td>3</td>
<td>2%</td>
<td>14.00%</td>
</tr>
<tr>
<td>5</td>
<td>8%</td>
<td>1</td>
<td>7%</td>
<td>8.25%</td>
</tr>
</tbody>
</table>
APPLICATION 5.6.2A (page 136)

A stock currently selling for $10 will either rise to $30 or fall to $0 in three months. How much would a three-month call sell for if its strike price were $20? The payoff of the call ($10 = $30 - $20) would be one-third the payoff of the stock ($10/$30 = 1/3). Therefore, the call must sell for $3.33 ($10 stock price × 1/3).

EXPLANATION

The key piece to understand in this application is that the payoff of the call ($10 or $30 - $20) is 1/3 ($10/$30) of the ending stock price of $30.

To solve this application, we need to find the payoff of the call by subtracting the future stock price of $30 by the strike price of $20 for a difference of $10. Then divide $10 (pay off the call) by $30 (future stock price), which equals 1/3. Then multiply the 1/3 by $10 (current stock price) for a product of $3.33, which is the price of the call.

CALCULATIONS

Step One: Press $30 → - → $20
Step Two: Press ÷ → $30
Step Three: Press x → $10
Step Four: Press = Answer: 3.33

WORKOUT AREA: Here are sample problems – cover one of the values and see if you can solve it using the others

<table>
<thead>
<tr>
<th>1 Year Call</th>
<th>1 Year Call Strike</th>
<th>Current Stock</th>
<th>1 Year Stock Price</th>
<th>1 Year Stock Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3.33</td>
<td>$20.00</td>
<td>$10.00</td>
<td>$30.00</td>
<td>$0.00</td>
</tr>
<tr>
<td>$10.00</td>
<td>$25.00</td>
<td>$20.00</td>
<td>$50.00</td>
<td>$0.00</td>
</tr>
<tr>
<td>$6.00</td>
<td>$40.00</td>
<td>$10.00</td>
<td>$100.00</td>
<td>$0.00</td>
</tr>
<tr>
<td>$9.00</td>
<td>$20.00</td>
<td>$15.00</td>
<td>$50.00</td>
<td>$0.00</td>
</tr>
<tr>
<td>$2.86</td>
<td>$30.00</td>
<td>$20.00</td>
<td>$35.00</td>
<td>$0.00</td>
</tr>
<tr>
<td>$1.67</td>
<td>$5.00</td>
<td>$2.50</td>
<td>$15.00</td>
<td>$0.00</td>
</tr>
<tr>
<td>$8.00</td>
<td>$30.00</td>
<td>$10.00</td>
<td>$150.00</td>
<td>$0.00</td>
</tr>
</tbody>
</table>
APPLICATION 5.7.3A (page 139)

Consider a simplified scenario in which all market interest rates are currently 5%, compounded semiannually. An investor has two bonds in a portfolio:

1. A $1,000 face value one-year, 5% coupon bond (with semiannual coupon payments), and
2. A $2,000 face value, five-year zero-coupon bond.

What is the duration of the investor’s portfolio?

First, note that the duration of the five-year zero-coupon bond is 5.0. To calculate the duration of the one-year bond, view the bond as a portfolio of two cash flows: a $25 cash flow (coupon) due in six months with a present value (i.e., market value) of $24.39 (found as $25/1.025), and a $1,025 cash flow (principal plus interest) with a value of $975.61 (found as $1,025/1.025/1.025). The duration of the one-year bond is: $\left(\frac{24.39/1,000}{1,000}\right) \times 0.50 + \left(\frac{975.61/1,000}{1,000}\right) \times 1.00 = 0.988$.

The current value of the portfolio is found by summing the values of the $1,000 coupon bond ($1,000) and the zero-coupon bond [$1,562.40, found as $2,000 \times (1.025)^{-10}$], for a total value of $2,562.30. The duration of the portfolio is formed as a value-weighted average of the durations of its assets: $\left(\frac{1,000/2,562.30}{2,562.30}\right) \times 0.988 + \left(\frac{1,562.40/2,562.30}{2,562.30}\right) \times 5.0] = 3.43$ years.

EXPLANATION

First, the duration of a zero-coupon bond is equal to its maturity. In the case, the zero-coupon bond matures in 5 years, so its duration is 5.0. The duration of a fixed-coupon bond is the weighted average of the longevities of the cash flows to a coupon bond where the weight of each of the bond’s cash flows is the proportion of the bond’s total value attributable to that cash flow.

To determine the duration of the one-year, 5% coupon bond, we can use equation 5.16:

\[
D = \frac{\sum_{t=1}^{T} \frac{tC(t)}{(1+y)^t}}{P_0}
\]

\[
D = \frac{0.50 \times 25}{(1.025)^1} + \frac{1.0 \times (1,000 + 25)}{(1.025)^2} \frac{1,000}{2,562.30}
\]

\[
D = 0.988
\]

Note that we use 0.50 for the first coupon and 1.0 for the second coupon and repayment of principal. This is because they coincide with proportion of the timeline of payments – the final payment at maturity is 1.0 and the coupon payments are in proportion to the final payment – in this case, six months relative to one.
year maturity.

Next, we must combine the two bonds in a value-weighted portfolio, based on today’s market values. The current market interest rate across all maturities is 5%, so the one-year, 5% coupon bond is trading at par ($1,000). The five-year, zero-coupon bond will return $2,000 at maturity, so the current market value is as follows.

\[
Price \ of \ Zero \ Coupon \ Bond = \frac{\$2,000}{(1.025)^{10}} = \$1,562.40
\]

Next, we must combine the two bonds in the portfolio to get a total portfolio value of $2,562.40 ($1,000 + $1,562.40).

Next, we must find the weighted average duration between the two bonds. The coupon-paying bond represents 39.03% of the portfolio ($1,000 / $2,562.40), while the zero-coupon bond represents 60.97% of the portfolio ($1,562.40 / $2,562.40).

Finally, we find the weighted average duration of the portfolio:

\[
Duration = 0.3903 \times 0.988 + 0.6097 \times 5.0
\]

\[
Duration = 3.43 \text{ years}
\]

**CALCULATIONS**

Press \(2^\text{ND} \rightarrow \text{CLR WORK}\)

*Note: this problem will include storage and recall, do not clear work until the end.*

To find the duration of the one-year, coupon bond:

Step One: Press 0.50 \(\rightarrow\) \(\times\) \(\rightarrow\) 25
Step Two: Press = “12.5”
Step Three: Press + \(\rightarrow\) (1.025)
Step Four: Press = “12.1951”
Step Five: Press STO \(\rightarrow\) 1
Step Six: Press 1000 \(\rightarrow\) + \(\rightarrow\) 25
Step Seven: Press = “1025”
Step Eight: Press + \(\rightarrow\) 1.025 \(\rightarrow\) \(x^2\)
Step Nine: Press = “975.6097”
Step Ten: Press + \(\rightarrow\) RCL \(\rightarrow\) 1
Step Eleven: Press = “987.804”
Step Twelve: Press + 1000
Step Thirteen: Press =
Answer = 0.988
STO \(\rightarrow\) 2

To find the price of the five-year, zero-coupon bond:

Step One: Press 2000 \(\rightarrow\)
Step Two: Press + \(\rightarrow\) 1.025 \(\rightarrow\) \(y^x\)
Step Three: Press 10
To find the weighted average duration of the portfolio:

Step One: Press 1000 → + → 1562.40
Step Two: Press = “2562.40”
Step Three: Press STO → 4
Step Four: Press 1000 → + → RCL → 4
Step Five: Press = “0.3903”
Step Six: Press STO → 5
Step Seven: Press 1562.40 → + → RCL → 4
Step Eight: Press = “0.6097”
Step Nine: Press STO → 6
Step Ten: Press RCL → 2
Step Eleven: Press x → RCL → 5
Step Twelve: Press + → RCL → 6
Step Thirteen: Press 5
Step Fourteen: Press =
Answer = 3.43
APPLICATION 5.7.4A (page 140)

Consider a simplified scenario in which an investor has two bonds in a portfolio:

(1) $1,000 of market value in a 10-year zero-coupon bond, and
(2) $1,000 of market value in a five-year zero-coupon bond.

If the investor wishes to be immunized to a horizon point of 7.0 years, what transactions should be executed?

Note that the investor’s current duration is 7.5, found as: \((0.50 \times 5) + (0.50 \times 10)\). To obtain the target duration of 7.0 years, she needs to select portfolio weights based on market values \((w)\) for the five-year bond and \((1 - w)\) for the 10-year bond, such that the portfolio’s duration is equal to the 7.0 year time horizon:

\[(w \times 5) + (1 - w) \times 10 = 7.0\]

\[5w = 10 - 7\]

\[w = 0.60\]

Thus, $200 of the 10-year bond should be sold to lower its weight to \((800/2,000 = 40\%)\) and used to purchase more of the five-year bond, to bring its weight to \((1,200/2,000 = 60\%)\), so that the duration is 7.0, found as: \((0.6 \times 5.0) + (0.4 \times 10)\).

EXPLANATION

With zero-coupon bonds, the math becomes fairly easy. There are no intermediate cash flows to impact their durations, and we know the duration will be equal to the length of maturity. This is simply a weighted average calculation where we are solving for the weights.

\[(w \times 5) + (1 - w) \times 10 = 7.0\]

\[5w = 10 - 7\]

\[w = 0.60\]

Note: it does not matter to which bond “w” or “1-w” is assigned. However, they must line up with the appropriate duration measure.

CALCULATIONS

Once we have simplified the algebra down to \(5w = 10 - 7\), we can do the following:

Step One: Press 10 → - → 7
Step Two: Press = “3”
Step Three: Press + → 5
Step Four: Press =
Answer = 0.60
WORKOUT AREA: Here are sample problems — cover one of the values and see if you can solve it using the others

<table>
<thead>
<tr>
<th>Target Duration</th>
<th>Bond 1</th>
<th>Bond 2</th>
<th>Weight of Bond 1</th>
<th>Weight of Bond 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>10</td>
<td>5</td>
<td>40.00%</td>
<td>60.00%</td>
</tr>
<tr>
<td>8</td>
<td>9</td>
<td>7</td>
<td>85.71%</td>
<td>14.29%</td>
</tr>
<tr>
<td>9</td>
<td>10</td>
<td>5</td>
<td>80.00%</td>
<td>20.00%</td>
</tr>
<tr>
<td>5</td>
<td>8</td>
<td>4</td>
<td>25.00%</td>
<td>75.00%</td>
</tr>
<tr>
<td>6.5</td>
<td>9</td>
<td>3</td>
<td>16.67%</td>
<td>83.33%</td>
</tr>
<tr>
<td>8</td>
<td>10</td>
<td>5</td>
<td>60.00%</td>
<td>40.00%</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td>3</td>
<td>66.67%</td>
<td>33.33%</td>
</tr>
</tbody>
</table>
APPLICATION 5.7.7A (page 142)

An investor has a $1,000,000 portfolio with long positions that form a duration of 5.0 years. The investor wishes to consider two alternatives: adding $1,000,000 in short positions to hedge the portfolio or adding $500,000 in short positions to hedge the portfolio. What securities would provide immunization under the two scenarios?

Solution: the $1,000,000 long position with a duration of 5.0 years can be hedged with $1,000,000 in short positions if the short positions have a duration of $\frac{5.0}{2}$ years (i.e., short $1,000,000$ of five-year zero-coupon bonds or other assets that would have a positive duration of 5.0 if held long). The negative position implicit in the short position will offset the positive duration exposure of the long position for an infinitesimal, parallel, and instantaneous shift in interest rates. In order to form a hedge with only $500,000$ of short positions, the positions would have to have a duration of $-10.0$ years, such as having $500,000$ of market value short sold in 10-year zero-coupon bonds. The proceeds of the short sales should be held in cash to avoid introducing further interest rate risk.

EXPLANATION

In order to completely hedge out the risk of the $1,000,000 portfolio of long positions, an effectively equal and opposite (i.e., short) position must be taken to bring the exposure down to 0.00. A good way to check this quantitatively is to multiply the duration x the position size, then check for answers where adding the duration x a negative position size would equal zero. We can use this equation below:

\[(1,000,000 \times 5.0) - (V \times D) = 0\]

\[V \times D = (1,000,000 \times 5.0) - 0 = 5,000,000\]

CALCULATIONS

To find the combined V and D, we do the following:

Step One: Press → 1,000,000 → x → 5
Step Two: Press = “5,000,000”
Step Three: Press - → 0
Step Four: Press =
Answer = 5,000,000
WORKOUT AREA: Here are sample problems – cover one of the values and see if you can solve it using the others.

<table>
<thead>
<tr>
<th>Long Position</th>
<th>Duration of Long Position</th>
<th>Target Duration</th>
<th>(V x D) Needed to Hedge Duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1,000,000.00</td>
<td>5</td>
<td>0</td>
<td>$(5,000,000.00)</td>
</tr>
<tr>
<td>$2,000,000.00</td>
<td>4</td>
<td>0</td>
<td>$(8,000,000.00)</td>
</tr>
<tr>
<td>$300,000,000.00 2 0  $(600,000,000.00)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$500,000.00  10  0  $(5,000,000.00)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$30,000.00  2  0  $(60,000.00)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$1,000,000.00  7  0  $(7,000,000.00)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
An investor has a $1,000,000 portfolio with long positions that form a duration of 5.0 years. The investor’s goal for the portfolio is to have a duration of 4.0 years because the portfolio is to be liquidated at that time to fund a project. The investor wishes to add short positions to hedge the portfolio to a duration of 4.0 years.

There are many solutions to this problem. For example, the investor could short $200,000 of five-year zero-coupon bonds and hold the proceeds from the short sale in cash. Any combination of dollar amount \( V \) and duration \( D \) that solves the following equation would lower the duration to 4.0 years:

\[
($1,000,000 \times 5.0) - (V \times D) = $4,000,000
\]

\[
V \times D = ($1,000,000 \times 5.0) - $4,000,000 = $1,000,000
\]

**EXPLANATION**

Similar to Application 5.7.7A, we must find a combination of duration and notional value to hedge the existing portfolio. The application uses $200,000 and 5.0 years duration, but we could easily use $1,000,000 and 1.0 year duration as well. For this problem, whatever combination that gets us to $1,000,000 will hedge the portfolio appropriately.

**CALCULATIONS**

To find the combined \( V \) and \( D \), we do the following:

- **Step One:** Press → 1,000,000 → \( x \) → 5
- **Step Two:** Press “5,000,000”
- **Step Three:** Press - → 4,000,000
- **Step Four:** Press =
- **Answer:** 1,000,000

**WORKOUT AREA:** Here are sample problems – cover one of the values and see if you can solve it using the others.

<table>
<thead>
<tr>
<th>Long Position</th>
<th>Duration of Long Position</th>
<th>Target Duration</th>
<th>(V x D) Needed to Hedge Duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1,000,000.00</td>
<td>5</td>
<td>4</td>
<td>$ (1,000,000.00)</td>
</tr>
<tr>
<td>$2,000,000.00</td>
<td>4</td>
<td>2</td>
<td>$ (4,000,000.00)</td>
</tr>
<tr>
<td>$300,000,000</td>
<td>2</td>
<td>1</td>
<td>$ (300,000,000.00)</td>
</tr>
<tr>
<td>$500,000.00</td>
<td>10</td>
<td>5</td>
<td>$ (2,500,000.00)</td>
</tr>
<tr>
<td>$30,000.00</td>
<td>2</td>
<td>1</td>
<td>$ (30,000.00)</td>
</tr>
<tr>
<td>$1,000,000.00</td>
<td>7</td>
<td>5</td>
<td>$ (2,000,000.00)</td>
</tr>
</tbody>
</table>
APPLICATION 5.8.2A (page 145)

Using the CAPM equation, when the risk-free rate is 2%, the expected return of the market is 10%, and the beta of asset i is 1.25, what is the expected return of asset i?

By placing each of these variables on the right side of Equation 5.17 and solving the left side, the expected return of asset i is 12%.

EXPLANATION

Apply equation 5.17 to solve the CAPM equation for the expected return of asset i. Subtract 10% (expected return of the market) by 2% for a difference of 8%. Multiply 8% by 1.25 (beta of the asset) for a product of 10%. Lastly, add the risk-free rate of 2% to 10% for a sum of 12%, which is the expected return of asset i.

CALCULATIONS

Step One: Press 0.1 → - → 0.02 Step
Two: Press x → 1.25
Step Three: Press + → 0.02
Step Four: Press =
Answer: 0.12

WORKOUT AREA: Here are sample problems – cover one of the values and see if you can solve it using the others

<table>
<thead>
<tr>
<th>Expected Return of</th>
<th>Risk-Free Rate</th>
<th>Expected Return of</th>
<th>Beta of Asset i</th>
</tr>
</thead>
<tbody>
<tr>
<td>12.00%</td>
<td>2.00%</td>
<td>10.00%</td>
<td>1.25</td>
</tr>
<tr>
<td>16.75%</td>
<td>2.50%</td>
<td>12.00%</td>
<td>1.5</td>
</tr>
<tr>
<td>27.00%</td>
<td>3.00%</td>
<td>15.00%</td>
<td>2</td>
</tr>
<tr>
<td>40.00%</td>
<td>4.00%</td>
<td>20.00%</td>
<td>2.25</td>
</tr>
<tr>
<td>25.00%</td>
<td>2.00%</td>
<td>25.00%</td>
<td>1</td>
</tr>
<tr>
<td>17.03%</td>
<td>1.50%</td>
<td>15.00%</td>
<td>1.15</td>
</tr>
<tr>
<td>6.25%</td>
<td>2.50%</td>
<td>10.00%</td>
<td>0.5</td>
</tr>
</tbody>
</table>
Applying 5.8.3A (page 147)

Returning to the previous example in which the risk-free rate is 2% and the beta of asset i is 1.25, if the actual return of the market is 22%, the ex post CAPM model would generate a return due to non-idiosyncratic effects of 27% for the asset: 2% + [1.25(22% − 2%)]. If the asset’s actual return is 30%, then the extra 3% would be attributable to idiosyncratic return, $\varepsilon_{it}$.

Explanations

This is similar to Application 5.8.2A, but instead of using expected return, actual returns are used in the model. We need to apply a slightly modified equation 5.19 as shown below:

$$R_{it} = R_{f} + \beta_{i}(R_{m} - R_{f}) + \varepsilon_{it}$$

Subtract 22% (actual return of the market) from 2% (the risk-free rate) for a difference of 20%. Multiply 20% by 1.25 for a product of 25%. Then add 2% to 25% for a sum of 27%, which is the asset return due to non-idiosyncratic effects.

If the asset actually returned 30% (i.e. $R_{it} = 30\%$), then 3% difference between 27% (asset return due to non-idiosyncratic effects) and 30% (actual asset return), would be attributable to idiosyncratic return, $\varepsilon_{it}$. Thus, $\varepsilon_{it} = 3\%$

Calculations

Step One: Press 0.22 → - → 0.02
Step Two: Press x → 1.25
Step Three: Press + → 0.02
Step Four: Press =
Answer: 0.27

To find the idiosyncratic return
Step One: Press 0.3 → - → 0.27
Step Two: Press =
Answer: 0.03
WORKOUT AREA: Here are sample problems – cover one of the values and see if you can solve it using the others

<table>
<thead>
<tr>
<th>Actual Return of the Asset</th>
<th>Actual Return of the Asset Attributable to Non-Idiosyncratic Effects</th>
<th>Risk-Free Rate</th>
<th>Actual Return of the Market</th>
<th>Beta of Asset i</th>
<th>$\varepsilon_{it}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>30.00%</td>
<td>27.00%</td>
<td>2.00%</td>
<td>22.00%</td>
<td>1.25</td>
<td>3.00%</td>
</tr>
<tr>
<td>23.00%</td>
<td>16.75%</td>
<td>2.50%</td>
<td>12.00%</td>
<td>1.5</td>
<td>6.25%</td>
</tr>
<tr>
<td>29.00%</td>
<td>27.00%</td>
<td>3.00%</td>
<td>15.00%</td>
<td>2</td>
<td>2.00%</td>
</tr>
<tr>
<td>22.00%</td>
<td>40.00%</td>
<td>4.00%</td>
<td>20.00%</td>
<td>2.25</td>
<td>-18.00%</td>
</tr>
<tr>
<td>41.00%</td>
<td>25.00%</td>
<td>2.00%</td>
<td>25.00%</td>
<td>1</td>
<td>16.00%</td>
</tr>
<tr>
<td>28.30%</td>
<td>17.03%</td>
<td>1.50%</td>
<td>15.00%</td>
<td>1.15</td>
<td>11.28%</td>
</tr>
<tr>
<td>10.60%</td>
<td>6.25%</td>
<td>2.50%</td>
<td>10.00%</td>
<td>0.5</td>
<td>4.35%</td>
</tr>
</tbody>
</table>
APPLICATION 6.2.2A (page 156)

A three-year riskless security trades at a yield of 3.4%, whereas a forward contract on a two-year riskless security that settles in three years trades at a forward rate of 2.4%. Assuming that the rates are continuously compounded, what is the no-arbitrage yield of a five-year riskless security?

Inserting 3.4% as the shorter-term rate in Equation 6.2 and 2.4% as the left side of Equation 6.2, the longer-term rate, \( R_T \), can be solved as 3.0%, noting that \( T = 5 \) and \( t = 3 \). Note that earning 3.0% for five years (15%) is equal to the sum of earning 3.4% for three years (10.2%) and 2.4% for two years (4.8%). The rates may be summed due to the assumption of continuous compounding.

EXPLANATION

We need to manipulate Equation 6.2 to solve for \( R_T \).

\[
F_{T-t} = \frac{(T \times R_T - t \times R_t)}{(T - t)}
\]

\[
F_{T-t} \times (T - t) = (T \times R_T - t \times R_t)
\]

\[
F_{T-t} \times (T - t) + t \times R_t = T \times R_t
\]

\[
R_T = \frac{F_{T-t} \times (T - t) + t \times R_t}{T}
\]

We have solved for \( R_T \). Now \( R_t = 3.4\% \), \( F_{T-t} = 2.4\% \), \( T = 5 \), and \( t = 3 \)

\[
R_T = \frac{0.024 \times (5 - 3) + 3 \times 0.034}{5}
\]

\[
R_T = \frac{0.024 \times (2) + 3 \times 0.034}{5}
\]

\[
R_T = \frac{0.048 + 3 \times 0.034}{5}
\]

\[
R_T = \frac{0.048 + 0.102}{5}
\]

\[
R_T = \frac{0.15}{5}
\]

\[
R_T = 0.03
\]

Therefore, \( R_T = 3.0\% \). If we multiple 3.0% by 5, the product is 15% which is the sum of the two products of 3.4% by 3 and 2.4% by 2. These rates can be summed because of the assumption of continuous compounding.
CALCULATIONS

Step One: Press 5 → - → 3
Step Two: Press x → 0.024
Step Three: Press = “0.048”
Step Four: Press 3 → x → 0.034
Step Five: Press + → 0.048 Step Six: Press ÷ → 5
Step Seven: Press =
Answer: 0.03

WORKOUT AREA - Here are sample problems – cover one of the values and see if you can solve it using the others

<table>
<thead>
<tr>
<th>F_{T-t}</th>
<th>T</th>
<th>R_{T}</th>
<th>t</th>
<th>R_{t}</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.40%</td>
<td>5</td>
<td>3.00%</td>
<td>3</td>
<td>3.40%</td>
</tr>
<tr>
<td>3.00%</td>
<td>7</td>
<td>3.57%</td>
<td>2</td>
<td>5.00%</td>
</tr>
<tr>
<td>3.00%</td>
<td>6</td>
<td>3.50%</td>
<td>3</td>
<td>4.00%</td>
</tr>
<tr>
<td>2.00%</td>
<td>5</td>
<td>2.90%</td>
<td>3</td>
<td>3.50%</td>
</tr>
<tr>
<td>5.00%</td>
<td>10</td>
<td>5.50%</td>
<td>5</td>
<td>6.00%</td>
</tr>
<tr>
<td>3.00%</td>
<td>5</td>
<td>4.20%</td>
<td>3</td>
<td>5.00%</td>
</tr>
<tr>
<td>2.00%</td>
<td>5</td>
<td>2.60%</td>
<td>3</td>
<td>3.00%</td>
</tr>
</tbody>
</table>
A stock sells for $100 and is certain not to make any cash distributions in the next year. A forward contract on that stock trades with a settlement in one year. Assuming that the interest rate corresponding to one year is 5% compounded continuously, what is the no-arbitrage price of this forward contract?

A one-year forward contract on the stock must trade at $105.13 using Equation 6.4. At settlement, a long position in the forward contract obligates the holder to pay $105.13 in exchange for delivery of the stock. If the holder of the forward contract places the stock’s initial value ($100) in an account (or as collateral) offering at 5% continuously compounded return, the $100 investment will enable the purchase of the stock at the end of the year without further cash.

EXPLANATION

The price of the forward contract needs to be the same as the expected price to purchase the stock in the spot market after one year assuming risk neutrality (i.e., the stock price grows at the 5% riskless rate). A stock sells for $100, so we need to pay $100 in the spot market for that stock. To finance that purchase it costs 5% continuously compounded, so purchase stock in the spot market and hold it one year costs $105.13.

Note, this stock is not expected to pay a dividend in the next year. If it did, it would decrease the cost of financing.

CALCULATIONS

Step One: Press 0.05
Step Two: Press LN
Step Three: Press $ \times \rightarrow 100$
Step Four: Press $=$

Answer = “105.13”

WORKOUT AREA –

<table>
<thead>
<tr>
<th>Price</th>
<th>Riskless Rate Continuously Compounded</th>
<th>Dividend Yield</th>
<th>Time</th>
<th>No-Arbitrage Forward Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>$100</td>
<td>5%</td>
<td>0%</td>
<td>1.00</td>
<td>$105.13</td>
</tr>
<tr>
<td>$105</td>
<td>4%</td>
<td>0%</td>
<td>1.00</td>
<td>$109.29</td>
</tr>
<tr>
<td>$50</td>
<td>6%</td>
<td>0%</td>
<td>4.00</td>
<td>$212.37</td>
</tr>
<tr>
<td>$30</td>
<td>3%</td>
<td>0%</td>
<td>1.00</td>
<td>$30.91</td>
</tr>
<tr>
<td>$200</td>
<td>5%</td>
<td>0%</td>
<td>1.00</td>
<td>$210.25</td>
</tr>
<tr>
<td>$175</td>
<td>3%</td>
<td>0%</td>
<td>2.00</td>
<td>$360.66</td>
</tr>
</tbody>
</table>
A stock sells for $50 and pays a continuous dividend yield of 1% per year. Assuming a 5% continuously compounded riskless rate, what is the no-arbitrage price of this forward contract with a time to settlement or delivery of 0.25 years?

The formula for the forward contract on a dividend paying risky asset is:

\[ F_t = P_0 \times e^{(r-q) \times T} \]

\[ F_t = 50 \times e^{(0.05-0.01) \times 0.25} \]

\[ F_t = 50.5025 \]

**EXPLANATION**

In order to solve this, we will need to utilize equation 6.5. To find the forward price (Ft), we need the current spot price (P0), the continuous dividend yield (q), the continuously compounded riskless rate (r), and the time until maturity (T). Note that in order to use equation 6.5, the riskless rate and dividend yield must both be continuously compounded. If they are not, the equation will not work.

\[ F_t = P_0 \times e^{(r-q) \times T} \]

\[ F_t = 50 \times e^{(0.05-0.01) \times 0.25} \]

\[ F_t = 50.5025 \]

**CALCULATIONS**

Step One: Press 0.05 → - → 0.01
Step Two: Press = “0.04”
Step Three: Press x → 0.25 → = → “0.01”
Step Five: Press 2ND → e^x
Step Six: Press = “1.01005”
Step Seven: Press x → 50
Step Eight: Press =
Answer = 50.5025

**WORKOUT AREA**

<table>
<thead>
<tr>
<th>Price</th>
<th>Continuous Div Yield</th>
<th>Continuous Riskless Rate</th>
<th>t</th>
<th>Forward Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>$50.00</td>
<td>1%</td>
<td>5%</td>
<td>0.25</td>
<td>$50.5025</td>
</tr>
<tr>
<td>$45.00</td>
<td>2%</td>
<td>3%</td>
<td>1.00</td>
<td>$45.4523</td>
</tr>
<tr>
<td>$40.00</td>
<td>2%</td>
<td>6%</td>
<td>2.00</td>
<td>$43.3315</td>
</tr>
<tr>
<td>$60.00</td>
<td>1%</td>
<td>5%</td>
<td>3.00</td>
<td>$67.6498</td>
</tr>
<tr>
<td>$42.00</td>
<td>2%</td>
<td>4%</td>
<td>0.75</td>
<td>$42.7148</td>
</tr>
<tr>
<td>$50.00</td>
<td>2%</td>
<td>5%</td>
<td>1.00</td>
<td>$51.5227</td>
</tr>
</tbody>
</table>
APPLICATION 6.3.6A (page 163)

If the spot price of an equity index that pays a dividend yields equal to the riskless rate is $500, what is the one-year forward price on the equity index?

The forward contract of every time to delivery has a forward price of exactly $500. Market participants would be indifferent between buying and selling the index in the spot market with instant delivery or in the forward market with delayed delivery because the interest payments and dividends offset each other.

EXPLANATION

Applying Equation 6.5, we find that the exponential of 0 equals 1. If the dividend yield equals the riskless rate, the $(r-q)$ will always be 0.00%. Regardless of the time horizon $(T)$, the exponent will still equal 0. Therefore, the spot price of the equity index of $500 multiplied by 1 equals $500, which is the one-year forward price on the equity index.

Using Equation 6.5:

$$F_t = P_0 \times e^{(r-q) \times T}$$

$$F_t = 500 \times e^{(0.02\times0.02) \times 1}$$

$$F_t = 500$$

CALCULATION

This is for illustrative purposes: assume the dividend yield equals 2% and the riskless rate equals 2%.

Step One: Press 0.02 → - → 0.02
Step Two: Press = “0.00”
Step Three: Press x → 1 → = → “0.00”
Step Five: Press 2ND → e^x
Step Six: Press = “1.00”
Step Seven: Press x → 500
Step Eight: Press =
Answer = 500
WORKOUT AREA: Here are sample problems – cover one of the values and see if you can solve it using the others.

<table>
<thead>
<tr>
<th>One-Year Forward Price of Index</th>
<th>Spot Price</th>
<th>Riskless Rate</th>
<th>Dividend Yield of Equity Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>$500.00</td>
<td>$500.00</td>
<td>5%</td>
<td>5%</td>
</tr>
<tr>
<td>$200.00</td>
<td>$200.00</td>
<td>3%</td>
<td>3%</td>
</tr>
<tr>
<td>$404.02</td>
<td>$400.00</td>
<td>2%</td>
<td>1%</td>
</tr>
<tr>
<td>$353.52</td>
<td>$350.00</td>
<td>5%</td>
<td>4%</td>
</tr>
<tr>
<td>$105.13</td>
<td>$100.00</td>
<td>7%</td>
<td>2%</td>
</tr>
<tr>
<td>$50.00</td>
<td>$50.00</td>
<td>4%</td>
<td>4%</td>
</tr>
</tbody>
</table>
APPLICATION 6.3.6B (page 164)

Assuming a continuously compounded annual interest rate of 5%, if the spot price of an equity index with 2% continuously paid dividends is $500, what would be the forward price on the equity index with settlement in three months?

The price of every forward contract on that index for every time to settlement would be $500e^{(0.05−0.02)T}. The three-month forward price would be $500e^{(0.03 \times 0.25)}$, or $503.76. Six-month and 12-month forward prices would be $507.56$ and $515.23$, respectively (found by inserting 0.50 and 1.00 for $T$, and 0.03 for $r−q$).

EXPLANATION

Similarly to Application 6.3.6a we need to utilize equation 6.5.

In this scenario we are varying $T$ for three-months (0.25), six-months (0.50), and twelve-months (1). Before we do that, let’s fill out equation 6.11 with the information we have:

\[500e^{(0.05−0.02)T}\]

Now we can solve for all three scenarios, by varying $T$ as outlined above.

**Three-Months:**

\[
\begin{align*}
&= 500e^{(0.03)(0.25)} \\
&= 503.76
\end{align*}
\]

**Six-Months:**

\[
\begin{align*}
&= 500e^{(0.03)(0.50)} \\
&= 507.56
\end{align*}
\]

**Twelve-Months:**

\[
\begin{align*}
&= 500e^{(0.03)(1)} \\
&= 515.23
\end{align*}
\]
CALCULATIONS

Three-Months
Step One: Press 0.05 → - → 0.02
Step Two: Press x → 0.25
Step Three: Press \(2^{nd}\) → \(e^x\)
Step Four: Press x → 500 Step
Five: Press = Answer: $503.76

Six-Months
Step One: Press 0.05 → - → 0.02 Step
Two: Press x → 0.50
Step Three: Press \(2^{nd}\) → \(e^x\)
Step Four: Press x → 500 Step
Five: Press = Answer: $507.56

Twelve-Months
Step One: Press 0.05 → - → 0.02 Step
Two: Press x → 1
Step Three: Press \(2^{nd}\) → \(e^x\)
Step Four: Press x → 500 Step
Five: Press = Answer: $515.23
WORKOUT AREA: Here are sample problems – cover one of the values and see if you can solve it using the others.

<table>
<thead>
<tr>
<th>3-Month Forward Price of Equity Index</th>
<th>6-Month Forward Price of Equity Index</th>
<th>One-year Forward Price of Equity Index</th>
<th>Spot Price of an Equity Index</th>
<th>Risk-Free Rate</th>
<th>Dividends of Equity Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>$503.76</td>
<td>$507.56</td>
<td>$515.23</td>
<td>$500.00</td>
<td>5.00%</td>
<td>2.00%</td>
</tr>
<tr>
<td>$250.00</td>
<td>$250.00</td>
<td>$250.00</td>
<td>$250.00</td>
<td>5.00%</td>
<td>5.0%</td>
</tr>
<tr>
<td>$100.25</td>
<td>$100.50</td>
<td>$101.01</td>
<td>$100.00</td>
<td>5.00%</td>
<td>2.0%</td>
</tr>
<tr>
<td>$49.75</td>
<td>$49.50</td>
<td>$49.01</td>
<td>$50.00</td>
<td>4.00%</td>
<td>4.0%</td>
</tr>
<tr>
<td>$75.28</td>
<td>$75.56</td>
<td>$76.13</td>
<td>$75.00</td>
<td>4.00%</td>
<td>1.0%</td>
</tr>
<tr>
<td>$24.94</td>
<td>$24.88</td>
<td>$24.75</td>
<td>$25.00</td>
<td>5.00%</td>
<td>5.0%</td>
</tr>
<tr>
<td>$363.08</td>
<td>$356.18</td>
<td>$362.47</td>
<td>$350.00</td>
<td>0.0%</td>
<td>0.0%</td>
</tr>
</tbody>
</table>
APPLICATION 6.3.6C (page 164)

Assuming a continuously compounded annual interest rate of 2%, if the spot price of an equity index with 3% continuously paid dividends is $500, what would be the forward price of a contract with settlement in three months?

The price of every forward contract of every time to delivery would be $500e^{(-0.01)T}$, with $(r-q) = -1\%$. The three-month forward price would be $500e^{-0.01 \times 0.25}$, or $498.75. Six-month and 12-month forward prices would be $497.51$ and $495.02$, respectively (found by inserting 0.50 and 1.00 for $T$).

EXPLANATION

Please see the explanation for Application 6.3.6A. Note that the applications only differ by whether the riskless rate is higher than the dividend yield or vice versa.
CALCULATIONS

Three-Months
Step One: Press 0.02 → - → 0.03
Step Two: Press x → 0.25
Step Three: Press $2^{nd}$ → $e^x$
Step Four: Press x → 500 Step
Five: Press = Answer: $498.75$

Six-Months
Step One: Press 0.02 → - → 0.03 Step
Two: Press x → 0.50
Step Three: Press $2^{nd}$ → $e^x$
Step Four: Press x → 500 Step
Five: Press = Answer: $497.51$

Twelve-Months
Step One: Press 0.02 → - → 0.03 Step
Two: Press x → 1
Step Three: Press $2^{nd}$ → $e^x$
Step Four: Press x → 500 Step
Five: Press = Answer: $495.02$
WORKOUT AREA: Here are sample problems – cover one of the values and see if you can solve it using the others

<table>
<thead>
<tr>
<th>3-Month Forward Index</th>
<th>6-Month Forward Price of Equity Index</th>
<th>One-year Forward Price of Equity Index</th>
<th>Spot Price of an Equity Index</th>
<th>Risk-Free Rate</th>
<th>Dividends of Equity Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>$498.75</td>
<td>$497.51</td>
<td>$495.02</td>
<td>$500.00</td>
<td>2.00%</td>
<td>3.00%</td>
</tr>
<tr>
<td>$100.00</td>
<td>$100.00</td>
<td>$100.00</td>
<td>$100.00</td>
<td>5.00%</td>
<td>5.0%</td>
</tr>
<tr>
<td>$150.19</td>
<td>$150.38</td>
<td>$150.75</td>
<td>$150.00</td>
<td>2.50%</td>
<td>2.0%</td>
</tr>
<tr>
<td>$75.09</td>
<td>$75.19</td>
<td>$75.38</td>
<td>$75.00</td>
<td>1.00%</td>
<td>0.5%</td>
</tr>
<tr>
<td>$94.76</td>
<td>$94.53</td>
<td>$94.05</td>
<td>$95.00</td>
<td>5.00%</td>
<td>6.0%</td>
</tr>
<tr>
<td>$35.35</td>
<td>$35.71</td>
<td>$36.43</td>
<td>$35.00</td>
<td>4.00%</td>
<td>0.0%</td>
</tr>
<tr>
<td>43.72</td>
<td>$44.44</td>
<td>$43.89</td>
<td>$45.00</td>
<td>2.50%</td>
<td>5.0%</td>
</tr>
</tbody>
</table>
APPLICATION 6.4.2A (page 167)

Consider a six-month forward contract on a commodity that trades at a spot price of $50. The commodity has market-wide convenience yields of 3%, storage costs of 2%, and financing costs (interest rates) of 7%. What is the price of the six-month forward contract on the commodity? The forward price is $51.52, found by placing $0.5(7% + 2% − 3%) in as the exponent of Equation 6.6, $50 as $P_0$, and solving for $F_T$.

EXPLANATION

Application 6.4.2A involves solving for the left side of Equation 6.6 given all of the values on the right side. Note that the solution involved the exponential function ($e^x$). To solve for $e^x$ simply enter the value of $x$ into the calculator and then press the $e^x$ button (example $e^{x+y}$ when $x$ has a value of 2.1 and $y$ has a value of 3.1 has the keystrokes: 2.1 + 3.1 = $e^x$).

Proficiency in applications involving Equation 11.2 may include the ability to solve for one of the values on the right side of Equation 11.2 give all of the other values (including the left side). This can be accomplished by rearranging the formula so that the missing value is alone on the left side. For $S$ this is easy: $S=F(T) e^{-(r+c- y)T}$ (note that a term with a negative exponent is equivalent to placing the term in a denominator (e.g., $e^{-(r+c-y)T} = 1/(e^{(r+c-y)T})$). For $r$, $c$, and $y$ the term must be brought out of the exponent by taking the natural logarithm of each side of the equation. Dividing $F(T)$ by $S$ in Equation 6.6 and taking the natural logarithm of each side produces: $\ln \frac{F(T)}{S} = (r+c-y)T$.

This resulting equation can be easily factored to solve for $r$, $c$, $y$, or $T$. For example, so solve for $r$ the equation is: $r = \frac{\ln[F(T)/S] - (c-y)T}{T}$. Consider a problem where $F(T) = $52, $S = $50, $c = 6\%$, $y = 4\%$ and $T = 0.25$. Therefore $r = \frac{\ln(52/50) - (.06-.04)*0.25}{0.25}$.

***. Note that the natural logarithm of $x$ is found with the keystrokes; $x \ln$.

CALCULATIONS

Step One: Press $0.07 \rightarrow + \rightarrow 0.02$ Step
Step Two: Press $-$ $→$ 0.03
Step Three: Press $x$ $→$ 0.5
Step Four: Press $2^{\text{nd}}$ $→$ $e^x$
Step Five: Press $x$ $→$ 50 Step
Six: Press $=$ Answer: $51.52$
WORKOUT AREA: Here are sample problems – cover one of the values and see if you can solve it using the others

<table>
<thead>
<tr>
<th>F(T)</th>
<th>S</th>
<th>r</th>
<th>c</th>
<th>y</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td>51.52</td>
<td>50</td>
<td>7.00%</td>
<td>2.00%</td>
<td>3.00%</td>
<td>0.50</td>
</tr>
<tr>
<td>40.59</td>
<td>38</td>
<td>3.00%</td>
<td>5.00%</td>
<td>2.00%</td>
<td>1.10</td>
</tr>
<tr>
<td>27.03</td>
<td>26</td>
<td>2.00%</td>
<td>2.00%</td>
<td>1.00%</td>
<td>1.30</td>
</tr>
<tr>
<td>13.34</td>
<td>14</td>
<td>1.00%</td>
<td>2.00%</td>
<td>4.00%</td>
<td>4.80</td>
</tr>
<tr>
<td>2.04</td>
<td>2</td>
<td>3.00%</td>
<td>5.00%</td>
<td>3.00%</td>
<td>0.37</td>
</tr>
<tr>
<td>10.00</td>
<td>10</td>
<td>2.00%</td>
<td>2.00%</td>
<td>4.00%</td>
<td>7.00</td>
</tr>
<tr>
<td>22.03</td>
<td>22</td>
<td>4.00%</td>
<td>2.00%</td>
<td>5.00%</td>
<td>0.15</td>
</tr>
</tbody>
</table>
APPLICATION 7.2.1A (page 199)

Let’s return to the example of JAC Fund’s $1 million holding of the ETF with an expected return of zero. Estimating roughly that the daily standard deviation of the ETF is 1.35%, for a 99% confidence interval, the 10-day VaR is found through substituting the known values into the equation:

\[ 2.33 \times \sigma \times \sqrt{\text{Days}} \times \text{Value} \]
\[ = 2.33 \times 1.35\% \times \sqrt{10} \times$1,000,000 \]

The first three values multiplied together produce the percentage change in the value that is being defined as a highly abnormal circumstance. In this case, the answer would be very roughly 10%, indicating that there is a 1% chance that the ETF could fall 10% or more in 10 business days. This percentage is then multiplied by the position’s value (the fourth term) to produce the dollar amount of the VaR. In the example, the 10% loss on the $1 million stock holdings would produce a VaR of approximately $100,000.

EXPLANATION

In order to solve this application we need to use equation 7.5. In this case, with a 99% confidence interval the z-score is 2.33. Following equation 7.5, we multiply 2.33 by 1.35% by the square root of 10 (the days in the period) to get a product of 9.94%. 9.94% represents the percentage change in the value. To complete this solution we multiply 9.94% by $1,000,000 for an answer of $99,469.44. The z-score is a value that is assumed to be provided rather than being memorized or calculated.

CALCULATIONS

Step One: Press 2.33 \( \rightarrow \) x \( \rightarrow \) 0.0135
Step Two: Press x \( \rightarrow \) 10 \( \rightarrow \) \( \sqrt{} \)\( x \)
Step Three: Press x \( \rightarrow \) 1,000,000
Step Four: Press =
Answer: 99469.44
Here are sample problems – cover one of the values (except confidence interval) and see if you can solve it using the others.

<table>
<thead>
<tr>
<th>Parametric VaR</th>
<th>Percentage Change in the Holdings Value</th>
<th>Daily Standard Deviation of Returns</th>
<th>Confidence Interval</th>
<th>Z-Score</th>
<th>Time Frame (days)</th>
<th>Funds Holding</th>
</tr>
</thead>
<tbody>
<tr>
<td>$99,469.44</td>
<td>9.95%</td>
<td>1.35%</td>
<td>99%</td>
<td>2.33</td>
<td>10</td>
<td>$1,000,000</td>
</tr>
<tr>
<td>$70,439.73</td>
<td>7.04%</td>
<td>1.35%</td>
<td>95%</td>
<td>1.65</td>
<td>10</td>
<td>$1,000,000</td>
</tr>
<tr>
<td>$1,209,222.86</td>
<td>24.18%</td>
<td>2.68%</td>
<td>99%</td>
<td>2.33</td>
<td>15</td>
<td>$5,000,000</td>
</tr>
<tr>
<td>$856,316.62</td>
<td>17.13%</td>
<td>2.68%</td>
<td>95%</td>
<td>1.65</td>
<td>15</td>
<td>$5,000,000</td>
</tr>
<tr>
<td>$361,576.66</td>
<td>18.08%</td>
<td>3.47%</td>
<td>99%</td>
<td>2.33</td>
<td>5</td>
<td>$2,000,000</td>
</tr>
<tr>
<td>$256,052.14</td>
<td>12.80%</td>
<td>3.47%</td>
<td>95%</td>
<td>1.65</td>
<td>5</td>
<td>$2,000,000</td>
</tr>
<tr>
<td>$700,165.00</td>
<td>70.02%</td>
<td>6.01%</td>
<td>99%</td>
<td>2.33</td>
<td>25</td>
<td>$1,000,000</td>
</tr>
</tbody>
</table>
APPLICATION 7.4.1A (page 205)

Consider a portfolio that earns 10% per year and has an annual standard deviation of 20% when the risk-free rate is 3%. The Sharpe ratio is \((10\% - 3\%)/20\%\), or 0.35. When using annual returns and an annual standard deviation of returns, the Sharpe ratio may be interpreted as the annual risk premium that the investment earned per percentage point in annual standard deviation.

In this case, the investment’s return exceeded the riskless rate by 35 basis points for each percentage point in standard deviation. In an analysis of past data, the mean return of the portfolio is used as an estimate of its expected return, and the historical standard deviation of the sample is used as an estimate of the asset’s true risk. Throughout the remainder of this analysis of performance measures, the analysis may be viewed as interchangeable between using historical estimates and using expectations.

EXPLANATION

The Sharpe ratio in this application is calculated by finding the difference between 10% and 3% (the portfolio return and the risk-free rate otherwise known as excess return), then dividing by 20% for a quotient of 0.35. This follows equation 7.6.

\[ SR = \frac{E(R_p) - R_f}{\sigma_p} \]

CALCULATIONS

Step One: Press 0.1 → - → 0.03
Step Two: Press ÷ → 0.2
Step Three: Press = Answer:
0.35

WORKOUT AREA: Here are sample problems — cover one of the values and see if you can solve it using the others

<table>
<thead>
<tr>
<th>Sharpe Ratio</th>
<th>Annual Return</th>
<th>Annual Standard Deviation</th>
<th>Risk-Free Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.35</td>
<td>10.00%</td>
<td>20.00%</td>
<td>3.00%</td>
</tr>
<tr>
<td>0.13</td>
<td>5.00%</td>
<td>23.00%</td>
<td>2.00%</td>
</tr>
<tr>
<td>0.33</td>
<td>12.00%</td>
<td>32.00%</td>
<td>1.50%</td>
</tr>
<tr>
<td>0.73</td>
<td>25.00%</td>
<td>30.00%</td>
<td>3.00%</td>
</tr>
<tr>
<td>0.38</td>
<td>15.00%</td>
<td>34.00%</td>
<td>2.00%</td>
</tr>
<tr>
<td>0.99</td>
<td>22.00%</td>
<td>19.00%</td>
<td>3.10%</td>
</tr>
<tr>
<td>0.20</td>
<td>3.00%</td>
<td>10.00%</td>
<td>1.00%</td>
</tr>
</tbody>
</table>
Ignoring compounding for simplicity, and assuming statistically independent returns through time, the Sharpe ratios based on semiannual returns and quarterly returns are, using the same annual values as illustrated earlier, as follows:

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Sharpe Ratio Calculation</th>
<th>Sharpe Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Annual:</td>
<td>(10% − 3%) / 20% = 0.350</td>
<td>0.350</td>
</tr>
<tr>
<td>Semiannual:</td>
<td>[(10% − 3%) / 2] / (20%√0.5) = 0.247</td>
<td>0.247</td>
</tr>
<tr>
<td>Quarterly:</td>
<td>[(10% − 3%) / 4] / (20%√0.25) = 0.175</td>
<td>0.175</td>
</tr>
</tbody>
</table>

Note that the Sharpe ratio declines from 0.350 to 0.175, which is a 50% decrease, as the time interval for measurement is reduced by 75%, from annual to quarterly.

EXPLANATION

Recall the values in Application 5.3.1a, a portfolio earns 10% per year and has an annual standard deviation of 20% when the risk-free rate is 3%. Let’s also assume that returns are statistically independent through time. The key for this application is the understand how to adjust an annual statistic for different time periods.

The solution for the annual Sharpe ratio is as explained for Application 5.3.1a.

The solution for the semiannual Sharpe ratio is calculated by finding the difference between 10% and 3% (the annual excess return), then dividing by 2 in order to find the semiannual excess return. Next, we need to divide the semiannual excess return by the product of the annual standard deviation multiplied by the square root of ½ or 0.5 (the semiannual standard deviation).

The end result is a semiannual Sharpe ratio of .247.

The solution for the quarterly Sharpe ratio is calculated by finding the difference between 10% and 3% (the annual excess return), then dividing by 4 in order to find the quarterly excess return. Next, we need to divide the quarterly excess return by the product of the annual standard deviation multiplied by the square root of ¼ or 0.25 (the semiannual standard deviation). The end result is a semiannual Sharpe ratio of .175.
CALCULATIONS

Annual Sharpe Ratio
Please see the calculation for Application 5.3.1a

Semiannual Sharpe Ratio
Step One: Press 0.1 → - → 0.03
Step Two: Press ÷ → 2
Step Three: Press = “0.035”
Step Four: Press 0.5 → √x
Step Five: Press x → 0.2
Step Six: Press = “0.1414”
Step Seven: Press 0.035 → ÷ → 0.1414
Answer: 0.247

Quarterly Sharpe Ratio
Step One: Press 0.1 → - → 0.03
Step Two: Press ÷ → 4
Step Three: Press = “0.0175”
Step Four: Press 0.25 → √x
Step Five: Press x → 0.2 Step
Six: Press = “0.1”
Step Seven: Press 0.0175 → ÷ → 0.1
Answer: 0.175

WORKOUT AREA: Here are sample problems – cover one of the values and see if you can solve it using the others

<table>
<thead>
<tr>
<th>Annual Sharpe</th>
<th>Semiannual Sharpe</th>
<th>Quarterly Sharpe</th>
<th>Annual Return</th>
<th>Annual Standard Deviation</th>
<th>Risk-Free</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.35</td>
<td>0.25</td>
<td>0.18</td>
<td>10.00%</td>
<td>20.00%</td>
<td>3.00%</td>
</tr>
<tr>
<td>0.13</td>
<td>0.09</td>
<td>0.07</td>
<td>5.00%</td>
<td>23.00%</td>
<td>2.00%</td>
</tr>
<tr>
<td>0.33</td>
<td>0.23</td>
<td>0.16</td>
<td>12.00%</td>
<td>32.00%</td>
<td>1.50%</td>
</tr>
<tr>
<td>0.73</td>
<td>0.52</td>
<td>0.37</td>
<td>25.00%</td>
<td>30.00%</td>
<td>3.00%</td>
</tr>
<tr>
<td>0.38</td>
<td>0.27</td>
<td>0.19</td>
<td>15.00%</td>
<td>34.00%</td>
<td>2.00%</td>
</tr>
<tr>
<td>0.99</td>
<td>0.70</td>
<td>0.50</td>
<td>22.00%</td>
<td>19.00%</td>
<td>3.10%</td>
</tr>
<tr>
<td>0.20</td>
<td>0.14</td>
<td>0.10</td>
<td>3.00%</td>
<td>10.00%</td>
<td>1.00%</td>
</tr>
</tbody>
</table>

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APPLICATION 7.4.3A (page 207)

Consider a portfolio that earns 10% per year and has a beta with respect to the market portfolio of 1.5 when the risk-free rate is 3%. The Treynor ratio is \((10\% - 3\%)/1.5\), or 0.0467 (4.67%). The Treynor ratio may be interpreted as the risk premium that the investment earns per unit of beta. In this example, the investment’s expected return is 4.67% higher than the riskless rate for each unit of beta.

EXPLANATION

To solve for the Treynor ratio, let’s apply equation 7.8,

\[
TR = \frac{E(R_p) - R_f}{\beta_p}
\]

10% minus 3% divided by 1.5 equals 4.67%. Once again we are finding the excess return (portfolio return minus the risk free rate) and dividing it by a measure of risk, in this case it is beta, which is a measure of systematic risk.

CALCULATIONS

Step One: Press 0.1 → - → 0.03
Step Two: Press ÷ → 1.5
Step Three: Press = Answer: 0.0467

WORKOUT AREA: Here are sample problems – cover one of the values and see if you can solve it using the others

<table>
<thead>
<tr>
<th>Treynor Ratio</th>
<th>Annual Return</th>
<th>Beta</th>
<th>Risk-Free Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.67%</td>
<td>10.00%</td>
<td>1.5</td>
<td>3.00%</td>
</tr>
<tr>
<td>9.67%</td>
<td>12.00%</td>
<td>1</td>
<td>2.33%</td>
</tr>
<tr>
<td>2.00%</td>
<td>4.00%</td>
<td>0.5</td>
<td>3.00%</td>
</tr>
<tr>
<td>6.67%</td>
<td>15.00%</td>
<td>1.8</td>
<td>3.00%</td>
</tr>
<tr>
<td>8.46%</td>
<td>25.00%</td>
<td>2.6</td>
<td>3.00%</td>
</tr>
<tr>
<td>12.78%</td>
<td>20.00%</td>
<td>1.33</td>
<td>3.00%</td>
</tr>
<tr>
<td>2.50%</td>
<td>10.00%</td>
<td>3</td>
<td>2.50%</td>
</tr>
</tbody>
</table>
APPLICATION 7.4.5A (page 209)

Consider a portfolio that earns 10% per year when the investor’s target rate of return is 8% per year. The semistandard deviation based on returns relative to the target is 16% annualized. The Sortino ratio would be (10% – 8%)/16%, or 0.125.

EXPLANATION

Apply equation 7.9 to find the Sortino ratio we must subtract 10% and 8%, then divide the difference by 16% for a quotient of 0.125. Once again we are finding the excess return (portfolio return minus the target rate of return) and dividing it by a measure of risk, in this case it is semistandard deviation, which is a measure of downside risk.

CALCULATIONS

Step One: Press 0.1 → - → 0.08
Step Two: Press ÷ → .16
Step Three: Press =

Answer: 0.125

WORKOUT AREA: Here are sample problems – cover one of the values and see if you can solve it using the others

<table>
<thead>
<tr>
<th>Sortino Ratio</th>
<th>Annual Return</th>
<th>Semistandard Deviation</th>
<th>Target Rate of Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>12.50%</td>
<td>10.00%</td>
<td>16.00%</td>
<td>8.00%</td>
</tr>
<tr>
<td>75.00%</td>
<td>25.00%</td>
<td>20.00%</td>
<td>10.00%</td>
</tr>
<tr>
<td>-10.00%</td>
<td>3.00%</td>
<td>10.00%</td>
<td>4.00%</td>
</tr>
<tr>
<td>8.70%</td>
<td>12.00%</td>
<td>23.00%</td>
<td>10.00%</td>
</tr>
<tr>
<td>105.56%</td>
<td>24.00%</td>
<td>18.00%</td>
<td>5.00%</td>
</tr>
<tr>
<td>110.00%</td>
<td>19.00%</td>
<td>10.00%</td>
<td>8.00%</td>
</tr>
<tr>
<td>-40.00%</td>
<td>8.00%</td>
<td>5.00%</td>
<td>10.00%</td>
</tr>
</tbody>
</table>
APPLICATION 7.4.6A (page 210)

If a portfolio consistently outperformed its benchmark by 4% per year, but its performance relative to that benchmark typically deviated from that 4% mean with an annualized standard deviation of 10%, then its information ratio would be 4%/10%, or 0.40.

EXPLANATION

To calculate the information ratio we need to divide 4% (the amount that the portfolio outperformed the benchmark per year) by 10% (the annual standard deviation of returns of the portfolio) for an answer of 0.40.

CALCULATIONS

Step One: Press 0.04 → ÷ → 0.1 Step
Two: Press =
Answer: 0.40

WORKOUT AREA: Here are sample problems – cover one of the values and see if you can solve it using the others

<table>
<thead>
<tr>
<th>Information Ratio</th>
<th>Portfolio Return – Benchmark Return</th>
<th>Standard Deviation of Excess Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.40</td>
<td>4.00%</td>
<td>10.00%</td>
</tr>
<tr>
<td>1.25</td>
<td>25.00%</td>
<td>20.00%</td>
</tr>
<tr>
<td>0.30</td>
<td>3.00%</td>
<td>10.00%</td>
</tr>
<tr>
<td>0.52</td>
<td>12.00%</td>
<td>23.00%</td>
</tr>
<tr>
<td>1.33</td>
<td>24.00%</td>
<td>18.00%</td>
</tr>
<tr>
<td>1.90</td>
<td>19.00%</td>
<td>10.00%</td>
</tr>
<tr>
<td>1.60</td>
<td>8.00%</td>
<td>5.00%</td>
</tr>
</tbody>
</table>
APPLICATION 7.5.1A (page 211)

A portfolio is expected to earn 7% annualized return when the riskless rate is 4% and the expected return of the market is 8%. If the beta of the portfolio is 0.5, the alpha of the portfolio is 1%, found by substituting into Equation 7.12 and solving:

\[ \alpha_p = \beta_p \left[ E(R_m) - R_f \right] - \left[ E(R_p) - R_f \right] \]

\[ \alpha_p = 7\% - 4\% - [0.5(8\% - 4\%)] = 1\% \]

EXPLANATION

In order to solve this application, we need to apply equation 7.12. First, subtracted 0.08 (the expected return of the market) by 0.04 (the riskless rate). Multiply the difference of 0.4 by 0.5 (the portfolio’s beta) for a product of 0.2. Subtract 0.07 by 0.04 for a difference of 0.03. Subtract 0.03 by 0.02 for a difference of 0.01 or 1% (Jensen’s alpha).

CALCULATIONS

Step One: Press 0.08 \( \rightarrow \) - \( \rightarrow \) 0.04
Step Two: Press \( \times \) \( \rightarrow \) 0.5
Step Three: Press = “0.02”
Step Four: Press 0.07 \( \rightarrow \) - \( \rightarrow \) 0.04
Step Five: Press = “0.03”
Step Six: Press 0.03 \( \rightarrow \) - \( \rightarrow \) 0.02
Step Seven: Press =
Answer: 0.01

WORKOUT AREA: Here are sample problems – cover one of the values and see if you can solve it using the others

<table>
<thead>
<tr>
<th>Jensen’s Alpha</th>
<th>Portfolio Expected Annualized Return</th>
<th>Risk-Free Rate</th>
<th>Expected Annual Return of the Market</th>
<th>Portfolio Beta</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>7.00%</td>
<td>4.00%</td>
<td>8.00%</td>
<td>0.5</td>
</tr>
<tr>
<td>0.01</td>
<td>15.00%</td>
<td>2.00%</td>
<td>10.00%</td>
<td>1.5</td>
</tr>
<tr>
<td>-0.12</td>
<td>10.50%</td>
<td>1.50%</td>
<td>12.00%</td>
<td>2</td>
</tr>
<tr>
<td>-0.07</td>
<td>8.30%</td>
<td>3.00%</td>
<td>15.00%</td>
<td>1</td>
</tr>
<tr>
<td>0.16</td>
<td>25.00%</td>
<td>2.00%</td>
<td>8.00%</td>
<td>1.2</td>
</tr>
<tr>
<td>0.13</td>
<td>20.00%</td>
<td>1.50%</td>
<td>5.00%</td>
<td>1.7</td>
</tr>
<tr>
<td>0.28</td>
<td>30.00%</td>
<td>4.00%</td>
<td>3.00%</td>
<td>2.4</td>
</tr>
</tbody>
</table>
APPLICATION 7.5.2A (page 213)

Consider a portfolio with $M^2 = 4\%$. The portfolio is expected to earn 10\%, whereas the riskless rate is only 2\%. What is the ratio of the volatility of the market to the volatility of the portfolio? Inserting the given rates generates $4\% = 2\% + [(\text{ratio of volatilities}) \times 8\%]$. The ratio of the volatility of the market to the volatility of the portfolio must be 25%.

EXPLANATION

In order to solve this problem, we need to manipulate equation 7.14, like so:

\[
M^2 = R_f + \{ (\sigma_m / \sigma_p) [E(R_p) - R_f] \}
\]

\[
M^2 - R_f = \{ (\sigma_m / \sigma_p) [E(R_p) - R_f] \}
\]

\[
\frac{(M^2 - R_f)}{[E(R_p) - R_f]} = \frac{(\sigma_m / \sigma_p)}
\]

Now, we can plug in the data provided $M^2 = 4\%$, $E(R_p) = 10\%$, and $R_f = 2\%$:

\[
\frac{(0.04 - 0.02)}{(0.1 - 0.02)} = \frac{\sigma_m}{\sigma_p}
\]

\[
0.02 / 0.08 = \frac{\sigma_m}{\sigma_p}
\]

\[
0.25 = \frac{\sigma_m}{\sigma_p}
\]

The ratio of the volatilities is

0.25
CALCULATIONS

Step One: Press 0.04 → - → 0.02
Step Two: Press = “0.02”
Step Three: Press 0.1 → - → 0.02
Step Four: Press = “0.08”
Step Five: Press 0.02 → ÷ → 0.08
Step Six: Press =
Answer: 0.25

WORKOUT AREA: Here are sample problems – cover one of the values and see if you can solve it using the others

<table>
<thead>
<tr>
<th>M²</th>
<th>Portfolio Expected Annualized Return</th>
<th>Risk-Free Rate</th>
<th>Ratio of the Volatility of the Market to the Volatility of the Portfolio</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.00%</td>
<td>10.00%</td>
<td>2.00%</td>
<td>25.00%</td>
</tr>
<tr>
<td>3.00%</td>
<td>15.00%</td>
<td>2.00%</td>
<td>7.69%</td>
</tr>
<tr>
<td>8.00%</td>
<td>10.50%</td>
<td>1.50%</td>
<td>72.22%</td>
</tr>
<tr>
<td>5.00%</td>
<td>8.30%</td>
<td>3.00%</td>
<td>37.74%</td>
</tr>
<tr>
<td>7.50%</td>
<td>20.00%</td>
<td>1.00%</td>
<td>34.21%</td>
</tr>
<tr>
<td>5.10%</td>
<td>30.00%</td>
<td>4.00%</td>
<td>4.23%</td>
</tr>
</tbody>
</table>
APPLICATION 7.5.2B (page 213)

Consider the following information

<table>
<thead>
<tr>
<th></th>
<th>Expected Return</th>
<th>Volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td>Market</td>
<td>12%</td>
<td>14%</td>
</tr>
<tr>
<td>Portfolio</td>
<td>14%</td>
<td>28%</td>
</tr>
<tr>
<td>Riskless Asset</td>
<td>2%</td>
<td>0%</td>
</tr>
</tbody>
</table>

$M^2$ is found by inserting four of the values into Equation 7.14:

\[
M^2 = R_f + \left(\frac{\sigma_m}{\sigma_p}\right)[E(R_p) - R_f]
\]

\[
M^2 = 2\% + \left(\frac{0.14}{0.28}\right)[0.14 - 0.02]
\]

\[
M^2 = 2\% + 6\% = 8\%
\]

EXPLANATION

We are given all the information we need for this problem. To find $M^2$, we must use equation 7.14:

\[
M^2 = R_f + \left(\frac{\sigma_m}{\sigma_p}\right)[E(R_p) - R_f]
\]

\[
M^2 = 2\% + \left(\frac{0.14}{0.28}\right)[0.14 - 0.02]
\]

\[
M^2 = 2\% + 6\% = 8\%
\]

CALCULATION

Step One: Press $0.14 \rightarrow - \rightarrow 0.02$
Step Two: Press = “0.12”
Step Three: Press $\times \rightarrow ( \rightarrow 0.14 \rightarrow + \rightarrow .28 \rightarrow )$
Step Four: Press = “0.06”
Step Five: Press $+ \rightarrow 0.02$
Step Six: Press =
Answer = 0.08

WORKOUT AREA

<table>
<thead>
<tr>
<th>$M^2$</th>
<th>Riskless Rate</th>
<th>Market Expected Return</th>
<th>Market Volatility</th>
<th>Portfolio Expected Return</th>
<th>Portfolio Volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.00%</td>
<td>2%</td>
<td>12%</td>
<td>14%</td>
<td>14%</td>
<td>28%</td>
</tr>
<tr>
<td>11.50%</td>
<td>3%</td>
<td>10%</td>
<td>15%</td>
<td>20%</td>
<td>30%</td>
</tr>
<tr>
<td>35.00%</td>
<td>5%</td>
<td>10%</td>
<td>30%</td>
<td>35%</td>
<td>30%</td>
</tr>
<tr>
<td>10.40%</td>
<td>2%</td>
<td>14%</td>
<td>18%</td>
<td>16%</td>
<td>30%</td>
</tr>
<tr>
<td>8.00%</td>
<td>4%</td>
<td>13%</td>
<td>15%</td>
<td>12%</td>
<td>30%</td>
</tr>
<tr>
<td>15.00%</td>
<td>10%</td>
<td>20%</td>
<td>30%</td>
<td>15%</td>
<td>30%</td>
</tr>
</tbody>
</table>
APPLICATION 8.2.1A (page 218)

Consider the Sludge Fund, a fictitious fund run by unskilled managers that generally approximates the S&P 500 Index but does so with an annual expense ratio of 100 basis points (1%) more than other investment opportunities that mimic the S&P 500. Using Equation 8.1 and assuming that the S&P 500 is a proxy for the market portfolio, the ex ante alpha of Sludge Fund would be approximately −100 basis points per year. This can be deduced from assuming that \( \beta_i = 1 \) and that \( \[E(R_{it}) - E(R_{mt})]\] = −1% due to the expense ratio. Sludge Fund could be expected to offer an ex ante alpha, meaning a consistently inferior risk-adjusted annual return, of −1% per year. This example illustrates that ex ante alpha can be negative to indicate inferior expected performance, although alpha is usually discussed in the pursuit of the superior performance associated with a positive alpha.

EXPLANATION

The application to this solution can be deduced as \( \alpha_i = -1\% \). Let’s consider the facts. The expected Fund return and the expected Market return are the same, except that the fund has an expense ratio of 1%. The beta of the fund to the market is 1. Therefore, the only difference between the market and the fund is a 1% expense ratio that weighs on the fund performance. Therefore, the \( \alpha_i = -1\% \).

Now let’s calculate this mathematically by manipulating equation 8.1:

\[
E(R_{i,t} - R_f) = \alpha_i + \beta_i \left[ E(R_{m,t}) - R_f \right]
\]

\[
\alpha_i = \beta_i \left[ E(R_{m,t}) - R_f \right] - E(R_{i,t} - \text{Expense Ratio} - R_f)
\]

With the modified equation 8.1, it becomes clearer how the expense ratio impacts alpha. Let’s solve for \( \alpha_i \).

\[
\alpha_i = \beta_i \left[ E(R_{m,t}) - R_f \right] - E(R_{i,t} - \text{Expense Ratio} - R_f)
\]

\[
\alpha_i = 1\left[0.1 - 0\right] - (0.1 + 0.01 - 0)
\]

\[
\alpha_i = 0.1 - (0.11)
\]

\[
\alpha_i = -0.01
\]

As you can see, \( \alpha_i = -1\% \).
CALCULATIONS

Step One: Press 1 → x → 0.1
Step Two: Press = “0.1”
Step Three: Press 0.1 → + → 0.01 - → 0
Step Four: Press = “0.11”
Step Five: Press 0.1 → - → 0.11
Step Six: Press =
Answer: -0.01 or -1%
APPLICATION 8.2.2A (page 219)

Consider the Trim Fund, a fund that tries to mimic the S&P 500 Index and has managers who are unskilled. Unlike the Sludge Fund from the previous section, Trim Fund has virtually no expenses. Although Trim Fund generally mimics the S&P 500, it does so with substantial error due to the random incompetence of its managers. However, the fund is able to maintain a steady systematic risk exposure of $\beta_i = 1$. Last year, Trim Fund outperformed the S&P 500 by 125 basis points. Using Equation 8.2, assuming that $\beta_i = 1$ and that $(R_{it} - R_{mt}) = +1.25\%$, it can be calculated that $\varepsilon_{it} = +1.25\%$. Thus, Trim Fund realized a return performance for the year that was 1.25\% higher than its benchmark, or its required rate of return. In the terms of this chapter, Trim Fund generated an ex post alpha of 125 basis points, even though the fund’s ex ante alpha was zero.

EXPLANATION

Let’s examine this application from a mathematic context as opposed to deducing the ex-post alpha from the given information.

To solve ex-post alpha we need to rearrange equation 8.2:

$$R_{it} - R_f = \beta_i (R_{mt} - R_f) + \varepsilon_{it}$$

$$R_{it} - R_f \over \beta_i = (R_{mt} - R_f) + \varepsilon_{it}$$

$$\varepsilon_{it} = \frac{R_{it} - R_f}{\beta_i} - R_{mt} + R_f$$

Now, not all the values are given. However, we know that $R_{mt} - R_f = 1.25\%$ so we can use example figures that reflect that difference. $\beta_i = 1$, $R_{ic} = 10\%$, $R_{mc} = 11.25\%$, and $R_f = 0\%$. Now, using these figures, we can calculate $\varepsilon_{it}$:

$$\varepsilon_{it} = \frac{0.1125 - 0}{1} - 0.10 + 0$$

As shown, $\varepsilon_{it} = 0.0125$

CALCULATIONS

This application uses deductive reasoning more than calculation. Some example numbers have been provided in the EXPLANATION section, which will be used here in the CALCULATIONS section.

Step One: Press $0.1125 \rightarrow - \rightarrow 0$
Step Two: Press $\div \rightarrow 1$
Step Three: Press $- \rightarrow 0.10 + \rightarrow 0$
Step Four: Press $=$ Answer: 0.0125
APPLICATION 8.3.7A (page 224)

Consider a regression with an alpha estimate of 0.5% (with a standard error of 0.3%) and a beta estimate of 1.1 (with a standard error of 0.3). Are the regression parameters statistically significant? The t-statistic of the alpha is 1.67, whereas the t-statistic of the beta is 3.67, each found by dividing the parameter estimates by the corresponding standard error. At a 5% confidence level, the t-statistic needs to exceed 1.96 to be deemed statistically significant (assuming a very large number of degrees of freedom). In this case, the alpha is not deemed to be significantly different from zero because the t-statistic is less than 1.96 (the critical value); however, the beta does differ significantly from zero, as its t-statistic exceeds 1.96.

EXPLANATION

We are looking for a 5% statistical significance of an alpha estimate and a beta estimate with a confidence level of 95%. We need to know that the alpha estimate is 0.5% with a standard error of 0.3 and the beta estimate is 1.1 with a standard error of 0.3. In addition, we need to know the z-score of a 5% confidence level is 1.96. Now, we need to calculate the t-statistic of the alpha estimate and beta estimate and compare those numbers with the z-score of a 95% confidence level or 1.96. If the estimate’s t-statistic exceeds the z-score for the set confidence interval, the estimate is statistically significant. If the estimate’s t-statistic is less than the z-score for the set confidence interval, the estimate is not deemed statistically significant.

To calculate the t-statistic for the alpha estimate we need to divide 0.5 by 0.3 which equals 1.67. 1.67 is less than the z-score of 1.96 for the set confidence interval. The alpha estimate is not deemed statistically significant.

To calculate the t-statistic for the beta estimate we need to divide 1.1 by 0.3 which equals 3.67. 3.67 is greater than the z-score of 1.96 for the set confidence level. The beta estimate is deemed statistically significant.

CALCULATIONS

Calculate the t-statistic for the alpha estimate

Step One: Press 0.5 → ÷ → 0.3  
Step Two: Press =  
Answer: 1.67

Calculate the t-statistic for the beta estimate

Step One: Press 1.1 → ÷ → 0.3  
Step Two: Press =  
Answer: 3.67
<table>
<thead>
<tr>
<th>Alpha Estimate Statistically Significant?</th>
<th>Beta Estimate Statistically Significant?</th>
<th>T-Statistic of Alpha</th>
<th>T-Statistic of Beta</th>
<th>Alpha Estimate Standard Error (%)</th>
<th>Beta Estimate</th>
<th>Beta Standard Error</th>
<th>Confidence Interval</th>
<th>Z-Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>not deemed to be significantly different from zero</td>
<td>does differ significantly from zero</td>
<td>1.67</td>
<td>3.67</td>
<td>0.50%</td>
<td>0.3</td>
<td>1.1</td>
<td>0.3</td>
<td>95%</td>
</tr>
<tr>
<td>does differ significantly from zero</td>
<td>does differ significantly from zero</td>
<td>1.67</td>
<td>3.67</td>
<td>0.50%</td>
<td>0.3</td>
<td>1.1</td>
<td>0.3</td>
<td>90%</td>
</tr>
<tr>
<td>not deemed to be significantly different from zero</td>
<td>does differ significantly from zero</td>
<td>1.67</td>
<td>3.67</td>
<td>0.50%</td>
<td>0.3</td>
<td>1.1</td>
<td>0.3</td>
<td>99%</td>
</tr>
<tr>
<td>does differ significantly from zero</td>
<td>does differ significantly from zero</td>
<td>3.33</td>
<td>1.67</td>
<td>1.00%</td>
<td>0.3</td>
<td>0.5</td>
<td>0.3</td>
<td>95%</td>
</tr>
<tr>
<td>does differ significantly from zero</td>
<td>does differ significantly from zero</td>
<td>5.00</td>
<td>5.00</td>
<td>1.50%</td>
<td>0.3</td>
<td>1.5</td>
<td>0.3</td>
<td>95%</td>
</tr>
<tr>
<td>does differ significantly from zero</td>
<td>does differ significantly from zero</td>
<td>6.67</td>
<td>4.00</td>
<td>2.00%</td>
<td>0.3</td>
<td>1.2</td>
<td>0.3</td>
<td>95%</td>
</tr>
<tr>
<td>does differ significantly from zero</td>
<td>does differ significantly from zero</td>
<td>-5.00</td>
<td>0.83</td>
<td>-1.50%</td>
<td>0.3</td>
<td>0.25</td>
<td>0.3</td>
<td>95%</td>
</tr>
</tbody>
</table>
APPLICATION 8.5.1A (page 228)

Consider the following data on Target Fund: $\beta = 1.5$ and its expected return is 14%. Assume that the expected return of the market is 11% and that the risk-free rate is 3%. During the next year, the market earns 8% and Target Fund earns 7%. What was: (1) the Fund’s ex ante alpha, (2) the Fund’s ex post alpha, (3) the Fund’s return that was skill, and (4) the Fund’s return that was luck?

(1) Inserting the market's expected return, the Fund’s beta, and the risk-free rate into the ex ante CAPM generates the expected return of an efficiently priced asset with a beta of 1.5:

$$E(R) = 3\% + [1.5 (11\% - 3\%)] \Rightarrow E(R) = 15\%$$

Because the Fund’s expected return is only 14%, it represents an ex ante alpha of $-1\%$.

(2) Given the market return was 5% more than the risk-free return of 3%, an asset with a beta of 1.5 should have earned 7.5% ($5\% \times 1.5$) more than the riskless rate (i.e., 10.5%). Because it earned 7%, it had an ex post alpha ($\varepsilon$) of $-3.5\%$.

(3) The answer to (1) means that the Fund is expected to underperform by 1%. Therefore, $-1\%$ of the actual subsequent return was skill.

(4) The Fund underperformed by $-3.5\%$. Since the expected loss on the fund is $-1\%$ based on skill (perhaps due to fees), the return attributed to bad luck (negative idiosyncratic return) was $-2.5\%$.

EXPLANATION

Here, we will use different variations of the CAPM to calculate alpha. In this example, we are told that the expected market return is 11% and the risk-free rate is 3%. According to the CAPM, a fund with a beta of 1.5 would be expected to return 15% ($3\% + [1.5 (11\% - 3\%)]$). Any difference in deviation will represent a positive or negative alpha, therefore because the fund is expected to return 14%, this would represent a $-1\%$ ex ante alpha.

The market actually returned 8%, or 5% more than the risk-free rate, which was less than expected. The fund, with a beta of 1.5, only returned 7%. According to the CAPM, a fund with a beta of 1.5 should have returned 10.5% ($3\% + [1.5 (8\% - 3\%)]$). Since the fund only returned 7%, this represents an ex post alpha of $-3.5\%$.

To determine luck vs. skill, we must compare the differences between the ex-post alpha and ex ante alpha. When we solved for the ex ante alpha, we determined the fund had an ex-ante alpha of $-1\%$ - in other words, we expected the fund to underperform by $-1\%$, which represents the fund manager’s skill. Any difference in alpha between ex-ante and ex-post will be attributable to luck. Since the ex-post alpha was $-3.5\%$, we can attribute $-2.5\%$ to luck.
CALCULATIONS

To determine the ex ante alpha:

Step One: Press → 0.11 → - → 0.03
Step Two: Press = “0.08”
Step Three: Press x → 1.5
Step Four: Press + → 0.03
Step Five: Press = “0.15”
Step Six: Press 0.14 → - → 0.15
Step Seven: Press =
Answer: -0.01

To determine the ex post alpha:

Step One: Press → 0.08 → - → 0.03
Step Two: Press = “0.05”
Step Three: Press x → 1.5
Step Four: Press + → 0.03
Step Five: Press = “0.105”
Step Six: Press 0.07 → - → 0.105
Step Seven: Press =
Answer: -0.035

WORKOUT AREA: Here are sample problems – cover one of the values and see if you can solve it using the others.

<table>
<thead>
<tr>
<th>Ex Ante Alpha</th>
<th>Ex Post Alpha</th>
<th>Expected Fund Return</th>
<th>Expected Market Return</th>
<th>Actual Fund Return</th>
<th>Actual Market Return</th>
<th>Beta</th>
<th>Risk Free Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1.00%</td>
<td>-3.50%</td>
<td>14%</td>
<td>11%</td>
<td>7%</td>
<td>8%</td>
<td>1.50</td>
<td>3%</td>
</tr>
<tr>
<td>-1.00%</td>
<td>-5.00%</td>
<td>12%</td>
<td>13%</td>
<td>10%</td>
<td>15%</td>
<td>1.00</td>
<td>5%</td>
</tr>
<tr>
<td>-1.00%</td>
<td>3.00%</td>
<td>14%</td>
<td>15%</td>
<td>20%</td>
<td>17%</td>
<td>1.00</td>
<td>4%</td>
</tr>
<tr>
<td>-0.40%</td>
<td>1.60%</td>
<td>5%</td>
<td>5%</td>
<td>7%</td>
<td>5%</td>
<td>1.20</td>
<td>3%</td>
</tr>
<tr>
<td>-1.20%</td>
<td>-1.50%</td>
<td>7%</td>
<td>7%</td>
<td>8%</td>
<td>8%</td>
<td>1.30</td>
<td>3%</td>
</tr>
<tr>
<td>3.00%</td>
<td>5.00%</td>
<td>8%</td>
<td>5%</td>
<td>10%</td>
<td>5%</td>
<td>1.00</td>
<td>3%</td>
</tr>
</tbody>
</table>
APPLICATION 9.2.3A (page 259)

Using the same values except that the construction costs are fixed at $86,667 (the original expected value), find the value of the land. The math is the same except the up-state payoff to the option is $73,333 ($160,000 – $86,667) and the value of the option is $24,444 ($73,333 x 1/3). Thus, having fixed construction costs increases the volatility of the spread, which in turn increases the value of the option. The implication is that land values benefit from decreased correlation between construction costs and improved real estate values.

EXPLANATION

Equation 9.2 is required to solve this problem:

\[
\text{Current Value} = \text{Expected Value} = (\text{UpValue} \times \text{UpProb}) + [\text{DownValue} \times (1 - \text{UpProb})]
\]

The first point to understand with this application is that it assumes we are under no obligation to develop the land. That is important to note as it impacts the DownValue state. If we were under an obligation to develop, then we would subtract the value of the land if the economy falters ($70,000.00) by the construction cost if the economy falters ($86,667.00, which is the same construction cost if the economy improves).

However, we find that if we subtract $70,000.00 from $86,667.00 for a difference of ($16,667.00). Therefore, under any probability we would not want to lose money on an investment and since we do not have the obligation to develop the land, the downstate will be $0 with a probability of 2/3 (the probability of the economy falters as outlined in the application). Now, we need to address the expected value of the UpValue. The UpValue is the difference between the $160,000.00 (the value of land if the economy improves) and $86,667.00 (the construction cost if the economy improves) or $73,333.00.

To compute the expected value of the upstate we need to multiply the UpValue or up state payoff by 2/3 (the probability that the up state payoff will occur or in this application it is the probability that the economy improves) for an expected value of $24,444.33. Lastly, we need to sum the expected value of the up state payoff and the expected value of the down state payoff, $24,444.33 plus $0 equals an option price of the land of $24,444.33.
CALCULATIONS

Find the Option Price of the Land

Step One: Press 70000 → - → 86667
Step Two: Press = “-16667”
Step Three: Press 160000 → - → 86667
Step Four: Press = “73333”
Step Five: Press 0 → x → 0.6667
Step Six: Press = “0”
Step Seven: Press 73333.33 → x → 0.3333
Step Eight: Press = “24444.33”
Step Nine: Press 24444.33 → + → 0
Step Ten: Press =
Answer: 24444.33

WORKOUT AREA: Here are sample problems – cover one of the values in the leftmost column and see if you can solve it using the others

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$24,444.33</td>
<td>$100,000.00</td>
<td>$100,000.00</td>
<td>$160,000.00</td>
<td>$70,000.00</td>
<td>$86,667.00</td>
<td>33.33%</td>
<td>66.67%</td>
<td>$24,444.33</td>
<td>$11,111.33</td>
</tr>
<tr>
<td>$20,000.00</td>
<td>$100,000.00</td>
<td>$160,000.00</td>
<td>$70,000.00</td>
<td>$100,000.00</td>
<td>$86,667.00</td>
<td>33.33%</td>
<td>66.67%</td>
<td>$20,000.00</td>
<td>$6,666.67</td>
</tr>
<tr>
<td>$46,875.00</td>
<td>$100,000.00</td>
<td>$150,000.00</td>
<td>$75,000.00</td>
<td>$37,500.00</td>
<td>$50,000.00</td>
<td>25.00%</td>
<td>75.00%</td>
<td>$18,750.00</td>
<td>$28,125.00</td>
</tr>
<tr>
<td>$56,750.00</td>
<td>$100,000.00</td>
<td>$250,000.00</td>
<td>$125,000.00</td>
<td>$62,500.00</td>
<td>$25,000.00</td>
<td>0.00%</td>
<td>50.00%</td>
<td>$62,500.00</td>
<td>$31,250.00</td>
</tr>
<tr>
<td>$87,500.00</td>
<td>$100,000.00</td>
<td>$200,000.00</td>
<td>$100,000.00</td>
<td>$50,000.00</td>
<td>$25,000.00</td>
<td>0.00%</td>
<td>50.00%</td>
<td>$75,000.00</td>
<td>$12,500.00</td>
</tr>
<tr>
<td>$27,500.00</td>
<td>$100,000.00</td>
<td>$100,000.00</td>
<td>$50,000.00</td>
<td>$50,000.00</td>
<td>$25,000.00</td>
<td>10.00%</td>
<td>90.00%</td>
<td>$5,000.00</td>
<td>$22,500.00</td>
</tr>
<tr>
<td>$28,500.00</td>
<td>$100,000.00</td>
<td>$95,000.00</td>
<td>$47,500.00</td>
<td>$23,750.00</td>
<td>$23,750.00</td>
<td>20.00%</td>
<td>80.00%</td>
<td>$9,500.00</td>
<td>$19,000.00</td>
</tr>
</tbody>
</table>
APPLICATION 9.2.4A (page 260)

Land that remains undeveloped is estimated to generate an expected return of 5%, and land that is
developed is estimated to generate an expected single-period return of 25%. If the probability that a parcel
of land will be developed is 10% over the next period, what is its expected return? Inserting the values into
Equation 9.3 generates \([0.10 \times 0.25] + (0.90 \times 0.05)] = 7\%.

EXPLANATION

To solve this, we will need to use equation 9.3 (note: this equation is very
similar to 9.2):

\[
E(R_t) = [P_d \times E(R_d)] + [(1 - P_d) \times E(R_{nd})]
\]

Let’s begin by computing the undeveloped expected value. We know that the return of the
undeveloped parcel of land is 5% and the probability that the land remains undeveloped is 90%,
found by subtracting 10% (the probability that the land will be developed) from 1. If we
multiply 5% (the return of the undeveloped parcel of land) by 90% (the probability that the
land will remain undeveloped) the product is the expected value of the undeveloped parcel of
land or 4.5%. Now the expected return of the developed land is calculated by multiplying 25%
(the return of the land once developed) by 10% (the probability that the land will be
developed) for a product of 2.5% (the expected return of the developed land).

We are ready to compute the overall expected value of the parcel of land which is equal to
the sum of 2.5% and 4.5% or 7%. 7% is the expected value of the parcel of land.

CALCULATIONS

Find the expected return of the Land

Step One: Press 0.05 → x → 0.90
Step Two: Press = “0.045“
Step Three: Press 0.25 → x → 0.10
Step Four: Press = “0.025“
Step Five: Press 0.045 → + → 0.025 Step
Six: Press =
Answer: 0.07
WORKOUT AREA: Here are sample problems – cover one of the values and see if you can solve it using the others

<table>
<thead>
<tr>
<th>Expected Return</th>
<th>Undeveloped Land Return</th>
<th>Developed Land Return</th>
<th>Probability of Development</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.00%</td>
<td>5.00%</td>
<td>25.00%</td>
<td>10.00%</td>
</tr>
<tr>
<td>7.68%</td>
<td>5.50%</td>
<td>20.00%</td>
<td>15.00%</td>
</tr>
<tr>
<td>11.00%</td>
<td>10.00%</td>
<td>30.00%</td>
<td>5.00%</td>
</tr>
<tr>
<td>9.00%</td>
<td>2.50%</td>
<td>35.00%</td>
<td>20.00%</td>
</tr>
<tr>
<td>6.07%</td>
<td>4.00%</td>
<td>27.00%</td>
<td>9.00%</td>
</tr>
<tr>
<td>7.13%</td>
<td>6.00%</td>
<td>21.00%</td>
<td>7.50%</td>
</tr>
<tr>
<td>9.50%</td>
<td>8.00%</td>
<td>33.00%</td>
<td>6.00%</td>
</tr>
</tbody>
</table>
APPLICATION 9.2.4B (page 261)

Land that remains undeveloped is estimated to generate an expected return of 5%, and land that is developed is estimated to generate an expected single-period return of 25%. If 20% of land in a database is developed in a particular year, by how much will an index based on land that remains undeveloped understate the average return on all land? Inserting the realized values into Equation 9.3 in place of the expected values generates that the mean return of a portfolio (with 20% development) is 

\[
[(0.20 \times 0.25) + (0.80 \times 0.05)] = 9\%.
\]

The historical index of returns based on land that remained undeveloped was 5%. The negative survivorship bias was 4%.

EXPLANATION

We are calculating the expected returns of a land portfolio that contains both developed and undeveloped land. Let's begin by computing the undeveloped land expected returns. We know that the expected return of undeveloped parcels of land is 5.0% and the proportion of that the land remains undeveloped is 80% (found by subtracting 20.0% from 1). If we multiply 5.0% (the expected return of the undeveloped parcels of land) by 80.0% (the proportion of land left undeveloped) the product is the expected return of the undeveloped parcels of land or 4.0%. Now the expected return of the developed land is calculated by multiplying 25.0% (the expected return of the land once developed) by 20.0% (the proportion of land developed) for a product of 5% (the expected return of the developed land). We are ready to compute the overall return of the land portfolio which is equal to the sum of 4.0% (expected return on undeveloped parcels of land) and 5.0% (expected return on developed parcels of land) or 9.0%. 9% is the expected value of land portfolio. Since the historical average index return of land that remained undeveloped is 5%, the index will understate the returns of the land portfolio by 4%. The answer is found by subtracting 5% (historical average index return of land that remained undeveloped) from 9% (returns of the land portfolio) for a difference of 4%.
CALCULATIONS

Find the expected return of the Land

Step One: Press 0.05 $\rightarrow$ x $\rightarrow$ 0.80
Step Two: Press $\times 0.04$
Step Three: Press 0.25 $\rightarrow$ x $\rightarrow$ 0.20
Step Four: Press $\times 0.05$
Step Five: Press 0.04 $\rightarrow$ + $\rightarrow$ 0.05
Step Six: Press $=$
Answer: 0.09

WORKOUT AREA: Here are sample problems – cover one of the values and see if you can solve it using the others

<table>
<thead>
<tr>
<th>Expected Return</th>
<th>Undeveloped Land Return</th>
<th>Developed Land Return</th>
<th>Probability of Development</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.00%</td>
<td>5.00%</td>
<td>25.00%</td>
<td>20.00%</td>
</tr>
<tr>
<td>7.68%</td>
<td>5.50%</td>
<td>20.00%</td>
<td>15.00%</td>
</tr>
<tr>
<td>12.38%</td>
<td>6.50%</td>
<td>30.00%</td>
<td>25.00%</td>
</tr>
<tr>
<td>14.70%</td>
<td>6.00%</td>
<td>35.00%</td>
<td>30.00%</td>
</tr>
<tr>
<td>18.55%</td>
<td>7.00%</td>
<td>40.00%</td>
<td>35.00%</td>
</tr>
<tr>
<td>6.53%</td>
<td>4.50%</td>
<td>45.00%</td>
<td>5.00%</td>
</tr>
<tr>
<td>6.30%</td>
<td>4.00%</td>
<td>27.00%</td>
<td>10.00%</td>
</tr>
</tbody>
</table>
APPLICATION 9.4.1A (page 264)

If the annual revenue in Exhibit 9.4 is expected to rise to $40,000 and the market cap rate rises to 8%, then with all other values remaining constant, the farmland's price would rise to $400,000 \[ \frac{($40,000 - $6,000 - $2,000)}{0.08} \]. With a price of $360,000 and an annual operating income of $40,000, what would the cap rate be?

From Equation 9.4:

\[
\frac{$360,000}{40,000} = \frac{cap \ rate}{\text{Value of Real Estate}}
\]

\[
\text{Cap rate} = \frac{40,000}{360,000} = 11.11\%
\]

EXPLANATION

There is a lot of extra information provided in this application, but the key aspects are contained in the last sentence. The price or value of the farmland is $360,000 and the annual income is $40,000, so manipulating equation 9.4 to read:

\[
\text{Cap Rate} = \frac{\text{Annual Operating Income}}{\text{Value of Real Estate}}
\]

We can solve for Cap rate:

\[
\text{Cap Rate} = \frac{\text{Annual Operating Income}}{\text{Value of Real Estate}} \quad \text{Cap Rate} = \frac{40,000}{360,000}
\]

\[
\text{Cap Rate} = 11.11\%
\]

The cap rate is 11.11%

CALCULATIONS

Find the Cap rate

Step One: Press 40000 → ÷ → 360000

Step Two: Press =

Answer: 0.1111
WORKOUT AREA: *Here are sample problems – cover one of the values and see if you can solve it using the others*

<table>
<thead>
<tr>
<th>Cap Rate</th>
<th>Annual Oper. Inc.</th>
<th>Farmland Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>11.11%</td>
<td>$40,000.00</td>
<td>$360,000.00</td>
</tr>
<tr>
<td>12.33%</td>
<td>$45,000.00</td>
<td>$365,000.00</td>
</tr>
<tr>
<td>13.51%</td>
<td>$50,000.00</td>
<td>$370,000.00</td>
</tr>
<tr>
<td>14.67%</td>
<td>$55,000.00</td>
<td>$375,000.00</td>
</tr>
<tr>
<td>15.79%</td>
<td>$60,000.00</td>
<td>$380,000.00</td>
</tr>
<tr>
<td>16.88%</td>
<td>$65,000.00</td>
<td>$385,000.00</td>
</tr>
<tr>
<td>8.75%</td>
<td>$35,000.00</td>
<td>$400,000.00</td>
</tr>
</tbody>
</table>
A fund manager follows a strategy that is expected to generate equally likely outcomes of +7%, +3%, +2%, or -4% per period. The manager enters into financial derivatives at zero initial cost that cap the fund’s returns at 3% while providing a downside protective floor of a 0% return. What is the reduction in the fund’s true volatility from using the financial derivatives? A true mean and volatility can be calculated because the probabilities provided are known rather than estimated based on a sample. The true mean is 2%. The true variance without the derivative strategy is simply the average of the squared deviations \( \frac{1}{4} \left[ (0.07)^2 + (0.03)^2 + 0 + (-0.04)^2 \right] = 0.00155 \) for a volatility of 0.03937. The true variance with the derivative strategy is \( \frac{1}{4} \left[ (0.03)^2 + (0.03)^2 + 0 + (0.00)^2 \right] = 0.00015 \), for a volatility of 0.01225. The derivative-protected strategy has a volatility that is roughly 31% of the original strategy.

**EXPLANATION**

What you are trying to determine here is the impact of the financial derivatives contract on the variance of the total portfolio. To find this, we must first find the arithmetic mean returns for the portfolio \textit{without} the derivatives contract, followed by the portfolio \textit{with} the derivatives contract. The problem states that the upside of the portfolio is capped at 3% and the downside is capped at 0%. Therefore, any returns outside the bands of 0% to 3% should be replaced by those returns (e.g., a 7% return now becomes a 3% return and a -4% return becomes 0%):

Mean return (without derivatives contract) = \[ \frac{0.07 + 0.03 + 0.02 + (-0.04)}{4} \] = 0.02

Mean return (with derivatives contract) = \[ \frac{0.03 + 0.03 + 0.02 + 0.00}{4} \] = 0.02

In this example, the arithmetic mean returns are the same. Next, we can use these means to calculate the variance of the portfolio:

Variance of portfolio without derivatives contract

\[
\frac{1}{4} \left[ (0.07-0.02)^2 + (0.03-0.02)^2 + (0.02-0.02)^2 + (-0.04-0.02)^2 \right] = 0.00155.
\]

To find the standard deviation, we simply need to take the square root of this to get a standard deviation of 0.03937 or 3.937%.

Variance of portfolio with the derivatives contract

\[
\frac{1}{4} \left[ (0.03-0.02)^2 + (0.03-0.02)^2 + (0.02-0.02)^2 + (0.00-0.02)^2 \right] = 0.00015.
\]

To find the standard deviation, we simply need to take the square root of this to get a standard deviation of 0.01225 or 1.225%.

Finally, we can find the reduction of the volatility using the new strategy by dividing the new volatility by the old one = \( \frac{0.01225}{0.03937} = 0.3112 \)

**CALCULATION**

There is a much easier way to calculate these numbers using your BAII-plus calculator. To do this, we will be using the “DATA” (located on the “7” key) and “STAT” (located on the “8” key”) functions.

For the portfolio without a derivatives contract:
Step One: Press “2ND” → “DATA”
Step Two: Press “2ND” → CLR WORK
Step Three: Press → “X01” 0.07 → ENTER
Step Four: Press ↓ → “Y01” 1 → ENTER
Step Five: Press ↓ → “X02” 0.03 → ENTER
Step Six: Press ↓ → “Y02” 1 → ENTER
Step Seven: Press ↓ → “X03” 0.02 → ENTER
Step Eight: Press ↓ → “Y03” 1 → ENTER
Step Nine: Press ↓ → “X04” 0.04 → |+| → ENTER
Step 10: Press ↓ → “Y04” 1 → ENTER
Step 11: Press “@ND” → “STAT”
Step 12: Press ↓ until you see the following: $\bar{x} = 2.000$ (this is your arithmetic mean return)
Step 13: Press ↓ until you see the following: $\sigma_x = 0.03937$ (this is your standard deviation)

For the portfolio with a derivatives contract:

Step One: Press “2ND” → “DATA”
Step Two: Press “2ND” → CLR WORK
Step Three: Press → “X01” 0.03 → ENTER
Step Four: Press ↓ → “Y01” 1 → ENTER
Step Five: Press ↓ → “X02” 0.03 → ENTER
Step Six: Press ↓ → “Y02” 1 → ENTER
Step Seven: Press ↓ → “X03” 0.02 → ENTER
Step Eight: Press ↓ → “Y03” 1 → ENTER
Step Nine: Press ↓ → “X04” 0.00 → +|−| → ENTER
Step 10: Press ↓ → “Y04” 1 → ENTER
Step 11: Press “@ND” → “STAT”
Step 12: Press ↓ until you see the following: $\bar{x} = 2.000$ (this is your arithmetic mean return)
Step 13: Press ↓ until you see the following: $\sigma_x = 0.01225$ (this is your standard deviation)

To calculate the volatility reduction:

Step One: Press 0.01225 → ÷ → 0.03937
Step Two: Press =
Answer: 0.3112
WORKOUT AREA: Here are sample problems – cover one of the values and see if you can solve it using the others.

<table>
<thead>
<tr>
<th>Returns</th>
<th>Average</th>
<th>Derivative Max/Min</th>
<th>Volatility (no derivatives)</th>
<th>Volatility (using derivatives)</th>
<th>% Non-Derivative Volatility to Derivative Volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.07, 0.03, 0.02, -0.04</td>
<td>2%</td>
<td>0, 0.03</td>
<td>0.03937</td>
<td>0.01225</td>
<td>31.11%</td>
</tr>
<tr>
<td>0.1, 0.03, 0.04, 0</td>
<td>4%</td>
<td>0, 0.15</td>
<td>0.03631</td>
<td>0.03631</td>
<td>100.00%</td>
</tr>
<tr>
<td>0.08, 0.05, 0.01, -0.06</td>
<td>2%</td>
<td>-0.05, 0.05</td>
<td>0.05244</td>
<td>0.04093</td>
<td>78.04%</td>
</tr>
<tr>
<td>0.07, 0.09, 0.01, 0.03</td>
<td>5%</td>
<td>-0.03, 0.08</td>
<td>0.03162</td>
<td>0.02861</td>
<td>90.48%</td>
</tr>
<tr>
<td>-0.04, 0.06, -0.03, -0.02</td>
<td>-1%</td>
<td>-0.1, 0.05</td>
<td>0.03961</td>
<td>0.03536</td>
<td>89.26%</td>
</tr>
</tbody>
</table>
APPLICATION 9.6.2A (page 273)

An analyst observes a stale return series over a period of 50 weeks and finds a mean weekly return of 0.24%. The analyst notes that the returns of the week prior to the most recent 50 returns (week 0) was 2.50% and the return of the most recent period (week 50) was 5.00%. What is the mean return of the true returns for weeks 1 to 50 based on the analyst’s assumption that $\alpha = 0.60$?

Substituting into Equation 9.6:

$$0.24\% = u + \left( \frac{1}{50} \right) [(1 - 0.6)2.50\% - 0.6 \times 5.00\%]$$

$$u = 0.24\% - \left( \frac{1}{50} \right)(1.00\% - 3.00\%) = 0.24\% - \left( \frac{-2.00\%}{50} \right) = 0.28\%$$

The large period 0 and T returns caused a relatively minor error (0.04%) from using a stale mean to estimate a true mean.

EXPLANATION

In order to solve this problem, we need to use Equation 9.6:

$$u^* = u + \left( \frac{1}{T} \right) [(1 - \alpha)R_{i,0} - \alpha R_{i,T}]$$

From there, the calculations for this problem are located in the Application 9.6.2A box.

CALCULATION

Step One: Press 0.01 → * → 0.03
Step Two: Press = “-0.02”
Step Three: Press x → 1 → ÷ → 50
Step Four: Press = “-0.0004”
Step Five: Press 0.0024 → - → 0.0004 → +|
Step Six: Press =
Answer: 0.0028
WORKOUT AREA: Here are sample problems – cover one of the values and see if you can solve it using the others.

<table>
<thead>
<tr>
<th>u*</th>
<th>u</th>
<th>T</th>
<th>α</th>
<th>r(i,0)</th>
<th>r(i, T)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.24%</td>
<td>0.2800%</td>
<td>50</td>
<td>0.6</td>
<td>2.50%</td>
<td>5%</td>
</tr>
<tr>
<td>0.30%</td>
<td>0.3380%</td>
<td>50</td>
<td>0.7</td>
<td>3.00%</td>
<td>4%</td>
</tr>
<tr>
<td>0.40%</td>
<td>0.4300%</td>
<td>60</td>
<td>0.6</td>
<td>3.00%</td>
<td>5%</td>
</tr>
<tr>
<td>0.35%</td>
<td>0.4050%</td>
<td>40</td>
<td>0.6</td>
<td>2.00%</td>
<td>5%</td>
</tr>
<tr>
<td>0.50%</td>
<td>0.5560%</td>
<td>50</td>
<td>0.6</td>
<td>2.00%</td>
<td>6%</td>
</tr>
<tr>
<td>0.20%</td>
<td>0.2400%</td>
<td>50</td>
<td>0.6</td>
<td>2.50%</td>
<td>5%</td>
</tr>
</tbody>
</table>
An analyst observes a stale return series for an index based on appraised values and finds an annualized volatility of 16% over the same time period in which an index based on market values of otherwise identical assets exhibited an annualized volatility of 27.7%. Based on the assumption that the returns from the series using appraisals is based on an equally weighted average of \( N \) data points (including the contemporaneous data point), how many data points are being averaged in order to estimate an appraised value? Substituting into Equation 9.8:

\[
16.0\% = 27.7\% / \sqrt{N}
\]

\[N \approx 3\]

**EXPLANATION**

In order to solve this problem, we need to use Equation 9.8:

\[
\sigma(r_{i,t}^*) = \frac{\sigma(r_{i,t})}{\sqrt{N}}
\]

We are given \( \sigma(r_{i,t}^*) = 16\% \) and \( \sigma(r_{i,t}) = 27.7\% \) so we simply need to solve for \( N \).

\[
\frac{\sigma(r_{i,t}^*)}{\sigma(r_{i,t})} = \sqrt{N}
\]

\[
N = \left( \frac{\sigma(r_{i,t})}{\sigma(r_{i,t}^*)} \right)^2 = \left( \frac{0.277}{0.16} \right)^2
\]

\[N = 2.997 \approx 3.0\]

**CALCULATION**

Step One: Press 0.16 ➔ + ➔ 0.277
Step Two: Press = “0.5776”
Step Three: Press ➔ x²
Step Four: Press =
Answer: 2.997
WORKOUT AREA: Here are sample problems – cover one of the values and see if you can solve it using the others.

<table>
<thead>
<tr>
<th>Appraisal Volatility</th>
<th>Market Volatility</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>16%</td>
<td>27.70%</td>
<td>3.00</td>
</tr>
<tr>
<td>14%</td>
<td>30.00%</td>
<td>4.59</td>
</tr>
<tr>
<td>18%</td>
<td>20.00%</td>
<td>1.23</td>
</tr>
<tr>
<td>16%</td>
<td>32.00%</td>
<td>4.00</td>
</tr>
<tr>
<td>19%</td>
<td>35.00%</td>
<td>3.39</td>
</tr>
<tr>
<td>4%</td>
<td>8.00%</td>
<td>4.00</td>
</tr>
</tbody>
</table>
APPLICATION 10.1.7A (page 288)

A plain-vanilla note from a particular issuer carries a coupon rate of 7%. The firm issues a CLN with a coupon of 4%. The CLN contains an implicit call option on the S&P GSCI (currently at 1,500) with a strike price set 10% out-of-the-money, at 1,650. How much would the CLN distribute as a principal payment on a $1,000,000 note under the following four scenarios: The S&P GSCI value at the notes maturity is: 1,500, 1,600, 1,700, or 1,800? The principal payment is simply $1,000,000 for the two scenarios in which the implicit call option finishes out of the money (S&P GSCI is 1,500 or 1,600). For S&P GSCI = 1,700, the payout is $1,000,000 × [1 + (1,700 – 1,650)/1,650] = $1,030,303. For S&P GSCI = 1,800, the payout is $1,000,000 × [1 + (1,800 – 1,650)/1,650] = $1,090,909

EXPLANATION

We can look at this problem like we would a regular call option on a stock. Whenever the closing price is higher than the strike price, the implicit call option will finish in the money. To solve this problem, we can use the following equation:

$$ Principal \times \left\{ \left[ 1 + \max \left( 0, \frac{GSCI_T - GSCI_X}{GSCI_X} \right) \right] \right\} $$

For the two scenarios where the final price was below the strike price of 1,605 (1,500 and 1,600), we can determine that the principal payment is simply the original $1,000,000.

For the other two, we can calculate the new principal payment as follows:

$$ \$1,000,000 \times \left\{ \left[ 1 + \max \left( 0, \frac{1,700 - 1,650}{1,650} \right) \right] \right\} = \$1,000,000 \times (1.0303) = \$1,030,303 $$

$$ \$1,000,000 \times \left\{ \left[ 1 + \max \left( 0, \frac{1,800 - 1,650}{1,650} \right) \right] \right\} = \$1,000,000 \times (1.0909) = \$1,090,909 $$

Note: this problem does not ask for coupon payments. This application focuses only on the principal payment.

CALCULATION

These calculations will only focus on the two in-the-money options.

When the GSCI is at 1,700

Step One: Press 1700 → + → 1650
Step Two: Press = “1.0303”
Step Three: Press x → 1,000,000
Step Four: Press =
Answer: 1030303
When the GSCI is at 1,800

Step One: Press 1800 → + → 1650
Step Two: Press “1.0900”
Step Three: Press x → 1,000,000
Step Four: Press =
Answer: 1090909

WORKOUT AREA: Here are sample problems – cover one of the values and see if you can solve it using the others.

<table>
<thead>
<tr>
<th>Principal</th>
<th>Strike Price</th>
<th>Final Index Value</th>
<th>New Principal Payment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$ 1,000,000.00</td>
<td>1650</td>
<td>1800</td>
<td>$ 1,090,909.09</td>
</tr>
<tr>
<td>$ 1,000,000.00</td>
<td>5000</td>
<td>4500</td>
<td>$ 1,000,000.00</td>
</tr>
<tr>
<td>$ 1,000,000.00</td>
<td>4000</td>
<td>4500</td>
<td>$ 1,125,000.00</td>
</tr>
<tr>
<td>$ 1,000,000.00</td>
<td>100</td>
<td>125</td>
<td>$ 1,250,000.00</td>
</tr>
<tr>
<td>$ 1,000,000.00</td>
<td>6000</td>
<td>2000</td>
<td>$ 1,000,000.00</td>
</tr>
<tr>
<td>$ 1,000,000.00</td>
<td>1500</td>
<td>1000</td>
<td>$ 1,000,000.00</td>
</tr>
<tr>
<td>$ 1,000,000.00</td>
<td>6000</td>
<td>8000</td>
<td>$ 1,333,333.33</td>
</tr>
<tr>
<td>$ 1,000,000.00</td>
<td>3000</td>
<td>2000</td>
<td>$ 1,000,000.00</td>
</tr>
</tbody>
</table>
The spot price of a commodity is $10 while its six-month futures price is $10.12. Given that the annual financing rate is 3%, the annual spoilage rate is 2%, and the storage cost per month is $0.02, what is the implied annual convenience yield?

Use the table in Exhibit 10.1 to find the cost of carry, assuming that the futures price of $10.12 must equal the spot price of $10.00 plus the cost of carry. The six-month financing cost is $10.00 × 0.03 × 6/12, or $0.15; the six-month spoilage cost is $10.00 × 0.02 × 6/12, or $0.10; and the storage cost ($0.02 × 6) is $0.12. The convenience yield (CY) must satisfy the following equation:

\[
\text{Futures Price} = \text{Spot Price} + \text{Financing Cost} + \text{Spoilage Cost} + \text{Storage Cost} - \text{Convenience Yield}
\]

Convenience Yield = \(5\%\) per year

Note that the computations are illustrated with simple interest rather than compounded interest for simplicity and because of the relatively short period of time. Of course, any one of the variables in the relationship could be solved given the values of all the other variables.

EXPLANATION

Remember, when calculating the forward price of a futures contract, any “costs” to the owner of the commodity are additive, while any “benefits” to the owner are subtracted. This problem provides you with all variables, except for the convenience yield (a “benefit”). Since this is a six-month contract, any annualized numbers provided will need to be multiplied by 6/12 or 0.5:

\[
\text{Futures Price} = \text{Spot Price} + \text{Financing Cost} + \text{Spoilage Cost} + \text{Storage Cost} - \text{Convenience Yield}
\]

We can further break down the Convenience Yield:

\[
\text{Convenience Yield} = CY \text{ (given as a percentage)} \times \text{Spot Price} \times \text{Time}
\]

\[= CY \text{ (given as a percentage)} \times 10 \times \frac{6}{12}
\]

So, we really need to find the CY as a percentage. We can rearrange the equation by substituting the components of Convenience Yield into the rest of the problem:

\[
CY = \frac{\text{Spot Price} + \text{Financing Cost} + \text{Spoilage Cost} + \text{Storage Cost} - \text{Futures Price}}{10 \times \frac{6}{12}}
\]

\[
CY = \frac{10 + 0.15 + 0.10 + 0.12 - 10.12}{10 \times \frac{6}{12}} = 0.05
\]

The convenience yield is 5\%
CALCULATIONS

Step One: Press 10 → + → 0.15 → + → 0.10 → 0.12
Step Two: Press = “10.37”
Step Three: Press - → 10.12
Step Four: Press = “0.25”
Step Five: Press ÷ → (10 → x → 0.5 → )
Step Six: Press = “5”
Step Seven: Press =
Answer: 0.05

WORKOUT AREA: Here are sample problems — cover one of the values and see if you can solve it using the others.

<table>
<thead>
<tr>
<th>Spot Price</th>
<th>Futures Price</th>
<th>Time (Months)</th>
<th>Financing Cost</th>
<th>Spoilage Cost</th>
<th>Storage Cost</th>
<th>Convenience Yield</th>
</tr>
</thead>
<tbody>
<tr>
<td>$ 10.00</td>
<td>$ 10.12</td>
<td>6</td>
<td>3.00%</td>
<td>2.00%</td>
<td>$ 0.02</td>
<td>5.00%</td>
</tr>
<tr>
<td>$ 50.00</td>
<td>$ 55.00</td>
<td>12</td>
<td>4.00%</td>
<td>3.00%</td>
<td>$ 0.05</td>
<td>-1.80%</td>
</tr>
<tr>
<td>$ 10.00</td>
<td>$ 10.05</td>
<td>6</td>
<td>4.00%</td>
<td>3.00%</td>
<td>$ 0.02</td>
<td>8.40%</td>
</tr>
<tr>
<td>$ 10.00</td>
<td>$ 10.10</td>
<td>6</td>
<td>2.50%</td>
<td>2.00%</td>
<td>$ 0.02</td>
<td>4.90%</td>
</tr>
<tr>
<td>$ 11.00</td>
<td>$ 11.05</td>
<td>6</td>
<td>3.00%</td>
<td>2.00%</td>
<td>$ 0.08</td>
<td>12.82%</td>
</tr>
<tr>
<td>$ 11.50</td>
<td>$ 11.60</td>
<td>10</td>
<td>3.00%</td>
<td>2.00%</td>
<td>$ 0.02</td>
<td>6.04%</td>
</tr>
<tr>
<td>$ 11.75</td>
<td>$ 11.82</td>
<td>6</td>
<td>3.00%</td>
<td>2.00%</td>
<td>$ 0.02</td>
<td>5.85%</td>
</tr>
<tr>
<td>$ 12.00</td>
<td>$ 12.20</td>
<td>6</td>
<td>3.00%</td>
<td>2.00%</td>
<td>$ 0.02</td>
<td>3.67%</td>
</tr>
</tbody>
</table>
APPLICATION 10.2.8A (page 294)

Consider a calendar spread that is long the two-year forward contract and short the one-year forward contract on a physical commodity with a spot price of $100. Assume that the number of contracts in the long position equals the number of contracts in the short position. The trader put the spread on in anticipation that storage costs, c, will rise. Assume that the forward prices adhere to Equation 6.8 and that \( r = 2\% \), \( c = 3\% \), and \( y = 5\% \). Note that these values were chosen for the simplicity that \( r + c - y = 0\% \) so that the forward prices equal the spot prices. (a) What would the profit or loss be to the trader if spot prices rose $1? (b) What would the profit or loss be to the trader if the storage costs rose one percentage point (from 3% to 4%)? (a) Changes in the spot price will not affect calendar spreads as long as none of the carrying costs change from \( r + c - y = 0\% \). All forward prices will continue to match the spot price, and the basis of all contracts will remain zero. The trader is hedged against changes in the spot price by holding an equal number of long and short contracts. (b) In the second scenario, when storage costs rise from 3% to 4%, \( r + c - y \) will no longer equal 0, and forward prices will rise relative to spot prices. In this example, the longer delivery date of the long position (two years) will cause the forward price of the two-year forward to rise in price by more than the one-year forward, netting the trader a profit from correctly speculating that the storage costs would rise. Specifically, the two-year forward rises from $100 to $102.020, and the one-year forward rises from $100 to $101.005, netting the trader a profit of $1.015 from being long the two-year forward and short the one-year forward. Note that the values are based on continuous compounding.

EXPLANATION

Let’s take stock of the situation. We have two positions short and long with the same number of contracts in each (let’s say 1,000 units of the underlying commodity). In addition, the spot price equals the forward price because \( r + c - y = 0\% \). Therefore, if there is a $1 spot price increase, the long position would profit by $1,000, but the short position would lose $1,000, breaking even.

Now using the same situation, if the storage costs increase to 4% from 3% \( r + c - y = 1\% \). Thus the spot price is not equal to the forward price as it was in the prior situation. Using equation 6.8,

\[
F(T) = e^{(r+c-y)T}S
\]

The two-year long forward price increases to:

\[
F(T) = e^{(r+c-y)T}S = e^{(0.02+0.04-0.05)2} \times 100 = 102.020
\]

\[
F(T) = e^{0.012} \times 100 = 100.0\]

\[
F(T) = 102.02
\]

The one-year short forward price increases to:

\[
F(T) = e^{(r+c-y)T}S = e^{(0.02+0.04-0.05)1} \times 100 = 100.04
\]

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\[ F(T) = e^{0.01 \times 1} \times 100 \]
\[ F(T) = 101.01 \]

This nets the trader a $1.01 ($2.02 - $1.01) profit because the trader is long the two-year forward contract that increased by $2.02 and short the one-year forward contract that increased by $1.01. Note that the above numbers are rounded.

A spread can be viewed as being the same as a trade that has established a long position and a short position with the same number of contracts. A trader with long and short positions of equal total size in terms of number of contracts is protected from a parallel shift (the quantity \( r+c-y \) stays constant) in the forward curve, but not necessarily from a non-parallel shift. A rise or fall in \( r+c-y \) will disproportionately affect longer term contracts. A calendar spread is exposed to a shift in \( r+c-y \).

**CALCULATIONS**

**Price of two-year long position**
- **Step One:** Press 0.02 → + → 0.04
- **Step Two:** Press - → 0.05 Step
- **Three:** Press x → 2
- **Step Four:** Press 2^{nd} → e^x
- **Step Five:** Press x → 100 Step
- **Six:** Press = Answer: $102.02

**Price of one-year short position**
- **Step One:** Press 0.02 → + → 0.04 Step
- **Two:** Press - → 0.05
- **Step Three:** Press x → 1
- **Step Four:** Press 2^{nd} → e^x
- **Step Five:** Press x → 100 Step
- **Six:** Press = Answer: $101.01
WORKOUT AREA: Here are sample problems – cover one of the values and see if you can solve it using the others. Assume for simplicity that at the start \( r+c-y=0 \) so that all forward prices equal the spot and that the trader has a spread with an equal number of long and short contracts. Although this is hedged if carrying costs remain constant (since all forward prices would move up or down by the same amount), when carrying costs change, the forward prices can shift by different amounts causing the hedge to fail (i.e., the value of the spread changes). Verify that you can calculate the last three columns given the first four columns and the fact that the positions were established with \( r+c-y=0 \) so that the forward prices were $50 for the first three problems.

<table>
<thead>
<tr>
<th>Spot Price</th>
<th>( r+c-y )</th>
<th>Maturity of Long</th>
<th>Maturity of Short</th>
<th>Price Long</th>
<th>Price Short</th>
<th>Profit or Loss</th>
</tr>
</thead>
<tbody>
<tr>
<td>$50</td>
<td>rises 1%</td>
<td>1</td>
<td>2</td>
<td>$50.50</td>
<td>$51.01</td>
<td>($0.51)</td>
</tr>
<tr>
<td>$50</td>
<td>rises 2%</td>
<td>2</td>
<td>1</td>
<td>$52.04</td>
<td>$51.01</td>
<td>$1.03</td>
</tr>
<tr>
<td>$50</td>
<td>falls 1%</td>
<td>1</td>
<td>3</td>
<td>$49.50</td>
<td>$48.52</td>
<td>$0.98</td>
</tr>
<tr>
<td>$40</td>
<td>falls 3%</td>
<td>4</td>
<td>2</td>
<td>$35.48</td>
<td>$37.67</td>
<td>($2.19)</td>
</tr>
<tr>
<td>$40</td>
<td>rises 3% no change</td>
<td>5</td>
<td>3</td>
<td>$46.47</td>
<td>$43.77</td>
<td>$2.70</td>
</tr>
<tr>
<td>$25</td>
<td>no change</td>
<td>3</td>
<td>5</td>
<td>$25.00</td>
<td>$25.00</td>
<td>$0.00</td>
</tr>
</tbody>
</table>
APPLICATION 10.3.2A (page 298)

Consider a one-year, fully collateralized commodity futures position on a commodity that experiences a 3% decline in its spot price. Over that same year, the basis in the commodity fell from the spot price exceeding the futures price by 2% to a basis of 0%. The riskless interest rate was 4% and the storage cost of the commodity was 2.4% per year. Compute the spot return, collateral yield, excess return, roll yield (i.e., roll return), and the return on a fully collateralized position.

The spot return is the change in the spot price, which is given as -3%. The collateral yield on a fully collateralized position would be equal to the riskless rate, which is given as 4%. The excess return of the futures contract is the percentage price change in the futures contract. Since the spot price fell 3% and the futures price rose relative to the spot price from being 2% lower to being equal, the excess return on the futures contract was -1%. The roll yield or roll return is the 2% gain in the futures contract relative to the spot price from the basis going from the futures price being 2% less than the spot price to being equal. The return on a fully collateralized position may be calculated using either Equation 10.6 (Collateral Yield + Excess Return = 4% + (-1%) = 3%) or Equation 10.7 (Spot Return + Collateral Yield + Roll Yield = -3% + 4% + 2% = 3%). The 2.4% storage costs are subsumed within the roll yield and are superfluous to the exercise.

EXPLANATION

Here, we are tasked with calculating the different components of return for a commodity futures contract. The variables we need to solve this are here:

- Spot Return: this is simply the change in the spot price, or -3%
- Collateral Yield: the collateral yield is equal to the riskless rate, or 4%
- Excess Return: this can be broken into the change in the spot price and the change in the basis. The spot price return was -3%, while the change in basis was equal to +2% (the futures price was 2% lower than the spot price, but now it is equal to it). Therefore, the excess return is -1%
- Roll yield: The roll yield represents the change in basis, which we just solved as +2%.

Note, there are two different ways to calculate the return of the fully collateralized position, which is a good way to check our work:

- Equation 10.6 calculates the return by adding the following: Collateral Yield + Excess Return
- Equation 10.7 expands this equation by adding the following: Spot Return + Collateral Yield + Roll Return

Note, excess return can be calculated by adding the Spot Return and Roll Yield.

Either way, we get a 3% return. Using Equation 10.6:

\[ \text{Return on Fully Collateralized Position} = \text{Collateral Yield} + \text{Excess Return} = 4\% + (-1\%) = 3\% \]
Using Equation 10.7:

\[ \text{Return on Fully Collateralized Position} = \text{Spot Return} + \text{Collateral Yield} + \text{Roll Yield} \]
\[ = (-3\%) + 4\% + 2\% = 3\% \]

**CALCULATIONS**

To calculate the excess return:

Step One: Press \(0.03\) \(\rightarrow\) \(+\) \(\rightarrow\) \(-\) \(\rightarrow\) \(+\) \(\rightarrow\) 0.02
Step Two: Press =
Answer: -0.01

To calculate the fully collateralized return using equation 10.6:

Step One: Press 0.04 \(\rightarrow\) \(+\) \(\rightarrow\) 0.01 \(\rightarrow\) \(+\) \(-\)
Step Two: Press =
Answer: 0.03

To calculate the fully collateralized return using equation 10.7:

Step One: Press 0.03 \(\rightarrow\) \(+\) \(-\) \(\rightarrow\) \(+\) 0.04 \(\rightarrow\) \(+\) \(\rightarrow\) 0.02
Step Two: Press =
Answer: 0.03

**WORKOUT AREA:** Here are sample problems – cover one of the values and see if you can solve it using the others.

<table>
<thead>
<tr>
<th>Spot Return</th>
<th>Excess Return</th>
<th>Roll Yield</th>
<th>Collateral Yield</th>
<th>Full Collateralized Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3%</td>
<td>-1%</td>
<td>2%</td>
<td>4%</td>
<td>3%</td>
</tr>
<tr>
<td>-3%</td>
<td>-3%</td>
<td>0%</td>
<td>1%</td>
<td>-2%</td>
</tr>
<tr>
<td>-4%</td>
<td>1%</td>
<td>5%</td>
<td>-5%</td>
<td>-4%</td>
</tr>
<tr>
<td>7%</td>
<td>15%</td>
<td>8%</td>
<td>-5%</td>
<td>10%</td>
</tr>
<tr>
<td>4%</td>
<td>7%</td>
<td>3%</td>
<td>1%</td>
<td>8%</td>
</tr>
</tbody>
</table>
APPLICATION 11.4.3.A (page 340)

Loosely following some of the values indicated earlier in this section for films, assume that the probability of substantial success for an investment in IP \((p)\) is 6%, the rate at which expected cash flows diminish each year after their initial potential \((g)\) is 5%, and the required rate of return \((r)\) is 12%. How much would this investment in IP be worth per dollar of projected possible first-year cash flow \((CF_1)\)? This example normalizes the analysis to a value of $1 for \(CF_1\). Using Equation 11.1 produces \((0.06 \times 1)/[0.12 - (-0.05)]\), which equals approximately $0.35. Roughly estimated, the value of the IP might be only 35 cents for each dollar of initial annual cash inflow that would be generated, assuming that the initial cash flow is a potential cash flow and therefore represents a very successful outcome.

EXPLANATION

Essentially, this formula is the perpetuity growth model (albeit with a negative growth rate) multiplied by the probability of success for the investment as shown in equation 11.1:

\[
V_{ip,0} = p \times \frac{CF_1}{(r - g)}
\]

To solve this equation to find out how much this investment in IP will be worth per dollar of first-year cash flows, we need to plug in the appropriate value, \(p = 6\%\), \(g = -5\%\), \(r = 12\%\), and \(CF_1 = 1\).

\[
V_{ip,0} = 0.06 \times \frac{1}{(0.12 - (-0.05))}
\]

\[
V_{ip,0} = 0.06 \times \frac{1}{(0.17)}
\]

\[
V_{ip,0} = 0.06 \times 5.88
\]

\[
V_{ip,0} = 0.35
\]
CALCULATIONS

Step One: Press 0.12 → - → 0.05 → +|
Step Two: Press = “0.17”
Step Three: Press 1 → + → 0.17
Step Four: Press x → 0.06
Step Five: Press =

Answer: 0.35

WORKOUT AREA: Here are sample problems – cover one of the values and see if you can solve it using the others

<table>
<thead>
<tr>
<th>Value per Dollar of IP</th>
<th>p</th>
<th>g</th>
<th>r</th>
<th>CF1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0.82</td>
<td>14.00%</td>
<td>-5.00%</td>
<td>12.00%</td>
<td>$1.00</td>
</tr>
<tr>
<td>$0.28</td>
<td>5.00%</td>
<td>-2.00%</td>
<td>16.00%</td>
<td>$1.00</td>
</tr>
<tr>
<td>$0.83</td>
<td>10.00%</td>
<td>-3.00%</td>
<td>9.00%</td>
<td>$1.00</td>
</tr>
<tr>
<td>$0.53</td>
<td>8.00%</td>
<td>1.00%</td>
<td>16.00%</td>
<td>$1.00</td>
</tr>
<tr>
<td>$2.50</td>
<td>20.00%</td>
<td>2.00%</td>
<td>10.00%</td>
<td>$1.00</td>
</tr>
<tr>
<td>$0.08</td>
<td>1.00%</td>
<td>-5.00%</td>
<td>8.00%</td>
<td>$1.00</td>
</tr>
<tr>
<td>$6.00</td>
<td>12.00%</td>
<td>-1.00%</td>
<td>1.00%</td>
<td>$1.00</td>
</tr>
</tbody>
</table>
Assume that Equation 11.2 is an appropriate valuation model and that $\frac{CF_1}{V_{ip,0}}$ is 3.0, $p$ is 0.06, and $g$ is $-0.05$. What is the investment's annual rate of return? Inserting the values generates the result that $r$ is 13%.

**EXPLANATION**

To solve this application we need to use equation 11.2:

$$r = p \times \left( \frac{CF_1}{V_{ip,0}} \right) + g$$

With equation 11.2 at hand, we need to plug in the given figures and solve for $r$. In this application $p=0.06$, $g = -0.05$, and $\frac{CF_1}{V_{ip,0}} = 3.0$.

$$r = 0.06 \times 3.0 + (-0.05)$$

$$r = 0.18 + (-0.05)$$

$$r = 0.13$$

The answer is $r = 13\%$.

**CALCULATIONS**

Step One: Press 3.0 → x → 0.06 → +|-|-
Step Two: Press + → 0.05 → +|-|-
Step Three: Press =
Answer: 0.13

**WORKOUT AREA:** Here are sample problems – cover one of the values and see if you can solve it using the others.

<table>
<thead>
<tr>
<th>Annual Rate of Return</th>
<th>$p$</th>
<th>$g$</th>
<th>$\frac{CF_1}{V_{ip}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>13.00%</td>
<td>6.00%</td>
<td>-5.00%</td>
<td>3.00</td>
</tr>
<tr>
<td>16.00%</td>
<td>5.00%</td>
<td>-4.00%</td>
<td>4.00</td>
</tr>
<tr>
<td>47.00%</td>
<td>10.00%</td>
<td>-3.00%</td>
<td>5.00</td>
</tr>
<tr>
<td>9.00%</td>
<td>8.00%</td>
<td>1.00%</td>
<td>1.00</td>
</tr>
<tr>
<td>52.00%</td>
<td>25.00%</td>
<td>2.00%</td>
<td>2.00</td>
</tr>
<tr>
<td>-3.00%</td>
<td>1.00%</td>
<td>-5.00%</td>
<td>2.00</td>
</tr>
<tr>
<td>59.00%</td>
<td>12.00%</td>
<td>-1.00%</td>
<td>5.00</td>
</tr>
</tbody>
</table>
APPLICATION 12.4.1A (page 364)

Assume that a borrower takes out a $100,000, 25-year mortgage (300 months), at a 6% annual nominal interest rate (a monthly interest rate of 6%/12, or 0.5%). What is the mortgage’s monthly payment?

The monthly payments (principal plus interest) can be calculated using Equation 12.1 directly, as follows:

\[ MP = 100,000 \times \left\{ \frac{0.005}{1 - (1.005)^{-300}} \right\} = 644.30 \]

Using a financial calculator, the monthly mortgage payment is calculated by inputting the following values: \( n \) (number of periods) = 12 \times 25 = 300 months, \( i \) (interest rate per period) = 6%/12 = 0.5%, \( PV \) (present value) = \( \pm $100,000 \), \( FV \) (future value) = $0, and solving for (compute) \( PMT \) (payment).

The \( PV \) is entered as either a positive or a negative number, depending on the calculator that is used. Note that some financial calculators require that the interest rate of 0.5% be entered as .005 and some as .5. Also, some financial calculators require prior clearing, use, or output of negative numbers and may or may not require input of other values, such as the \( FV \). Spreadsheets contain functions analogous to the financial calculator functions that are demonstrated throughout this chapter. In Excel, the payment can be calculated using \( = \text{pmt} \) (annual rate/12, number of months, loan amount). Note that payment amounts in practice are rounded to the nearest cent.

EXPLANATION

The mathematical way to solve this problem is using equation 12.1 as follows:

\[ MP = MB \times \left\{ \frac{i}{[1 - (1+i)^{-n}]} \right\} \]
\[ MP = 100,000 \times \left\{ \frac{0.005}{[1 - (1.005)^{-300}]} \right\} \]
\[ MP = 100,000 \times \left\{ \frac{0.005}{[1 - 0.22396568]} \right\} \]
\[ MP = 100,000 \times \left\{ \frac{0.005}{0.77603432} \right\} \]
\[ MP = $100,000 \times 0.0064430 \]
\[ MP = $644.30 \]

The monthly payment is $644.30 (rounded). However, there is a way to use the time value of money (TVM) functions on the TI-BA II Plus to solve this problem as well.
CALCULATIONS

Step One: Press 2nd → CLR TVM
Step Two: Press 12 → x → 25 → = → N
Step Three: Press 6 → ÷ → 12 → = → I/Y
Step Four: Press 100000 → +|- → PV
Step Five: Press 0 → FV
Step Six: Press CPT → PMT
Answer: 644.30

WORKOUT AREA: Here are sample problems – cover one of the values and see if you can solve it using the others

<table>
<thead>
<tr>
<th>Loan Monthly Payment</th>
<th>Loan</th>
<th>Annual Interest</th>
<th>Time Frame (Years, Monthly)</th>
<th>Amortization Balance</th>
</tr>
</thead>
<tbody>
<tr>
<td>$644.30</td>
<td>$100,000.00</td>
<td>6.00%</td>
<td>25</td>
<td>$0.00</td>
</tr>
<tr>
<td>$632.07</td>
<td>$100,000.00</td>
<td>6.50%</td>
<td>30</td>
<td>$0.00</td>
</tr>
<tr>
<td>$1,085.26</td>
<td>$100,000.00</td>
<td>5.50%</td>
<td>10</td>
<td>$0.00</td>
</tr>
<tr>
<td>$1,509.27</td>
<td>$100,000.00</td>
<td>7.00%</td>
<td>7</td>
<td>$0.00</td>
</tr>
<tr>
<td>$739.69</td>
<td>$100,000.00</td>
<td>4.00%</td>
<td>15</td>
<td>$0.00</td>
</tr>
<tr>
<td>$632.65</td>
<td>$100,000.00</td>
<td>4.50%</td>
<td>20</td>
<td>$0.00</td>
</tr>
<tr>
<td>$1,774.74</td>
<td>$100,000.00</td>
<td>2.50%</td>
<td>5</td>
<td>$0.00</td>
</tr>
</tbody>
</table>
APPLICATION 12.4.1B (page 365)

What would be the outstanding mortgage balance at the start of month 61 in terms of remaining principal of a $100,000, 25-year mortgage (300 months), at a 6% annual nominal interest rate?

As shown in Exhibit 12.5, the outstanding mortgage balance at the start of month 61 in terms of remaining principal is $89,932.18, five years after the loan has been taken out. This amount does not correspond exactly to a present value computation of the balance using the exact payment amount of $644.30 (using a financial calculator: \( n = 12 \times 20 = 240 \), \( i = 6.0\%/12 = 0.5\% \), \( PMT = 644.30 \), \( FV = 0 \), solve for \( PV \)). The reason is that mortgage payments are values that in practice are rounded to the nearest cent, and mortgage amortization computations (such as Exhibit 12.5) are based on this rounded payment amount ($644.30) rather than a more exact payment amount ($644.3014).

For simplicity, this discrepancy caused by rounding error is disregarded in the computations that follow. Notice that over time, the proportion of interest payment to principal payment declines, and increasingly a larger portion of the total payment is allocated to paying down the principal.

EXPLANATION

We can use equation 12.1 to solve this equation once again, after a bit of manipulating:

\[
MP = MB \times \left( \frac{i}{[1-(1+i)^{-n}]} \right) \\
\frac{MP}{[i/1-(1+i)^{-n}]} = MB
\]

After we have manipulated the equation, we can solve for the mortgage balance:

As you can see owing to the minor rounding of the payment, we can arrive at a nontrivially different mortgage balance when compounded through the years.

CALCULATIONS

Step One: Press 2nd \( \rightarrow \) CLR TVM
Step Two: Press 12 \( \rightarrow \) x \( \rightarrow \) 20 \( \rightarrow \) = \( \rightarrow \) N Step
Three: Press 6 \( \rightarrow \) ÷ \( \rightarrow \) 12 \( \rightarrow \) = \( \rightarrow \) I/Y Step
Four: Press 644.3021 \( \rightarrow \) PMT
Step Five: Press 0 \( \rightarrow \) FV Step
Six: Press CPT → PV

Answer: 89,932.18

WORKOUT AREA: Here are sample problems – cover one of the values and see if you can solve it using the others

<table>
<thead>
<tr>
<th>Loan Monthly Payment</th>
<th>Initial Loan Value</th>
<th>Annual Interest Rate</th>
<th>Month of Mortgage Life</th>
<th>Original Maturity</th>
<th>Remaining Principal</th>
</tr>
</thead>
<tbody>
<tr>
<td>$644.30</td>
<td>$100,000.00</td>
<td>6.00%</td>
<td>61</td>
<td>25</td>
<td>$89,932.09</td>
</tr>
<tr>
<td>$706.78</td>
<td>$100,000.00</td>
<td>7.00%</td>
<td>30</td>
<td>25</td>
<td>$96,111.76</td>
</tr>
<tr>
<td>$908.70</td>
<td>$100,000.00</td>
<td>10.00%</td>
<td>40</td>
<td>25</td>
<td>$96,543.64</td>
</tr>
<tr>
<td>$584.59</td>
<td>$100,000.00</td>
<td>5.00%</td>
<td>50</td>
<td>25</td>
<td>$90,892.54</td>
</tr>
<tr>
<td>$474.21</td>
<td>$100,000.00</td>
<td>3.00%</td>
<td>10</td>
<td>25</td>
<td>$97,961.80</td>
</tr>
<tr>
<td>$768.91</td>
<td>$100,000.00</td>
<td>8.50%</td>
<td>15</td>
<td>30</td>
<td>$99,111.70</td>
</tr>
<tr>
<td>$675.21</td>
<td>$100,000.00</td>
<td>6.50%</td>
<td>20</td>
<td>25</td>
<td>$97,335.16</td>
</tr>
</tbody>
</table>
APPLICATION 12.4.1C (page 366)

Suppose that the market interest rate for the mortgage in Exhibit 12.1 rises to 7.5%. What is the market value of the mortgage, assuming it is the start of month 61?

The market value is equal to $79,978.33 (using a financial calculator: \( n = 12 \times 20 = 240, \ i = 7.5\%/12 = 0.625\%, \ PMT = $644.30, \ FV = \$0, \ solve \ for \ PV \)). At a new and lower market interest rate of 4.5%, the market value of the mortgage is equal to $101,841.56 (found as before except that \( i = 4.5\%/12 = 0.375\% \)). These values illustrate that the market value of fixed-rate mortgages, as fixed-income securities, varies inversely with market interest rates.

EXPLANATION

Please see the explanation for Application 12.4.1B. Note that this application changes the computation of the mortgage’s value during its life from the remaining principal to the market value based on a new market interest rate. Otherwise, the computation is the same.

CALCULATIONS

Step One: Press 2nd \( \rightarrow \) CLR TVM

Step Two: Press 12 \( \rightarrow \times \) 20 \( \rightarrow \) = \( \rightarrow \) N Step

Three: Press 4.5 \( \rightarrow \div \) 12 \( \rightarrow \) = \( \rightarrow \) I/Y Step Four:

Press 644.30 \( \rightarrow \) PMT

Step Five: Press 0 \( \rightarrow \) FV Step Six:

Press CPT \( \rightarrow \) PV Answer:

101841.56
WORKOUT AREA: Here are sample problems – cover one of the values in the market value column and see if you can solve it using the others

<table>
<thead>
<tr>
<th>Loan Monthly</th>
<th>Initial Loan</th>
<th>Annual Interest</th>
<th>Month of Mortgage</th>
<th>Original Maturity</th>
<th>Market Value</th>
<th>New Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>$644.30</td>
<td>$100,000.00</td>
<td>6.00%</td>
<td>61</td>
<td>25</td>
<td>$97,627.97</td>
<td>5.00%</td>
</tr>
<tr>
<td>$706.78</td>
<td>$100,000.00</td>
<td>7.00%</td>
<td>30</td>
<td>25</td>
<td>$125,984.98</td>
<td>4.00%</td>
</tr>
<tr>
<td>$908.70</td>
<td>$100,000.00</td>
<td>10.00%</td>
<td>40</td>
<td>25</td>
<td>$78,841.64</td>
<td>13.00%</td>
</tr>
<tr>
<td>$584.59</td>
<td>$100,000.00</td>
<td>5.00%</td>
<td>50</td>
<td>25</td>
<td>$83,483.12</td>
<td>6.00%</td>
</tr>
<tr>
<td>$474.21</td>
<td>$100,000.00</td>
<td>3.00%</td>
<td>10</td>
<td>25</td>
<td>$69,369.36</td>
<td>6.50%</td>
</tr>
<tr>
<td>$768.91</td>
<td>$100,000.00</td>
<td>8.50%</td>
<td>15</td>
<td>30</td>
<td>$133,284.24</td>
<td>5.50%</td>
</tr>
<tr>
<td>$675.21</td>
<td>$100,000.00</td>
<td>6.50%</td>
<td>20</td>
<td>25</td>
<td>$93,170.65</td>
<td>7.00%</td>
</tr>
</tbody>
</table>
APPLICATION 12.4.2A (page 367)

Consider a $100,000 mortgage that is structured as a 10/15 interest-only mortgage, with an annual rate of 6%. What would the payments be for the first 10 and the last 15 years? For the first 10 years, the monthly payments, which are interest only, would be $500 ($100,000 x 6.0%/12). Between years 11 and 25, the monthly fixed payment necessary to fully amortize the mortgage for the remaining 15 years would be $843.86 (using a financial calculator: n = 12 x 15 = 180, i = 6%/12 = 0.5%, PV = +/- $100,000, FV = $0, solve for PMT).

EXPLANATION

There are two separate problems in this application:

1) We need to determine the interest only payments for the first 10 years of the mortgage
2) We need to determine the fixed payment that will amortize the mortgage for the remaining 15 years.

For the first 10 years, since it is interest only all we need to do is calculate the monthly interest rate from the annual interest of 6% by dividing it by 12 for a monthly rate of 0.5%. Then take the monthly interest rate of 0.5% and multiply it by 100000, the value of the mortgage, for a monthly payment of $500.

For an explanation on how to solve for the amortized payment, see Application 12.4.1A.
CALCULATIONS

Find payments for the first 10 years Step One:
Press .06 → ÷ → 12 Step Two: Press x
→ 100000 Step Three: Press =
Answer: 500

Find payments for the last 15 years
Step One: Press 2nd → CLR TVM
Step Two: Press 12 → x → 15 → = → N Step
Three: Press 6 → ÷ → 12 → = → I/Y Step Four:
Press 100000 → +|→ PV Step Five: Press 0 →
FV
Step Six: Press CPT → PMT Answer:
843.86

WORKOUT AREA: Here are sample problems – cover one of the values and see if you can solve it using the others

<table>
<thead>
<tr>
<th>Loan Monthly Payment (First 10 Years)</th>
<th>Loan Monthly Payment (Last 15 Years)</th>
<th>Loan Value</th>
<th>Annual Interest Rate</th>
<th>25 Year (10/15 Interest Only Mortgage)</th>
<th>Amortization Balance</th>
</tr>
</thead>
<tbody>
<tr>
<td>$500.00</td>
<td>$843.86</td>
<td>$100,000.0</td>
<td>6.0%</td>
<td>10 15</td>
<td>$0.00</td>
</tr>
<tr>
<td>$812.50</td>
<td>$1,306.66</td>
<td>$150,000.0</td>
<td>6.5%</td>
<td>10 15</td>
<td>$0.00</td>
</tr>
<tr>
<td>$916.67</td>
<td>$1,634.17</td>
<td>$200,000.0</td>
<td>5.5%</td>
<td>10 15</td>
<td>$0.00</td>
</tr>
<tr>
<td>$1,562.50</td>
<td>$2,317.53</td>
<td>$250,000.0</td>
<td>7.5%</td>
<td>10 15</td>
<td>$0.00</td>
</tr>
<tr>
<td>$2,125.00</td>
<td>$2,954.22</td>
<td>$300,000.0</td>
<td>8.5%</td>
<td>10 15</td>
<td>$0.00</td>
</tr>
<tr>
<td>$333.33</td>
<td>$477.83</td>
<td>$50,000.0</td>
<td>8.0%</td>
<td>10 15</td>
<td>$0.00</td>
</tr>
<tr>
<td>$1,604.17</td>
<td>$2,859.79</td>
<td>$350,000.0</td>
<td>5.5%</td>
<td>10 15</td>
<td>$0.00</td>
</tr>
</tbody>
</table>
APPLICATION 12.4.3A (page 369)

What would the monthly payment be for the mortgage in Exhibit 12.6 in the second year, when the mortgage’s rate climbs to 10.0%? Note that it is necessary to decrease the mortgage’s original principal to reflect amortization and decrease the months remaining by 12, to 288.

From Exhibit 12.6, the monthly mortgage payment that the borrower would have to make during the second year, for which a higher index rate of 8.5% applies, is equal to $903.36 \( (n = 12 \times 24 = 288, i = 10%/12, PV = +/-98,470.30, FV = 0, \) solve for \( PMT \).) Notice that the increase in interest rates between the first year and the second year has caused a substantial increase (27.81%) in the monthly payment that the borrower is obligated to make.

EXPLANATION

Please see the explanation for Application 12.4.1A.

CALCULATIONS

Step One: Press 2nd → CLR TVM
Step Two: Press 12 → x → 24 → = → N Step
Three: Press 10 → ÷ → 12 → = → I/Y Step Four:
Press 98470.30 → +|- → PV Step Five: Press 0 → FV
Step Six: Press CPT → PMT
Answer: 903.36
WORKOUT AREA: Here are sample problems – cover one of the values and see if you can solve it using the others. NOTE: The loan values in rows 2-7 are arbitrary. Also note that all rows assume an initial mortgage maturity of 25 years and that the number in the fourth column represents the year number. The 4 in row 4 requests the mortgage payment for the first month of the fourth year. The first month of the fourth year begins with 264 months remaining: \((25 \times 12) - (3 \times 12)\).

<table>
<thead>
<tr>
<th>Loan Monthly Payment</th>
<th>Loan Value</th>
<th>Interest Rate Per Year</th>
<th>Time: Year # (start of Year)</th>
<th>Amortization Balance</th>
</tr>
</thead>
<tbody>
<tr>
<td>$903.36</td>
<td>$98,470.30</td>
<td>10.00%</td>
<td>2</td>
<td>$0.00</td>
</tr>
<tr>
<td>$782.05</td>
<td>$100,000.00</td>
<td>8.00%</td>
<td>2</td>
<td>$0.00</td>
</tr>
<tr>
<td>$882.77</td>
<td>$100,000.00</td>
<td>9.50%</td>
<td>2</td>
<td>$0.00</td>
</tr>
<tr>
<td>$871.17</td>
<td>$100,000.00</td>
<td>9.00%</td>
<td>4</td>
<td>$0.00</td>
</tr>
<tr>
<td>$917.39</td>
<td>$100,000.00</td>
<td>10.00%</td>
<td>2</td>
<td>$0.00</td>
</tr>
<tr>
<td>$826.06</td>
<td>$100,000.00</td>
<td>8.50%</td>
<td>3</td>
<td>$0.00</td>
</tr>
<tr>
<td>$774.51</td>
<td>$100,000.00</td>
<td>7.50%</td>
<td>4</td>
<td>$0.00</td>
</tr>
</tbody>
</table>
APPLICATION 12.4.4A (page 371)

To illustrate balloon payments, assume that the borrower and the lender in the original example decide that the $100,000 loan made at the fixed rate of 6% per year compounded monthly for 25 years will amortize to a $70,000 balance on the 25-year maturity date rather than being fully amortized to $0. This amount of $70,000 is known as a balloon payment and will be due at the end of 25 years. In this case, the monthly payment would be equal to $543.29 (using a financial calculator: \( n = 12 \times 25 = 300, i = 6%/12 = 0.5\%, PV = +/-$100,000, FV = $70,000, solve for PMT \)). Notice that the $543.29 monthly payment is less than the $644.30 payment that was computed for the case of the fully amortizing loan, even though the interest rates in both mortgages are equal to 6%.

EXPLANATION

Please see the explanation for Application 12.4.1A. The key is that the present value of the mortgage payments not only include the present value of the payments (entered as with the PMT key) but also the present value of the balloon payment (entered with the FV key). The PMT and FV must be of the same sign.

CALCULATIONS

Step One: Press 2nd → CLR TVM
Step Two: Press 12 → x → 25 → = → N
Step Three: Press 6 → ÷ → 12 → = → I/Y
Step Four: Press 100000 → +|→ PV
Step Five: Press 70000 → FV
Step Six: Press CPT → PMT
Answer: 543.29
**WORKOUT AREA:** Here are sample problems – cover one of the values and see if you can solve it using the others.

<table>
<thead>
<tr>
<th>Loan Monthly Payment</th>
<th>Loan</th>
<th>Fixed Per Year</th>
<th>Time Frame (Years, Monthly)</th>
<th>Amortization Balance (Balloon Payment)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$543.29</td>
<td>$100,000.00</td>
<td>6.00%</td>
<td>25</td>
<td>$70,000.00</td>
</tr>
<tr>
<td>$643.62</td>
<td>$100,000.00</td>
<td>6.50%</td>
<td>20</td>
<td>$50,000.00</td>
</tr>
<tr>
<td>$603.83</td>
<td>$100,000.00</td>
<td>7.00%</td>
<td>30</td>
<td>$75,000.00</td>
</tr>
<tr>
<td>$706.46</td>
<td>$100,000.00</td>
<td>5.00%</td>
<td>10</td>
<td>$55,000.00</td>
</tr>
<tr>
<td>$605.13</td>
<td>$100,000.00</td>
<td>5.50%</td>
<td>7</td>
<td>$85,000.00</td>
</tr>
<tr>
<td>$375.00</td>
<td>$100,000.00</td>
<td>4.50%</td>
<td>25</td>
<td>$100,000.00</td>
</tr>
<tr>
<td>$892.19</td>
<td>$100,000.00</td>
<td>7.50%</td>
<td>8</td>
<td>$65,000.00</td>
</tr>
</tbody>
</table>
APPLICATION 12.6.2A (page 378)

Mortgage B experiences a CPR of 2% in its 20th month. How would this prepayment rate be expressed using the PSA benchmark? Mortgage B has a PSA prepayment speed of 50% in month 20. Mortgage B’s prepayment rate of 2% is 50% of the 4% benchmark. The 4% benchmark is 0.2% \times 20 months, since the month number is less than 30.

EXPLANATION

This application may seem slightly complicated, but it is actually quite easy. Let’s take stock of what we know. Mortgage B has a CPR of 2% in month 20. We know enough to solve this problem. The PSA benchmark is 0.2% multiplied by the number of months into the life of the mortgage, in this case 20. (The benchmark hits its 6% maximum at month 30). The 20 months times the 0.2% per month is a product of 4%. Dividing 2% by 4% the PSA prepayment rate is 50%.

CALCULATIONS

Step One: Press 0.02 → + → 0.04 Step
Two: Press =
Answer: 0.50

WORKOUT AREA: Here are sample problems – cover one of the values and see if you can solve it using the others

<table>
<thead>
<tr>
<th>Mortgage CPR</th>
<th>Month</th>
<th>PSA Benchmark</th>
<th>PSA Prepayment Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00%</td>
<td>2</td>
<td>0.40%</td>
<td>250.0%</td>
</tr>
<tr>
<td>1.00%</td>
<td>7</td>
<td>1.40%</td>
<td>71.4%</td>
</tr>
<tr>
<td>3.00%</td>
<td>4</td>
<td>0.80%</td>
<td>375.0%</td>
</tr>
<tr>
<td>2.00%</td>
<td>15</td>
<td>3.00%</td>
<td>66.7%</td>
</tr>
<tr>
<td>1.00%</td>
<td>19</td>
<td>3.80%</td>
<td>26.3%</td>
</tr>
<tr>
<td>6.00%</td>
<td>40</td>
<td>6.00%</td>
<td>100.0%</td>
</tr>
<tr>
<td>2.00%</td>
<td>20</td>
<td>4.00%</td>
<td>50.0%</td>
</tr>
</tbody>
</table>
APPLICATION 12.6.2B (page 378)

Mortgage C experiences a PSA rate of 200% in each month and is now five years old. What is its CPR? The PSA standard is 6% at 30 months and beyond, and 200% of 6% is 12%. Since the mortgage is already at or beyond month 60, the CPR for the mortgage is now 12%.

EXPLANATION

At month 60 (5 years), the PSA benchmark is 6% (every month after 30 is 6%). The PSA rate is 200%. Therefore, the CPR rate is 200% multiplied by 6% or 12%.

CALCULATIONS

Step One: Press 2 → x → 0.06
Step Two: Press =
Answer: 0.12

WORKOUT AREA: Here are sample problems – cover one of the values and see if you can solve it using the others

<table>
<thead>
<tr>
<th>Five year Mortgage C CPR</th>
<th>Mortgage C PSA Rate Each Month</th>
<th>PSA at 30</th>
</tr>
</thead>
<tbody>
<tr>
<td>12.00%</td>
<td>200.00%</td>
<td>6.00%</td>
</tr>
<tr>
<td>15.00%</td>
<td>250.00%</td>
<td>6.00%</td>
</tr>
<tr>
<td>9.00%</td>
<td>150.00%</td>
<td>6.00%</td>
</tr>
<tr>
<td>6.00%</td>
<td>100.00%</td>
<td>6.00%</td>
</tr>
<tr>
<td>18.00%</td>
<td>300.00%</td>
<td>6.00%</td>
</tr>
<tr>
<td>21.00%</td>
<td>350.00%</td>
<td>6.00%</td>
</tr>
<tr>
<td>24.00%</td>
<td>400.00%</td>
<td>6.00%</td>
</tr>
</tbody>
</table>
APPLICATION 13.2.4A (page 392)

Assume that a real estate project has a current market value of $125 million and expected annual cash flows from rent, net of operating expenses, of $10 million. What is its cap rate?

The answer is found using Equation 13.1: Cap Rate = ($10 million/ $125 million) = 8%.

EXPLANATION

To solve this problem, we will have to use equation 13.1:

\[
\text{Cap Rate} = \frac{\text{Net Operating Income}}{\text{Value}}
\]

We are given the variables needed to solve this problem, so it's a simple as substituting them into the equation. (Note, these figures are in millions)

\[
\text{Cap Rate} = \frac{10}{125} = 0.08 = 8\%
\]

CALCULATIONS

Step One: Press 10 \(\rightarrow\) \(\div\) \(\rightarrow\) 125
Step Two: Press =
Answer: 0.08

WORKOUT AREA: Here are sample problems – cover one of the values and see if you can solve it using the others

<table>
<thead>
<tr>
<th>NOI</th>
<th>Building Value</th>
<th>Cap Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>125</td>
<td>8.00%</td>
</tr>
<tr>
<td>15</td>
<td>100</td>
<td>15.00%</td>
</tr>
<tr>
<td>20</td>
<td>360</td>
<td>5.56%</td>
</tr>
<tr>
<td>5</td>
<td>50</td>
<td>10.00%</td>
</tr>
<tr>
<td>8</td>
<td>180</td>
<td>4.44%</td>
</tr>
</tbody>
</table>
APPLICATION 13.2.4B (page 393)

An investor is considering the purchase of a core real estate property that offers $350,000 per year in net operating income. The investor’s analysis of cap rates on similar properties finds an average cap rate of 7%. What is the value of the property based on Equation 13.2 and the investor’s estimates of the net operating income and appropriate cap rate?

The answer is found using Equation 13.2:

\[ \text{Value} = \frac{\text{NOI}}{\text{Cap Rate}} = \frac{$350,000}{0.07} = $5,000,000 \]

EXPLANATION

To solve this problem, we will have to use equation 13.2, which is a variation of equation 13.1:

\[ \text{Value} = \frac{\text{NOI}}{\text{Cap Rate}} \]

Again, we are given the variables needed to solve this problem, so it’s a simple as substituting them into the equation. (Note, these figures are in millions)

\[ \text{Value} = \frac{$350,000}{0.07} = $5,000,000 \]

CALCULATIONS

Step One: Press 350,000 → ÷ → 0.07
Step Two: Press =
Answer: 5,000,000

WORKOUT AREA:

Please refer to the Workout Area in Application 13.2.4A.
 APPLICATION 13.3.2A (page 399)

Assume that U.S. Treasury notes with a seven-year maturity are currently yielding 5.8%, that the liquidity premium is 1% per year, and that the required or anticipated risk premium for the systematic risk of the real estate project is 2.2% per year. With these numbers, the required rate of return for this real estate project is 9%. The required return using Equation 13.6 is found as $(1.058 \times 1.01 \times 1.022) - 1$, or 9.21%. Using Equation 13.7, the three rates sum to 9% (i.e., $5.8\% + 1.0\% + 2.2\% = 9\%$).

EXPLANATION

There are two ways to solve this application. Equation 13.6 is the exact required rate of return, while equation 13.7 is an approximate required rate of return. Let's start by calculating the required rate of return using 13.7:

\[
r \approx R_f + E(R_{LP}) + E(R_{RP})
\]

\[
r \approx 0.058 + 0.01 + 0.022
\]

\[
r \approx 0.09
\]

The approximate required rate of return is 9%. Now, let's calculate the exact required rate of return using equation 13.6:

\[
r = [1 + R_f][1 + E(R_{LP})][1 + E(R_{RP})] - 1
\]

\[
r = [1 + 0.058][1 + 0.022][1 + 0.01] - 1
\]

\[
r = [1.058][1.022][1.01] - 1
\]

\[
r = [1.081276][1.01] - 1
\]

\[
r = [1.09208876] - 1
\]

\[
r = 0.09208876
\]

The exact required rate of return is 9.21%.
CALCULATIONS

Find the approximate required rate of return

Step One: Press 0.058 → +
Step Two: Press 0.01 → + → 0.022 Step
Three: Press =
Answer: 0.09

Find the exact required rate of return Step

One: Press 1 → + → 0.058 Step Two:
Press = “1.058”
Step Three: Press 1 → + → 0.022 Step
Four: Press = “1.022”
Step Five: Press 1 → + → 0.01 Step
Six: Press = “1.01”
Step Seven: Press 1.058 → x → 1.022 Step Eight:
Press x → 1.01
Step Nine: Press - → 1
Answer: 0.0921

WORKOUT AREA: Here are sample problems based on the approximation model—cover one of the values and see if you can solve it using the others

<table>
<thead>
<tr>
<th>Required Rate of Return for this Real Estate Project</th>
<th>Yield of 7-Year U.S. Treasury Notes</th>
<th>Liquidity Per Year</th>
<th>Required or Anticipated Risk Premium for the Systematic Risk of the Real Estate Project</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.00%</td>
<td>5.80%</td>
<td>1.00%</td>
<td>2.20%</td>
</tr>
<tr>
<td>9.40%</td>
<td>5.60%</td>
<td>1.50%</td>
<td>2.30%</td>
</tr>
<tr>
<td>8.80%</td>
<td>5.30%</td>
<td>1.00%</td>
<td>2.50%</td>
</tr>
<tr>
<td>9.60%</td>
<td>4.60%</td>
<td>2.00%</td>
<td>3.00%</td>
</tr>
<tr>
<td>8.45%</td>
<td>5.10%</td>
<td>1.25%</td>
<td>2.10%</td>
</tr>
<tr>
<td>10.95%</td>
<td>6.70%</td>
<td>1.15%</td>
<td>3.10%</td>
</tr>
<tr>
<td>11.25%</td>
<td>7.30%</td>
<td>1.75%</td>
<td>2.20%</td>
</tr>
</tbody>
</table>
APPLICATION 13.3.3 (page 401)

Investment A offers $80 per year in taxable income and an additional final non-taxable cash flow in five years of $1,000. An investor in a 40% tax bracket requires a pre-tax return of 8% and an after-tax return of 4.8% on investments. What is the value of Investment A on both a pre-tax basis and an after-tax basis? On a pre-tax basis, Investment A is worth $1,000, found on a financial calculator as \( PMT = 80, FV = 1,000, N = 5, I = 8\% \), solve for \( PV \). On an after-tax basis, the $80 annual income is worth $48 [$80 \times (100\% - 40\%)] . On an after-tax basis, Investment A is also worth $1,000, found on a financial calculator as \( PMT = 48, FV = 1,000, N = 5, I = 4.8\% \), solve for \( PV \).

EXPLANATION:

In both cases the problem is a simple “bond” problem of discounting a combination of an annuity and a lump sum.

CALCULATIONS

Find the value of Investment A on a pre-tax basis Step One:

 Press 2nd → CLR TVM
 Step Two: Press 5 → N Step
 Three: Press 8 → I/Y
 Step Four: Press 80 → PMT Step
 Five: Press 1000 → FV Step Six:
 Press CPT → PV Answer: 1000

Find the value of Investment A on a post-tax basis Step One:

 Press 2nd → CLR TVM
 Step Two: Press 5 → N
 Step Three: Press 4.8 → I/Y Step
 Four: Press 1 → - → 40 Step Five: x → 80 → = → PMT Step Six: Press
 1000 → FV Step Seven: Press CPT → PV Answer: 1000
WORKOUT AREA: Here are sample problems – cover one of the values in the two leftmost columns and see if you can solve it using the others

<table>
<thead>
<tr>
<th>Value of Investment A on a Pre-Tax Basis</th>
<th>Value of Investment A on a After-Tax Basis</th>
<th>Taxable Income From Investment A Per Year</th>
<th>Non-Taxable Income From Investment A in 5 Years</th>
<th>Invest or Tax Brack</th>
<th>Required Pre-Tax Return</th>
<th>Required After Tax Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1,000.00</td>
<td>$1,000.00</td>
<td>$80.00</td>
<td>$1,000.00</td>
<td>40.00%</td>
<td>8.00%</td>
<td>4.80%</td>
</tr>
<tr>
<td>$1,059.11</td>
<td>$1,044.29</td>
<td>$100.00</td>
<td>$1,000.00</td>
<td>30.00%</td>
<td>8.50%</td>
<td>5.95%</td>
</tr>
<tr>
<td>$1,110.54</td>
<td>$1,140.91</td>
<td>$85.00</td>
<td>$1,200.00</td>
<td>40.00%</td>
<td>9.00%</td>
<td>5.40%</td>
</tr>
<tr>
<td>$1,289.89</td>
<td>$1,293.42</td>
<td>$95.00</td>
<td>$1,300.00</td>
<td>40.00%</td>
<td>7.50%</td>
<td>4.50%</td>
</tr>
<tr>
<td>$1,918.00</td>
<td>$1,931.86</td>
<td>$120.00</td>
<td>$2,000.00</td>
<td>20.00%</td>
<td>7.00%</td>
<td>5.60%</td>
</tr>
<tr>
<td>$1,973.46</td>
<td>$2,049.66</td>
<td>$150.00</td>
<td>$2,200.00</td>
<td>40.00%</td>
<td>9.50%</td>
<td>5.70%</td>
</tr>
<tr>
<td>$2,240.47</td>
<td>$2,261.42</td>
<td>$135.00</td>
<td>$2,500.00</td>
<td>10.00%</td>
<td>8.00%</td>
<td>7.20%</td>
</tr>
</tbody>
</table>
APPLICATION 13.5.5A (page 405)

Private real estate fund A has $100 million of assets and $50 million of debt. Private real estate fund B has $20 million of equity and $30 million of debt. What is the LTV and debt-to-equity ratio of each of these geared funds? Fund A is 50% debt, and has an LTV of 50% and a debt-to-equity ratio of 1.0. Fund B is 60% debt, and has an LTV of 60% and a debt-to-equity ratio of 1.5.

EXPLANATION

In order to calculate debt to equity, we need to know the total debt of each fund and the equity of each fund. Fund A has $50 million of debt and $50 million of equity as determined by using the equations Assets = Debt + Equity ($100 million - $50 million = $50 million). Therefore, the debt to equity ratio of Fund A is 1.0. Fund B’s debt to equity ratio can be calculated the same way. Fund has $20 million of equity and $30 million of debt. Therefore the debt to equity ratio of Fund B is 1.5.

The Loan to Value (LTV) ratio for each fund can be calculated by dividing the amount of debt by the amount of assets. Fund A has a LTV ratio of 50% ($50 million divided by $100 million). Fund B doesn’t provide the value of assets, but we know that debt plus equity equals the amount of assets. In this case, we add $20 million (Fund B’s equity) plus $30 million (Fund B’s debt) for Fund B’s assets of $50 million. Therefore, the LTV ratio for Fund B is $30 million divided by $50 million for quotient of 60%.

CALCULATIONS

Find the LTV and Debt to Equity for Fund A Step One:

Press 50 → ÷ → 100

Step Two: Press = “0.5” (LTV ratio for Fund A) Step

Three: Press 100 → - → 50

Step Four: Press = “50”

Step Five: Press 50 → ÷ → 50 Step

Six: Press =

Answer: 1 (Debt to Equity for Fund A) Find the

LTV and Debt to Equity for Fund B

Step One: Press 20 → + → 30 Step

Two: Press = “50”

195
Step Three: Press $30 \div 50$
Step Four: Press $= 0.6$ (LTV ratio for Fund B) Step Five:
Press $30 \div 20$
Step Six: Press $=$
Answer: 1.5 (Debt to Equity for Fund B)

WORKOUT AREA: Here are sample problems – cover one of the values and see if you can solve it using the others

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0.50</td>
<td>33.33%</td>
<td>1.50</td>
<td>60.00%</td>
<td>$100.00</td>
<td>$50.00</td>
<td>$20.00</td>
<td>$30.00</td>
</tr>
<tr>
<td>0.50</td>
<td>33.33%</td>
<td>2.33</td>
<td>70.00%</td>
<td>$150.00</td>
<td>$75.00</td>
<td>$15.00</td>
<td>$35.00</td>
</tr>
<tr>
<td>0.50</td>
<td>33.33%</td>
<td>4.00</td>
<td>80.00%</td>
<td>$200.00</td>
<td>$100.00</td>
<td>$10.00</td>
<td>$40.00</td>
</tr>
<tr>
<td>0.60</td>
<td>37.50%</td>
<td>9.00</td>
<td>90.00%</td>
<td>$250.00</td>
<td>$150.00</td>
<td>$5.00</td>
<td>$45.00</td>
</tr>
<tr>
<td>0.58</td>
<td>36.84%</td>
<td>2.00</td>
<td>66.67%</td>
<td>$300.00</td>
<td>$175.00</td>
<td>$25.00</td>
<td>$50.00</td>
</tr>
<tr>
<td>0.71</td>
<td>41.67%</td>
<td>1.83</td>
<td>64.71%</td>
<td>$350.00</td>
<td>$250.00</td>
<td>$30.00</td>
<td>$55.00</td>
</tr>
<tr>
<td>0.69</td>
<td>40.74%</td>
<td>1.71</td>
<td>63.16%</td>
<td>$400.00</td>
<td>$275.00</td>
<td>$35.00</td>
<td>$60.00</td>
</tr>
</tbody>
</table>
APPLICATION 14.2.1A (page 420)

TTMAR Hedge Fund has a 1.5 and 30 fee arrangement, with no hurdle rate and a NAV of $200 million at the start of the year. At the end of the year, before fees, the NAV is $253 million. Assuming that management fees are computed on start-of-year NAVs and are distributed annually, find the annual management fee, the incentive fee, and the ending NAV after fees, assuming no redemptions or subscriptions. The annual management fee is simply 1.5% of $200 million, or $3 million. After the management fee of $3 million, the fund earned a profit of $50 million ($253 – $3 – $200). The incentive fee on the profit is $15 million ($50 x 30% = $15). Therefore, the ending NAV after distribution of fees to the fund manager is $235 million ($253 – $3 – $15).

EXPLANATION

In this application, we need to determine the management fee and the incentive fee for TT MAR Hedge Fund, which has a 1.5% management fee and a 30% incentive fee. To start lets calculate the management fee, which is .015 (management fee percentage) multiplied by $200 million (beginning NAV) for a product of $3 million, which is the management fee. To calculate the incentive fee, we need to solve for the fund’s profit minus the management fee. To do that we subtract from $253 million (ending NAV) the $3 million management fee and the $200 million (beginning NAV) for a difference of $50 million, this is the fund’s profit after management fees. Solving for the incentive fee, we multiply the 30% incentive fee percentage by the $50 million profit after management fees for a $15 million incentive fee. Therefore, the ending NAV after distribution of fees to the fund manager is the difference between $253 million (ending NAV), $3 million (management fee), and $15 million (incentive fee), which is $235 million.
CALCULATIONS

Management fee
Step One: Press 200 → x → 0.015
Step Two: Press = Answer: 3

Incentive Fee
Step One: Press 253 → - → 3
Step Two: Press - → 200
Step Three: Press x → 0.30
Step Four: Press =
Answer: 15

Ending NAV
Step One: Press 253 → - → 3
Step Two: Press - → 15
Step Three: Press =
Answer: 235

WORKOUT AREA: Here are sample problems – cover one of the values and see if you can solve it using the others

<table>
<thead>
<tr>
<th>Annual Management Fee (millions)</th>
<th>Incentive Fee (millions)</th>
<th>Ending NAV After Fees (millions)</th>
<th>Fund Profit (millions)</th>
<th>Management Fee (millions)</th>
<th>Performance Fee (millions)</th>
<th>Hurdle Rate</th>
<th>Beginning NAV (millions)</th>
<th>Ending NAV (millions)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3.0</td>
<td>$15.0</td>
<td>$235.0</td>
<td>$50.0</td>
<td>1.5%</td>
<td>30.0%</td>
<td>0.0%</td>
<td>$200</td>
<td>$253</td>
</tr>
<tr>
<td>$4.0</td>
<td>$9.2</td>
<td>$236.8</td>
<td>$46.0</td>
<td>2.0%</td>
<td>20.0%</td>
<td>0.0%</td>
<td>$200</td>
<td>$250</td>
</tr>
<tr>
<td>$4.5</td>
<td>$19.1</td>
<td>$376.4</td>
<td>$95.5</td>
<td>1.5%</td>
<td>20.0%</td>
<td>0.0%</td>
<td>$300</td>
<td>$400</td>
</tr>
<tr>
<td>$3.8</td>
<td>$29.4</td>
<td>$416.8</td>
<td>$196.3</td>
<td>1.5%</td>
<td>15.0%</td>
<td>0.0%</td>
<td>$250</td>
<td>$450</td>
</tr>
<tr>
<td>$2.0</td>
<td>$44.4</td>
<td>$203.6</td>
<td>$148.0</td>
<td>2.0%</td>
<td>30.0%</td>
<td>0.0%</td>
<td>$100</td>
<td>$250</td>
</tr>
<tr>
<td>$1.0</td>
<td>$0.0</td>
<td>$100.0</td>
<td>$0.0</td>
<td>1.0%</td>
<td>30.0%</td>
<td>0.0%</td>
<td>$100</td>
<td>$101</td>
</tr>
<tr>
<td>$2.0</td>
<td>$0.5</td>
<td>$102.6</td>
<td>$3.0</td>
<td>2.0%</td>
<td>15.0%</td>
<td>0.0%</td>
<td>$100</td>
<td>$105</td>
</tr>
</tbody>
</table>
APPLICATION 14.2.1B (page 421)

VVMAR Hedge Fund has a 1.5 and 30 fee arrangement, with no hurdle rate and a NAV of $200 million at the start of the year. At the end of the year, after fees, the NAV is $270 million. Assuming that management fees are computed on start-of-year NAVs and are distributed annually, find the annual management fee, the incentive fee, and the ending NAV before fees, assuming no redemptions or subscriptions. The incentive fee represents 30% of the total profits and so represents the proportion 30%/70% to the net profits to limited partners. Since the profit to the limited partners is $70 million, the incentive fee to the manager must be $30 million (i.e., $70 million x 30%/70%). Thus, the NAV after management fees but before incentive fees must be $300 million. The management fees are 1.5% of the starting NAV: 1.5% x $200 million = $3 million, inferring an ending NAV of $303 million before fees. To recap: $303 million is reduced to $300 million by the 1.5% management fee on the starting value of $200 million. The fund therefore earned a profit of $100 million after management fees ($300 million – $200 million). The incentive fee to the manager was 30% of $100 million, or $30 million. The profit after fees to the limited partners was $70 million, leaving a NAV of $270 million after all fees.

EXPLANATION

In this application, we face the challenge of being given the AFTER FEE end-of-year NAV. We need to back out the management fee and the incentive fee for TTMAR Hedge Fund, which has a 1.5% management fee and a 30% incentive fee. The management fee is easy to determine because it is based on beginning net asset value. But the incentive fee is based on profits.

To start let’s calculate the management fee, which is .015 (management fee percentage) multiplied by $200 million (beginning NAV) for a product of $3 million, which is the management fee. In order to determine the incentive fee let’s consider the split between limited partners (investors) and the general partners (fund manager), which is 30% and 70%. Since the limited partners are receiving 270 million (ending NAV after fees) -$200 million (beginning NAV) or $70 million. That implies that the incentive fee to the general partners is $30 million, or 30% of the $100 million profit after the management fee. Therefore, the ending NAV before fees was $200 million (beginning NAV) plus $3 million (management fee) plus $30 million (incentive fee), for a sum of $303 million.
CALCULATIONS

Management fee
Step One: Press 200 → x → 0.015
Step Two: Press = Answer: 3

Incentive Fee
Step One: Press 270 → - → 200
Step Two: Press x → 0.3
Step Three: Press = “21”
Step Four: Press 1 → - → 0.30
Step Five: Press = “0.70”
Step Six: Press 21 → ÷ → 0.70
Answer: 30

Ending NAV
Step One: Press 200 → + → 3
Step Two: Press + → 70 + → 30
Step Three: Press =
Answer: 303

WORKOUT AREA: Here are sample problems – solve for columns 1, 2, 4 and 9 using the rest.

<table>
<thead>
<tr>
<th>Annual Management Fee (millions)</th>
<th>Incentive Fee (millions)</th>
<th>Ending NAV After Fees (millions)</th>
<th>Fund Profit (millions)</th>
<th>Management Fee (percent)</th>
<th>Performance Fee (percent)</th>
<th>Hurdle Rate (percent)</th>
<th>Beginning NAV (millions)</th>
<th>Ending NAV (millions)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3.00</td>
<td>$30.00</td>
<td>$270.00</td>
<td>$300.00</td>
<td>1.50%</td>
<td>30.00%</td>
<td>0.00%</td>
<td>$200.00</td>
<td>$303.00</td>
</tr>
<tr>
<td>$2.00</td>
<td>$50.00</td>
<td>$300.00</td>
<td>$350.00</td>
<td>2.00%</td>
<td>20.00%</td>
<td>0.00%</td>
<td>$100.00</td>
<td>$352.00</td>
</tr>
<tr>
<td>$2.25</td>
<td>$100.00</td>
<td>$450.00</td>
<td>$550.00</td>
<td>1.50%</td>
<td>25.00%</td>
<td>0.00%</td>
<td>$150.00</td>
<td>$552.25</td>
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<td>$552.94</td>
<td>3.00%</td>
<td>15.00%</td>
<td>0.00%</td>
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<td>$150.00</td>
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<td>$1.13</td>
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<td>$200.00</td>
<td>$222.06</td>
<td>1.50%</td>
<td>15.00%</td>
<td>0.00%</td>
<td>$75.00</td>
<td>$223.18</td>
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<tr>
<td>$1.00</td>
<td>$26.47</td>
<td>$250.00</td>
<td>$276.47</td>
<td>1.00%</td>
<td>15.00%</td>
<td>0.00%</td>
<td>$100.00</td>
<td>$277.47</td>
</tr>
</tbody>
</table>
APPLICATION 14.2.5A (page 428)

Consider a $1 billion hedge fund with a 20% incentive fee at the start of a new incentive fee computation period. If the hedge fund computes incentive fees annually and begins the year very near its high-water mark, what would be the value of the incentive fee over the next year for annual asset volatilities of 10%, 20%, and 30% using the at-the-money incentive fee approximation formula?

Inserting $i = 20\%$, $NAV = \$1$ billion, $T = 1$, and the three given volatilities generates approximations of $\$8$ million, $\$16$ million, and $\$24$ million.

EXPLANATION

Utilizing equation 14.3, we can solve for the annual asset volatilities of 10%, 20%, and 30%.

Incentive Fee Call Option Value = $i \times 40\% \times NAV \times \sigma_1$

The parameters are $i = 20\%$ and $NAV=\$1$ billion.

Solving for the 10% annual asset volatility:

Incentive Fee Call Option Value $\approx 0.20 \times 0.40 \times \$1,000,000,000 \times 0.10$

Incentive Fee Call Option Value $\approx 0.08 \times \$1,000,000,000 \times 0.10$

Incentive Fee Call Option Value $\approx \$80,000,000 \times 0.10$

Incentive Fee Call Option Value $\approx \$8,000,000$
Solving for the 20% annual asset volatility:

\[ \text{Incentive Fee Call Option Value} \approx 0.20 \times 0.40 \times 1,000,000,000 \times 0.20 \]

\[ \text{Incentive Fee Call Option Value} \approx 0.08 \times 1,000,000,000 \times 0.20 \]

\[ \text{Incentive Fee Call Option Value} \approx 80,000,000 \times 0.20 \]

\[ \text{Incentive Fee Call Option Value} \approx 16,000,000 \]

Solving for the 30% annual asset volatility:

\[ \text{Incentive Fee Call Option Value} \approx 0.20 \times 0.40 \times 1,000,000,000 \times 0.30 \]

\[ \text{Incentive Fee Call Option Value} \approx 0.08 \times 1,000,000,000 \times 0.30 \]

\[ \text{Incentive Fee Call Option Value} \approx 80,000,000 \times 0.30 \]

\[ \text{Incentive Fee Call Option Value} \approx 24,000,000 \]
10% annual asset volatility:
Step One: Press 0.20 → x → 0.40
Step Two: Press x → 0.10
Step Three: Press x → 1,000,000,000 Step
Four: Press =
Answer: 8,000,000 20%

20% annual asset volatility:
Step One: Press 0.20 → x → 0.40 Step Two:
Press x → 0.20
Step Three: Press x → 1,000,000,000 Step
Four: Press =
Answer: 16,000,000

30% annual asset volatility:
Step One: Press 0.20 → x → 0.40 Step Two:
Press x → 0.30
Step Three: Press x → 1,000,000,000 Step
Four: Press =
Answer: 24,000,000
WORKOUT AREA: Here are sample problems – cover one of the values and see if you can solve it using the others

<table>
<thead>
<tr>
<th>Option Value (millions)</th>
<th>Volatility</th>
<th>Incentive (millions)</th>
<th>NAV (millions)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$8.00</td>
<td>10%</td>
<td>20.00%</td>
<td>$1,000.00</td>
</tr>
<tr>
<td>$8.00</td>
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<td>20.00%</td>
<td>$500.00</td>
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<td>$6.00</td>
<td>30%</td>
<td>20.00%</td>
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<tr>
<td>$6.00</td>
<td>40%</td>
<td>25.00%</td>
<td>$150.00</td>
</tr>
</tbody>
</table>
APPLICATION 15.4.5 (page 467)

A stock price experiences the following 10 consecutive daily prices corresponding to days $-10$ to $-1$: 100, 102, 99, 97, 95, 100, 109, 103, 103, and 106. What are the simple (arithmetic) moving average prices on day 0 using 3-day and 10-day moving averages, as well as the 3-day moving average for days $-2$ and $-1$? Using the data, the three-day moving average on day 0 is $\frac{(103 + 103 + 106)}{3}$, or 104. For days $-2$ and $-1$, the three-day moving averages are 104 and 105, respectively. The 10-day moving average for day 0 is 101.4. Because the price on day $-1$ moved above the recent three-day moving averages, a classic interpretation of a simple moving average trading system would be that a long position should have been established.

EXPLANATION

The simple moving average is a sum of the prices divided by the number of days in the moving average. The key is to make sure you are starting on the correct day. In this application we need to find the 3-day simple moving average on Day 0, so the prices on Day -1, Day -2, and Day -3 are the added together and then the sum is divided by 3 (number of days in the simple moving average).

Therefore, the 3-day simple moving average on Day 0 is $\frac{(106 + 103 + 103)}{3} = 104$. The 3-day simple moving average on Day -1 is $\frac{(103 + 103 + 109)}{3} = 105$. The 3-day simple moving average on Day -2 is $\frac{(103 + 109 + 100)}{3} = 104$. Now, the 10-day simple moving average on Day 0 is $\frac{(100 + 102 + 99 + 97 + 95 + 100 + 109 + 103 + 103 + 106)}{10}$ or 101.4.

CALCULATIONS

Find the 3-day simple moving average on Day 0

Step One: Press 106 → + → 103
Step Two: Press + → 103
Step Three: Press + → 3
Step Four: Press =
Answer: 104
Find the 3-day simple moving average on Day -1 Step One:
Step One: Press 103 → + → 103
Step Two: Press + → 109
Step Three: Press ÷ → 3
Step Four: Press =
Answer: 105

Find the 3-day simple moving average on Day -2
Step One: Press 103 → + → 109
Step Two: Press + → 100
Step Three: Press ÷ → 3
Step Four: Press =
Answer: 104

Find the 10-day simple moving average on Day 0 Step One:
Step One: Press 100 → + → 102
Step Two: Press + → 99
Step Three: Press + → 97
Step Four: Press + → 95
Step Five: Press + → 100
Step Six: Press + → 109
Step Seven: Press + → 103
Step Eight: Press + → 103
Step Nine: Press + → 106
Step Ten: Press ÷ → 10
Step Eleven: Press =
Answer: 101.4
WORKOUT AREA: Here are sample problems – cover one of the values in the four rightmost columns and see if you can solve it using the others

<table>
<thead>
<tr>
<th>Stock Price</th>
<th>Day -10</th>
<th>Day -9</th>
<th>Day -8</th>
<th>Day -7</th>
<th>Day -6</th>
<th>Day -5</th>
<th>Day -4</th>
<th>Day -3</th>
<th>Day -2</th>
<th>Day -1</th>
<th>3-Day Moving Average</th>
<th>10-Day Moving Average</th>
<th>3-Day Moving Average</th>
<th>3-Day Moving Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>102</td>
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<td>109</td>
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<td>104.0</td>
<td>105.0</td>
<td></td>
</tr>
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<td>18</td>
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<td>22</td>
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<tr>
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<td>11</td>
<td>12</td>
<td>14</td>
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<td>17</td>
<td>18</td>
<td>21</td>
<td>18.7</td>
<td>13.7</td>
<td>15.3</td>
<td>16.7</td>
<td></td>
</tr>
</tbody>
</table>
A stock price experiences the following 10 consecutive daily prices corresponding to days \(-10\) to \(-1\): 100, 102, 99, 97, 95, 100, 109, 103, 103, and 106. What are the five-day weighted moving average prices on days \(-1\) and 0? The sum of the digits 1 through 5 is 15. The five-day weighted moving average on day 0 is as follows:

\[
\frac{(106 \times 5) + (103 \times 4) + (103 \times 3) + (109 \times 2) + (100 \times 1)}{15}, \text{ or 104.6}
\]

The five-day moving average on day \(-1\) is as follows:

\[
\frac{(103 \times 5) + (103 \times 4) + (109 \times 3) + (100 \times 2) + (95 \times 1)}{15}, \text{ or 103.27}
\]

**EXPLANATION**

The weighted moving average for \(N\) days is calculated by multiplying the most recent daily price by \(N\), and then the second most recent daily price in the series by the number \(N-1\), and so on ending with the least recent daily price multiplied by 1. Then the products are summed and divided by the sum of the numbers 1 to \(N\).

Let’s find the 5-day weighted moving average on Day 0. Day \(-1\), \(-2\), \(-3\), \(-4\), and \(-5\) prices are $106.00, $103.00, $103.00, $109.00, and $100.00. With those prices we need to weight them so: 

\[
(106 \times 5) + (103 \times 4) + (103 \times 3) + (109 \times 2) + (100 \times 1) = 1569.
\]

Now we need to divide that sum by the sum of the multiples: 

\[
5 + 4 + 3 + 2 + 1 = 15.
\]

\[
1569/15 = 104.6 \text{ is the 5-day weighted moving average on Day 0.}
\]

Let’s find the 5-day weighted moving average on Day \(-1\). Day \(-2\), \(-3\), \(-4\), \(-5\), and \(-6\) prices are $103.00, $103.00, $109.00, $100.00, and $95.00. With those prices we need to weight them so: 

\[
(103 \times 5) + (103 \times 4) + (109 \times 3) + (100 \times 2) + (95 \times 1) = 1549.
\]

Now we need to divide that sum by the sum of the multiples: 

\[
5 + 4 + 3 + 2 + 1 = 15.
\]

\[
1549/15 = 103.4 \text{ is the 5-day weighted moving average on Day -1.}
\]
CALCULATIONS

Find the 5-day weighted moving average on Day 0

Step One: Press 106 → x → 5
Step Two: Press = “530”
Step Three: Press 103 → x → 4
Step Four: Press = “412”
Step Five: Press 103 → x → 3
Step Six: Press = “309”
Step Seven: Press 109 → x → 2
Step Eight: Press = “218”
Step Nine: Press 100 → x → 1
Step Ten: Press = “100”
Step Eleven: Press 5 → + → 4
Step Twelve: Press + → 3
Step Thirteen: Press + → 2
Step Fourteen: Press + → 1
Step Fifteen: Press = “15”
Step Sixteen: Press 530 → + → 412
Step Seventeen: Press + → 309
Step Eighteen: Press + → 218
Step Nineteen: Press + → 100
Step Twenty: Press = “1569”
Step Twenty-One: Press 1569 → ÷ → 15
Answer: 104.6
Find the 5-day weighted moving average on Day -1

Step One:
Press 103 → x → 5

Step Two: Press = “515”

Step Three: Press 103 → x → 4

Step Four: Press = “412”

Step Five: Press 109 → x → 3

Step Six: Press = “327”

Step Seven: Press 100 → x → 2

Step Eight: Press = “200”

Step Nine: Press 95 → x → 1

Step Ten: Press = “95”

Step Eleven: Press 5 → + → 4

Step Twelve: Press + → 3

Step Thirteen: Press + → 2

Step Fourteen: Press + → 1

Step Fifteen: Press = “15”

Step Sixteen: Press 515 → + → 412

Step Seventeen: Press + → 327

Step Eighteen: Press + → 200

Step Nineteen: Press + → 95

Step Twenty: Press = “1549”

Step Twenty-One: Press 1549 → + → 15

Answer: 103.27
**WORKOUT AREA:** Here are sample problems – cover one of the values in the two rightmost columns and see if you can solve it using the others

<table>
<thead>
<tr>
<th>Stock Prices</th>
<th>Day -10</th>
<th>Day -9</th>
<th>Day -8</th>
<th>Day -7</th>
<th>Day -6</th>
<th>Day -5</th>
<th>Day -4</th>
<th>Day -3</th>
<th>Day -2</th>
<th>Day -1</th>
<th>5-Day Weighted Moving Average</th>
<th>5-Day Weighted Moving Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>102</td>
<td>99</td>
<td>97</td>
<td>95</td>
<td>100</td>
<td>109</td>
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<td>18</td>
<td>21</td>
<td></td>
<td>18.13</td>
<td>16.20</td>
</tr>
</tbody>
</table>
APPLICATION 15.4.6B (page 469):

A stock price experiences the following five consecutive daily prices corresponding to days $-5$ to $-1$: 100, 109, 103, 103, and 106. What are the exponential moving average prices on days $-1$ and 0 using $\lambda = 0.25$? Assume that the exponential moving average up to and including the price on day $-3$ was 100. The exponential moving average on day $-1$ is found as $0.25 \times 103$ (the day $-2$ price) plus $0.75 \times 100$ (the previous exponential moving average), which equals 100.75. The exponential moving average on day 0 is found as $0.25 \times 106$ (the day $-1$ price) plus $0.75 \times 100.75$ (the previous exponential moving average), or 102.0625.

EXPLANATION

Let’s find the 5-day exponential moving average on Day -1. Day -2 is $103. It is also important to note that the exponential moving average up to and including the price on day $-3$ is 100. With those prices we need to weight them so: $103 \times (0.25) + 100 \times (1 - 0.25) = 100.75$. 100.75 is the 5-day exponential moving average on Day -1.

Let’s find the 5-day exponential moving average on Day 0. Day -1 and -2 are $106.00$ and $103.00$. It is also important to note that the exponential moving average up to and including the price on day $-3$ is 100. With those prices we need to weight them so: $106 \times (0.25) + 100.75 \times (1 - 0.25) = 102.0625$. 102.0625 is the 5-day exponential moving average on Day 0. Note we used Day -1 exponential moving average to calculate the Day 0 exponential moving average.

CALCULATIONS

Find the 5-day exponential moving average on Day -1

Step One: Press 103 → x → 0.25
Step Two: Press = “25.75”
Step Three: Press 1 → - → 0.25
Step Four: Press x → 100
Step Five: Press = “75”
Step Six: Press 25.75 → + → 75
Step Seven: Press =
Answer: 100.75

Find the 5-day exponential moving average on Day 0 Step One:

Press 106 → x → 0.25

Step Two: Press = “26.5”

Step Three: Press 1 → - → 0.25

Step Four: Press x → 100.75 (Day -1 exponential moving average)

Step Five: Press = “75”

Step Six: Press 26.5 → + → 75.5625

Step Seven: Press =

Answer: 102.0625

WORKOUT AREA: Here are sample problems — cover the EMA values and see if you can solve it using the others

<table>
<thead>
<tr>
<th>Day -5</th>
<th>Day -4</th>
<th>Day -3</th>
<th>Day -2</th>
<th>Day -1</th>
<th>( \lambda )</th>
<th>EMA Day -1</th>
<th>EMA Day 0</th>
<th>Previous Moving Average Day 0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$100.00</td>
<td>$109.00</td>
<td>$103.00</td>
<td>$103.00</td>
<td>$106.00</td>
<td>0.25</td>
<td>$100.75</td>
<td>$101.50</td>
<td>100</td>
</tr>
<tr>
<td>$47.00</td>
<td>$55.00</td>
<td>$53.00</td>
<td>$51.00</td>
<td>$52.00</td>
<td>0.30</td>
<td>$50.30</td>
<td>$50.60</td>
<td>50</td>
</tr>
<tr>
<td>$34.00</td>
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<td>0.35</td>
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<td>33</td>
</tr>
<tr>
<td>$17.00</td>
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<td>$43.50</td>
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<td>0.25</td>
<td>$15.75</td>
<td>$16.50</td>
<td>15</td>
</tr>
</tbody>
</table>
A stock price experiences the following 10 consecutive daily high prices corresponding to days $t = 10$ to $-1$: 100, 102, 99, 98, 99, 104, 102, 103, 104, and 100. What is the day 0 price level that signals a breakout and possibly a long position, using these 10 days of data as representative of a trading range? A price of 105 exceeds the range of the past data and signals that a long position should be established. If the price series represented the low prices for each day, a current price of 97 would signal a breakout on the downside and would typically be interpreted as a sell signal.

**EXPLANATION**

The Upper Bound is determined by the highest daily price over the 10 consecutive day period, in this case $104.00 is the highest daily price. Therefore a price of 105 would signal that a long position should be established. The Lower Bound is determined by the lowest daily price over the 10 consecutive day period, in this case $98.00. Therefore a price of 97 would be interpreted as a sell signal.

**WORKOUT AREA:** Here are sample problems – cover the four rightmost columns of the values and see if you can solve it using the others.

<table>
<thead>
<tr>
<th>Day</th>
<th>Upper Bound</th>
<th>Lower Bound</th>
<th>Break Out</th>
</tr>
</thead>
<tbody>
<tr>
<td>-10</td>
<td>$100.00</td>
<td>$98.00</td>
<td>$97.00</td>
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<tr>
<td>-9</td>
<td>$102.00</td>
<td>$104.00</td>
<td>$105.00</td>
</tr>
<tr>
<td>-8</td>
<td>$99.00</td>
<td>$104.00</td>
<td>$100.00</td>
</tr>
<tr>
<td>-7</td>
<td>$99.00</td>
<td>$100.00</td>
<td>$98.00</td>
</tr>
<tr>
<td>-6</td>
<td>$104.00</td>
<td>$102.00</td>
<td>$104.00</td>
</tr>
<tr>
<td>-5</td>
<td>$100.00</td>
<td>$103.00</td>
<td>$104.00</td>
</tr>
<tr>
<td>-4</td>
<td>$104.00</td>
<td>$104.00</td>
<td>$104.00</td>
</tr>
<tr>
<td>-3</td>
<td>$98.00</td>
<td>$98.00</td>
<td>$98.00</td>
</tr>
<tr>
<td>-2</td>
<td>$98.00</td>
<td>$98.00</td>
<td>$98.00</td>
</tr>
<tr>
<td>-1</td>
<td>$98.00</td>
<td>$98.00</td>
<td>$98.00</td>
</tr>
<tr>
<td></td>
<td>$104.00</td>
<td>$104.00</td>
<td>$104.00</td>
</tr>
<tr>
<td></td>
<td>$98.00</td>
<td>$98.00</td>
<td>$98.00</td>
</tr>
<tr>
<td></td>
<td>$105.00</td>
<td>$105.00</td>
<td>$105.00</td>
</tr>
<tr>
<td></td>
<td>$97.00</td>
<td>$97.00</td>
<td>$97.00</td>
</tr>
</tbody>
</table>
APPLICATION 15.6.3A (page 480)

Consider a CTA with $30 million capital. The CTA has determined that 10% of this capital should be allocated to trading in the Brent Crude Oil market. The sizing function is estimated to be 0.8, which means the trader’s signal is strong and indicates a long position in this market. The size of each futures contract is 1,000 barrels, and assuming a current price of $50 per barrel, the notional value of each contract will be $50,000. Finally, assume that the annualized volatility target is 20% and that the annualized realized volatility using near-term futures prices of the past 30 days has been 30%.

Given these figures, the number of futures contracts based on Equation 15.5 will be as follows:

\[
Number of Futures Contracts = 0.8 \times \frac{10\% \times 30,000,000}{50,000} \times \frac{20\%}{30\%} = 32
\]

If oil markets become calmer, and the estimate of price volatility declines to 25%, the trader will need to rebalance his position to have 38 contracts in the portfolio.

EXPLANATION

To solve this problem, we must use Equation 15.5:

\[
Number of Futures Contracts = Sizing Function \times \frac{Risk Loading \times Equity}{Notional Value} \times \frac{RVol_T}{RVol_R}
\]

\[
Number of Futures Contracts = 0.8 \times \frac{10\% \times 30,000,000}{50,000} \times \frac{20\%}{30\%} = 32
\]

CALCULATIONS

Step One: Press 0.8 → x → 0.1 → x → 30,000,000 → 0.2
Step Two: Press = “480,000”
Step Three: Press ÷ ( → 50,000 → x → 0.3 →)
Step Four: Press = “15,000”
Step Five: Press =
Answer: 32
WORKOUT AREA: Here are sample problems – cover one of the columns and see if you can solve for it using the other information.

<table>
<thead>
<tr>
<th>Number of Contracts</th>
<th>Sizing Function</th>
<th>Risk Loading</th>
<th>Equity</th>
<th>Notional Value</th>
<th>Target Volatility</th>
<th>Realized Volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td>32</td>
<td>0.8</td>
<td>10%</td>
<td>$30,000,000.00</td>
<td>$50,000.00</td>
<td>20%</td>
<td>30%</td>
</tr>
<tr>
<td>54</td>
<td>0.9</td>
<td>15%</td>
<td>$30,000,000.00</td>
<td>$50,000.00</td>
<td>20%</td>
<td>30%</td>
</tr>
<tr>
<td>14</td>
<td>0.7</td>
<td>10%</td>
<td>$20,000,000.00</td>
<td>$100,000.00</td>
<td>30%</td>
<td>30%</td>
</tr>
<tr>
<td>6</td>
<td>0.6</td>
<td>5%</td>
<td>$2,000,000.00</td>
<td>$5,000.00</td>
<td>15%</td>
<td>30%</td>
</tr>
<tr>
<td>60</td>
<td>0.9</td>
<td>20%</td>
<td>$25,000,000.00</td>
<td>$50,000.00</td>
<td>20%</td>
<td>30%</td>
</tr>
<tr>
<td>252</td>
<td>0.8</td>
<td>35%</td>
<td>$30,000,000.00</td>
<td>$20,000.00</td>
<td>30%</td>
<td>50%</td>
</tr>
</tbody>
</table>
Continuing with the previous example in which the CTA’s capital is assumed to be $30 million, suppose the sizing function is 0.8 and the risk loading is 0.2%. The daily price volatility of oil is estimated to be $1.1, and each contract is for 1,000 barrels.

Given these figures, the trader will take a long position in futures contracts based on Equation 15.6:

\[
Number\ of\ Contracts = 0.8 \times \frac{0.2\% \times 30,000,000}{1.1 \times 1,000} \approx 44
\]

EXPLANATION

Using equation 15.6, we can solve for the number of contracts:

\[
Number\ of\ Contracts = \text{Sizing Function} \times \frac{\text{Risk Loading} \times \text{Capital}}{\text{PVol}_R \times \text{Contract}}
\]

\[
Number\ of\ Contracts = 0.8 \times \frac{0.2\% \times 30,000,000}{1.1 \times 1,000} \approx 44
\]

CALCULATIONS

Step One: Press 0.8 → x → 0.002 → x → 30,000,000
Step Two: Press = “480,000”
Step Three: Press + → (→ 1,000 →x → 1.1→)
Step Four: Press = “1,100”
Step Five: Press =
Answer: 43.636

WORKOUT AREA: Here are sample problems – cover one of the columns and see if you can solve for it using the other variables.

<table>
<thead>
<tr>
<th>Number of Contracts (Rounded)</th>
<th>Sizing Function</th>
<th>Risk Loading</th>
<th>Capital</th>
<th>Contract</th>
<th>Realized Volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td>44</td>
<td>0.8</td>
<td>0.20%</td>
<td>$30,000,000.00</td>
<td>$1,000.00</td>
<td>$1.10</td>
</tr>
<tr>
<td>25</td>
<td>0.9</td>
<td>0.20%</td>
<td>$30,000,000.00</td>
<td>$2,000.00</td>
<td>$1.10</td>
</tr>
<tr>
<td>5</td>
<td>0.7</td>
<td>0.20%</td>
<td>$20,000,000.00</td>
<td>$5,000.00</td>
<td>$1.05</td>
</tr>
<tr>
<td>10</td>
<td>0.6</td>
<td>0.20%</td>
<td>$50,000,000.00</td>
<td>$5,000.00</td>
<td>$1.20</td>
</tr>
<tr>
<td>15</td>
<td>0.9</td>
<td>0.20%</td>
<td>$25,000,000.00</td>
<td>$2,000.00</td>
<td>$1.50</td>
</tr>
<tr>
<td>2</td>
<td>0.8</td>
<td>0.20%</td>
<td>$30,000,000.00</td>
<td>$20,000.00</td>
<td>$1.10</td>
</tr>
</tbody>
</table>
APPLICATION 16.1.2A (page 503)

ABC Corp. has offered to purchase DEF Corp. for $25 per share. Immediately before the merger proposal announcement, DEF was trading at $18 per share. Immediately after the announcement, DEF is trading at $23 per share. Assuming that the share price of DEF would fall to $16 if the deal fails and that the riskless interest rate is 0%, describe a long position in DEF taken by an event-driven hedge fund both as a combination of positions in a risk-free bond and a binary call option and as a combination of positions including a binary put option. The hedge fund may be viewed as a long position in a riskless bond with a face value of $16 and a long position in a binary call option with a potential payout of $9 in case the merger is successful and shares of DEF rise to $25 per share. The hedge fund may also be viewed as a long position in a riskless bond with a face value of $25 and a short position in a binary put option with a potential payout of $9 in case the merger is not successful and shares of DEF decline to $16 per share.

EXPLANATION

This is a cash-for-stock merger. Essentially, if the ABC Corp. and DEF Corp. merger is successful, the payout to the hedge fund will be $25 (DEF Corp. price per share if acquired) versus $16 (DEF Corp. share price if merger fails). The hedge fund may be viewed as a long position in a riskless bond with a face value of $16 and a long position in a binary call option with a potential payout of $9 in case the merger is successful and shares of DEF rise to $25 per share. The hedge fund may also be viewed as a long position in a riskless bond with a face value of $25 and a short position in a binary put option with a potential payout of $9 in case the merger is not successful and shares of DEF decline to $16 per share.

WORKOUT AREA: Here are sample problems cover the two leftmost columns and solve the others.

<table>
<thead>
<tr>
<th>Share Price if Merger Fails</th>
<th>Share Price if Merger Occurs</th>
<th>Long Binary Call Max</th>
<th>Riskless Bond</th>
<th>Short Binary Put Min</th>
<th>Riskless Bond</th>
</tr>
</thead>
<tbody>
<tr>
<td>$16.00</td>
<td>$25.00</td>
<td>$9.00</td>
<td>$16.00</td>
<td>-$9.00</td>
<td>$25.00</td>
</tr>
<tr>
<td>$10.00</td>
<td>$15.00</td>
<td>$5.00</td>
<td>$10.00</td>
<td>-$5.00</td>
<td>$15.00</td>
</tr>
<tr>
<td>$57.00</td>
<td>$24.00</td>
<td>-$33.00</td>
<td>$57.00</td>
<td>$33.00</td>
<td>$24.00</td>
</tr>
<tr>
<td>$15.00</td>
<td>$18.00</td>
<td>$3.00</td>
<td>$15.00</td>
<td>-$3.00</td>
<td>$18.00</td>
</tr>
<tr>
<td>$17.00</td>
<td>$27.00</td>
<td>$10.00</td>
<td>$17.00</td>
<td>-$10.00</td>
<td>$27.00</td>
</tr>
<tr>
<td>$10.00</td>
<td>$23.00</td>
<td>$13.00</td>
<td>$10.00</td>
<td>-$13.00</td>
<td>$23.00</td>
</tr>
<tr>
<td>$12.00</td>
<td>$22.00</td>
<td>$10.00</td>
<td>$12.00</td>
<td>-$10.00</td>
<td>$22.00</td>
</tr>
</tbody>
</table>
APPLICATION 16.3.1A (page 518)

Prior to a merger announcement, MegaStock, trading at $102, plans to offer one share of MegaStock for 3.5 shares of MiniStock, trading at $20. After the announcement, MegaStock trades at $100, MiniStock jumps to $25, and an arbitrageur takes a traditional and hedged merger arbitrage position. Ignoring transaction costs, interest, and dividends, how much money would the arbitrageur earn per share of MegaStock if the merger consummates, and how much money would be lost if the deal fails and the prices revert to their preannouncement levels? The short position in one share of MegaStock generates proceeds of $100. Buying 3.5 shares of MiniStock costs $87.50. If the deal goes through, the arbitrageur pockets the $12.50 net proceeds as profit and delivers the exchanged shares to cover the short. If the deal fails, the arbitrageur sells the 3.5 shares at $20 for $70 in proceeds, buys back MegaStock at $102, and expends $32, which is a $19.50 loss relative to the proceeds of $12.50.

EXPLANATION

This is a stock-for-stock deal. Standard merger arbitrage is to have a long position in the company being acquired and short position the acquirer (i.e., most merger arb deals try to earn a risk premium by insuring against a deal failure).

First, we are to take a short position in the acquirer (MegaStock) that provides proceeds because we sold something we don’t own (ignoring financing costs). Now, we need to buy 3.5 shares of MiniStock (the company being acquired) for $87.50 (3.5 shares of MiniStock at $25) because that is how many shares will be exchanged for 1 share of Megastock (which we are short).

If the deal goes through, the merger arbitrageurs will pocket $12.50 or $100.00 (MegaStock share price after merger announcement) minus $87.50 or the product of 3.5 (number of shares of MiniStock for 1 share of MegaStock) and $25.00 (MiniStock share Price after the merger announcement).

If the deal falls through, the merger arbitrageurs will lose $19.50 or $100.00 (MegaStock share price after merger announcement) minus $87.50 or the product of 3.5 (number of shares of MiniStock for 1 share of MegaStock) and $25.00 (MiniStock share Price after the merger announcement), plus $70 or the product of 3.5 and $20.00 (MiniStock price before the merger announcement) minus $102 (the share price before merger announcement).
CALCULATIONS

Find the payoff if the deal goes through

Step One: Press 3.5 → x → 25
Step Two: Press = “87.50”
Step Three: Press 100 → - → 87.50
Step Four: Press =
Answer: 12.50

Find the payoff if the deal falls through Step

One: Press 3.5 → x → 25
Step Two: Press = “87.50”
Step Three: Press 3.5 → x → 20
Step Four: Press = “70”
Step Five: Press 100 → - → 87.50
Step Six: Press + → 70
Step Seven: Press - → 102
Step Eight: Press =
Answer: -19.50
**WORKOUT AREA:** Here are sample problems – cover values in the two rightmost columns and see if you can solve them using the others.

<table>
<thead>
<tr>
<th>Original Mega Share Price and if Merger fails</th>
<th>Original Mini Share Price and if Merger fails</th>
<th>MegaStock Share Price After Announced</th>
<th># of Shares of Mini per Share of Mega</th>
<th>MiniStock Share Price After Announced</th>
<th>Proceeds of Trades and Profit if Occurs</th>
<th>Loss if Merger Fails</th>
</tr>
</thead>
<tbody>
<tr>
<td>$102.00</td>
<td>$20.00</td>
<td>$100.00</td>
<td>3.5</td>
<td>$25.00</td>
<td>$12.50</td>
<td>-$19.50</td>
</tr>
<tr>
<td>$50.00</td>
<td>$40.00</td>
<td>$48.00</td>
<td>1</td>
<td>$42.00</td>
<td>$6.00</td>
<td>-$4.00</td>
</tr>
<tr>
<td>$200.00</td>
<td>$90.00</td>
<td>$196.00</td>
<td>2</td>
<td>$95.00</td>
<td>$6.00</td>
<td>-$14.00</td>
</tr>
<tr>
<td>$100.00</td>
<td>$10.00</td>
<td>$102.00</td>
<td>5</td>
<td>$19.00</td>
<td>$7.00</td>
<td>-$43.00</td>
</tr>
<tr>
<td>$50.00</td>
<td>$30.00</td>
<td>$50.00</td>
<td>1</td>
<td>$40.00</td>
<td>$10.00</td>
<td>-$10.00</td>
</tr>
<tr>
<td>$120.00</td>
<td>$20.00</td>
<td>$123.00</td>
<td>6</td>
<td>$24.00</td>
<td>-$21.00</td>
<td>-$21.00</td>
</tr>
<tr>
<td>$50.00</td>
<td>$80.00</td>
<td>$50.00</td>
<td>0.5</td>
<td>$85.00</td>
<td>$7.50</td>
<td>-$2.50</td>
</tr>
</tbody>
</table>
APPLICATION 16.4.4A (page 528)

A bond is purchased at 40% of face value. After bankruptcy, 30% of the bond's face value is ultimately recovered. Express the rate of return as a non-annualized rate, as an annualized rate based on a four-month holding period, and as an annualized rate based on a four-year holding period, ignoring compounding and assuming no coupon income. The non-annualized rate is –25%, found as a 10% loss on a 40% investment. The annualized rate based on a four-month holding period is –75%, found as \(-25\% \times (\frac{12}{4})\); and an annualized rate based on a four-year holding period is –6.25%, found as \(-\frac{25}{4}\).

EXPLANATION

In order to calculate the bond’s rate of return as a non-annualized rate we need to subtract the 40% (percentage of face value bond is purchased at) from 30% (percentage of the bond’s face value recovered after bankruptcy) and divide by 40% (percentage of the bond’s face value recovered after bankruptcy) for a quotient of -25%. The annual rate based on a 4 year holding period is found by dividing -25% by 4 for a quotient of -6.25%. To find the annualized rate based on a four-month holding period multiply -25% by 12/4 for a product of -75%.

CALCULATIONS

Find the rate of return as a non-annualized rate based

Step One: Press 0.30 \(\rightarrow\) – \(\rightarrow\) 0.40
Step Two: Press \(\div\) \(\rightarrow\) 0.40
Step Three: Press = Answer: -0.25

Find the rate of return as an annualized rate based on a 4 month holding period Step One:

Press 12 \(\rightarrow\) \(\div\) \(\rightarrow\) 4
Step Two: Press \(\times\) \(\rightarrow\) -0.25
Step Three: Press =
Answer: -0.75
Find the rate of return as an annualized rate based on a 4 year holding period

Step One: Press -0.25 → ÷ → 4

Step Two: Press =

Answer: -0.0625

WORKOUT AREA: Here are sample problems – cover one of the values in the three leftmost columns and see if you can solve it using the two rightmost columns

<table>
<thead>
<tr>
<th>Rate of Return as a Non-Annualized Rate</th>
<th>Rate of Return as an Annualized Rate based on a 4 Month Holding Period</th>
<th>Rate of Return as an Annualized Rate based on a 4 Year Holding Period</th>
<th>Percentage of Face Value Bond is Purchased At</th>
<th>Percentage of Bond’s Face Value Recovered After Bankruptcy</th>
</tr>
</thead>
<tbody>
<tr>
<td>-25.00%</td>
<td>-75.00%</td>
<td>-6.25%</td>
<td>40.00%</td>
<td>30.00%</td>
</tr>
<tr>
<td>-30.00%</td>
<td>-90.00%</td>
<td>-7.50%</td>
<td>50.00%</td>
<td>35.00%</td>
</tr>
<tr>
<td>22.22%</td>
<td>66.67%</td>
<td>5.56%</td>
<td>45.00%</td>
<td>55.00%</td>
</tr>
<tr>
<td>12.50%</td>
<td>37.50%</td>
<td>3.13%</td>
<td>40.00%</td>
<td>45.00%</td>
</tr>
<tr>
<td>11.11%</td>
<td>33.33%</td>
<td>2.78%</td>
<td>45.00%</td>
<td>50.00%</td>
</tr>
<tr>
<td>-14.29%</td>
<td>-42.86%</td>
<td>-3.57%</td>
<td>70.00%</td>
<td>60.00%</td>
</tr>
<tr>
<td>33.33%</td>
<td>100.00%</td>
<td>8.33%</td>
<td>75.00%</td>
<td>100.00%</td>
</tr>
</tbody>
</table>
APPLICATION 17.2.1A (page 537)

Consider a firm with a borrowing cost of 8% on unsecured, subordinated straight debt and a current stock price of $40. The firm may be able to issue three-year convertible bonds at an annual coupon rate of perhaps 4% by offering a conversion ratio such as 20. What is the bond’s strike price, and what does the conversion option allow the bond investors to do?

The conversion ratio of 20 is equivalent to a $50 strike price assuming that the bond’s face value is $1,000. On or before maturity, bond investors can opt to convert each $1,000 face value bond into 20 shares of the firm’s equity rather than receive the remaining principal and coupon.

EXPLANATION

The bond’s strike price is $50 found by dividing the $1,000.00 (convertible bond face value) by 20 (the conversion ratio). The conversion option allows the bond investor to convert each $1,000 face value convertible bond into 20 shares of the firm’s equity rather than receive the remaining principal and coupon.

CALCULATIONS

Step One: Press 1000 → ÷ → 20

Step Two: Press =

Answer: 50

WORKOUT AREA: Here are sample problems – cover the leftmost column and see if you can solve it using the others

<table>
<thead>
<tr>
<th>Strike Price of the Three-Year Convertible</th>
<th>Borrowing Cost on Unsecured Subordinated</th>
<th>Current Stock</th>
<th>Annual Coupon of the Three-Year Convertible Bonds</th>
<th>Conversion Ratio of the Three-Year Convertible</th>
<th>Convertible Bond Face</th>
</tr>
</thead>
<tbody>
<tr>
<td>$50.00</td>
<td>8.0%</td>
<td>$40.00</td>
<td>4.0%</td>
<td>20</td>
<td>$1,000.00</td>
</tr>
<tr>
<td>$66.67</td>
<td>7.5%</td>
<td>$50.00</td>
<td>5.0%</td>
<td>15</td>
<td>$1,000.00</td>
</tr>
<tr>
<td>$100.00</td>
<td>8.5%</td>
<td>$65.00</td>
<td>4.0%</td>
<td>10</td>
<td>$1,000.00</td>
</tr>
<tr>
<td>$40.00</td>
<td>9.0%</td>
<td>$55.00</td>
<td>4.5%</td>
<td>25</td>
<td>$1,000.00</td>
</tr>
<tr>
<td>$33.33</td>
<td>5.5%</td>
<td>$60.00</td>
<td>5.5%</td>
<td>30</td>
<td>$1,000.00</td>
</tr>
<tr>
<td>$28.57</td>
<td>6.0%</td>
<td>$45.00</td>
<td>3.0%</td>
<td>35</td>
<td>$1,000.00</td>
</tr>
<tr>
<td>$200.00</td>
<td>6.5%</td>
<td>$70.00</td>
<td>3.5%</td>
<td>5</td>
<td>$1,000.00</td>
</tr>
</tbody>
</table>
APPLICATION 17.2.1B (page 537)

Returning to the previous example of an 8% unsecured bond rate, a $40 stock price, and a conversion ratio of 20, and assuming that a three-year European-style call option—given a current stock price of $40, a strike price of $50, and other parameters, such as volatility and dividends—is valued at $5.14 per share according to the Black-Scholes option pricing model, what are the value of the convertible bond, the conversion value, and the conversion premium? Starting with the straight debt issue, the three-year bond in the example can be valued with a 4% coupon and an 8% discount rate, found from observing corporate bonds of similar credit risk, at $896.92, using a financial calculator with annual coupons and compounding for simplicity. Using representative calculator inputs $n = 3, i = 8, PMT = 40, and FV = 1,000 and computing $PV$ yields $896.92$. Adding the straight bond value of $896.92$ to the value of 20 options, $102.80$ (i.e., $5.14 \times 20$), yields a convertible bond valuation of $999.72$, a value that is very close to the bond’s face value of $1,000. The current stock price multiplied by the conversion ratio gives a conversion value of $800$ (i.e., $40 \times 20$). Therefore, this convertible bond is selling at a conversion premium of 24.97% [i.e., ($999.72 - $800)/$800].

EXPLANATION

Let’s first find the value of the convertible bond. This is the same as finding the present value in Application 17.2.1A by using the following parameters: $n = 3, i = 8, PMT = 40, and FV = 1,000$ for a present value of $896.92$. Now, we need to add the value of 20 options (conversion ratio of the three-year convertible bonds) to the present value. The value of 20 options is computed by multiplying 20 by $5.14$ (price of a three-year European-style call option) for a product of $102.80$. The sum of $896.92$ and $102.80$ of $999.72$ is the value of the convertible bond. The conversion value is $800.00$ found by multiplying 20 (conversion ratio of the three-year convertible bonds) and $40$ (current stock price). The conversion premium is 24.96 and is found by subtracting $800$ (the conversion value) from $999.72$ (value of the convertible bond) and dividing the difference by $800$ (the conversion value).
CALCULATIONS

Find the value of the convertible bond

   Step One: Press 2nd → CLR TVM
   Step Two: Press 5 → N
   Step Three: Press 8 → I/Y
   Step Four: Press 40 → PMT
   Step Five: Press 1000 → FV
   Step Six: Press CPT → PV
   Step Seven: “892.92”
   Step Eight: 20 → x → 5.14
   Step Nine: = → + → 892.92
   Step Ten: =
             Answer: 999.72

Find the conversion value

   Step One: 20 → x → 40
   Step Two: =
             Answer: 800

Find the conversion premium

   Step One: 999.72 → - → 800
   Step Two: Press ÷ → 800
   Step Three: Press =
             Answer: 0.2496
WORKOUT AREA: Here are sample problems – cover one of the values and see if you can solve it using the others.

<table>
<thead>
<tr>
<th>Application 19.2.1b</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value of the Convertible Bond</td>
</tr>
<tr>
<td>$999.72</td>
</tr>
<tr>
<td>$890.78</td>
</tr>
<tr>
<td>$927.98</td>
</tr>
<tr>
<td>$954.39</td>
</tr>
<tr>
<td>$923.99</td>
</tr>
<tr>
<td>$936.44</td>
</tr>
<tr>
<td>$1,310.27</td>
</tr>
</tbody>
</table>
APPLICATION 17.3.2A (page 553)

Consider a 30-day variance swap with a notional value of $250,000. The strike variance is 9.00. The realized variance of the index is 7.00. What would be the payment or payoff of the swap?

The variance swap payoff:

\[
250,000 \times (7.00 - 9.00) = -500,000
\]

The swap buyer received the realized variance and pays the strike variance, so in this example the swap buyer pays $500,000 to the variance swap payer.

EXPLANATION

This problem can be solved by using Equation 17.2a:

\[
\text{Variance Swap Payoff} = \text{Variance Notional Value} \times (\text{Realized Variance} - \text{Strike Variance})
\]

\[
= 250,000 \times (7.00 - 9.00)
\]

\[
= -500,000
\]

CALCULATIONS

Step One: Press 250,000 → x
Step Two: Press ( → 7 → - → 9 → )
Step Three Press = “-2”
Step Four: Press =
Answer: -500,000

WORKOUT AREA: Here are sample problems – cover one of the values and see if you can solve it using the others

<table>
<thead>
<tr>
<th>Notional Value</th>
<th>Strike Variance</th>
<th>Realized Variance</th>
<th>Variance Swap Payoff</th>
</tr>
</thead>
<tbody>
<tr>
<td>$250,000.00</td>
<td>9</td>
<td>7</td>
<td>$(500,000.00)</td>
</tr>
<tr>
<td>$250,000.00</td>
<td>8</td>
<td>8</td>
<td>-</td>
</tr>
<tr>
<td>$300,000.00</td>
<td>3</td>
<td>9</td>
<td>$1,800,000.00</td>
</tr>
<tr>
<td>$1,000,000.00</td>
<td>6</td>
<td>10</td>
<td>$4,000,000.00</td>
</tr>
<tr>
<td>$500,000.00</td>
<td>9</td>
<td>4</td>
<td>$(2,500,000.00)</td>
</tr>
<tr>
<td>$200,000.00</td>
<td>5</td>
<td>5</td>
<td>-</td>
</tr>
<tr>
<td>$150,000.00</td>
<td>4</td>
<td>1</td>
<td>$(450,000.00)</td>
</tr>
</tbody>
</table>
APPLICATION 17.3.2B (page 554)

Consider a 30-day volatility swap with a notional value of $500,000. The strike volatility is 17.50. The realized volatility of the reference asset is 18.25. What would be the payment of the swap?

The volatility swap payoff:

\[ 500,000 \times (18.25 - 17.50) = 375,000 \]

The swap buyer receives the realized volatility and pays the strike volatility, so in this example the swap payer pays $375,000 to the swap buyer.

EXPLANATION

Please see the explanation for APPLICATION 17.3.2A

CALCULATIONS

Step One: Press 250,000 → x
Step Two: Press ( → 18.25 → - → 17.50 → )
Step Three Press = “0.75”
Step Four: Press =
Answer: 375,000

WORKOUT AREA: Here are sample problems – cover one of the values and see if you can solve it using the others

See the WORKOUT AREA for Application 17.3.2A.
The payoff of a variance swap is $120,000. The strike variance is 9.00 and the realized variance is 10.00. What are the vega notional value and the variance notional value?

From the formula for the variance swap payoff (Equation 17.8):

\[
\text{Variance Swap Payoff} = \frac{\text{Vega Notional Value} \times (\text{Realized Variance} - \text{Strike Variance})}{2 \times \sqrt{\text{Strike Variance}}}
\]

\[
$120,000 = \frac{\text{Vega Notional Value} \times (10.00 - 9.00)}{2 \times \sqrt{9.00}}
\]

\[
\text{Vega Notional Value} = \frac{$120,000}{2 \times \sqrt{9.00}} = $720,000
\]

From the definition of variance notional value:

\[
\text{Variance Notional Value} = \frac{\text{Vega Notional Value}}{2\sqrt{\text{Strike Variance}}}
\]

\[
\text{Variance Notional Value} = \frac{$720,000}{2\sqrt{9.00}} = $120,000
\]

**EXPLANATION**

To solve this problem, we are given the necessary data points and can put them into Equation 17.8. The steps to solve this problem are as follows:

\[
\text{Variance Swap Payoff} = \frac{\text{Vega Notional Value} \times (\text{Realized Variance} - \text{Strike Variance})}{2 \times \sqrt{\text{Strike Variance}}}
\]

\[
\text{Variance Swap Payoff} \times 2 \times \sqrt{\text{Strike Variance}}
\]

\[
= \text{Vega Notional Value} \times (\text{Realized Variance} - \text{Strike Variance})
\]

\[
\text{Vega Notional Value} = \frac{\text{Variance Swap Payoff} \times (2 \times \sqrt{\text{Strike Variance}})}{(\text{Realized Variance} - \text{Strike Variance})}
\]

\[
\text{Vega Notional Value} = \frac{$120,000 \times 2 \times \sqrt{9.00}}{(10.00 - 9.00)} = $720,000
\]
CALCULATIONS

Step One: Press 120,000 → x → 2
Step Two: Press x → 9 → √
Step Three: Press = “720,000”
Step Four: Press + → ( → 10 → - → 9 → )
Step Four: Press =
Answer: 720,000

WORKOUT AREA: Here are sample problems – cover one of the values and see if you can solve it using the others

<table>
<thead>
<tr>
<th>Variance Swap Payoff</th>
<th>Vega Notional Value</th>
<th>Realized Variance</th>
<th>Strike Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>$ 120,000.00</td>
<td>$ 720,000.00</td>
<td>10</td>
<td>9</td>
</tr>
<tr>
<td>$ 150,000.00</td>
<td>$ 848,528.14</td>
<td>9</td>
<td>8</td>
</tr>
<tr>
<td>$ 256,000.00</td>
<td>$(418,046.25)</td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>$ 865,500.00</td>
<td>$ 1,632,002.45</td>
<td>11</td>
<td>8</td>
</tr>
<tr>
<td>$ 600,000.00</td>
<td>$ 848,528.14</td>
<td>12</td>
<td>8</td>
</tr>
<tr>
<td>$ 560,500.00</td>
<td>$ 1,372,939.00</td>
<td>8</td>
<td>6</td>
</tr>
</tbody>
</table>
APPLICATION 17.3.6A (page 558)

Suppose that the realized volatility of an asset has exactly five equally likely outcomes: 1%, 3%, 4%, 5%, or 7%. Calculate: (1) The expected value of the realized volatility, (2) the five equally likely realized variances corresponding to the five given outcomes (volatilities), (3) the expected value of the five variances, and (4) the value of volatility that corresponds to the value found in Step 3 (i.e., the volatility that corresponds with the expected variance).

1. The expectation of the five outcomes provided (i.e., realized volatilities) is 4%, found as the sum of the five outcomes divided by 5. 
2. The five variances (expressed as a decimal) corresponding to the five volatilities are: 0.0001, 0.0009, 0.0016, 0.0025, and 0.0049. 
3. The expected value of the five equally likely values found in Step 2 is the sum divided by five: \( \frac{0.0100}{5} = 0.0020 \). 
4. The value of volatility that corresponds to a variance of 0.0020 is found as its square root: \( 4.47\% \). Note that the expected variance—expressed as a volatility—differs substantially from the expected volatility (standard deviation). This issue needs to be considered in comparing derivatives with payoffs based on expected volatility with those based on expected variance.

EXPLANATION

The purpose of this exercise is to compare the difference between variance-based and volatility-based payoffs. To solve for these four problems, we must do the following:

To find the expected value of the outcomes (1), we will take a simple average of the five expected volatilities (Note: it is a simple average because they are all equally likely. If there was a higher probability associated with them, a simple average would not be appropriate).

\[
\text{Average} = \frac{1\% + 3\% + 4\% + 5\% + 7\%}{5} = 4\%
\]

To determine the variances (2), we simply have to square each volatility (i.e., standard deviation)

\[
\text{Variance} = 0.01^2 = 0.0001 \\
\text{Variance} = 0.03^2 = 0.0009 \\
\text{Variance} = 0.04^2 = 0.0016 \\
\text{Variance} = 0.05^2 = 0.0025 \\
\text{Variance} = 0.07^2 = 0.0049
\]

Next, we must find the expected value of the variances, which is a simple average of the five variances found above (Note: it is a simple average because they are all equally likely. If there was a higher probability associated with them, a simple average would not be appropriate).

\[
\text{Average} = \frac{0.0001 + 0.0009 + 0.0016 + 0.0025 + 0.0049}{5} = 0.0020
\]

Finally, to find the value of the volatility (4) using the expected value of the variances from the previous step, we simply take the square root of the variance:

\[
\text{Standard Deviation} = \sqrt{0.002} = 0.0447 = 4.47\%
\]
CALCULATIONS

To find the expected value of the volatilities:

Step One: Press 0.01 → + → 0.03 → + → 0.04 → + → 0.05 + → 0.07
Step Two: Press = “0.20”
Step Three: Press ÷ 5
Step Four: Press =
Answer: 0.04

To find the variances:

Step One: Press 0.01 → x^2
Step Two: Press =
Answer: 0.0001

Step One: Press 0.03 → x^2
Step Two: Press =
Answer: 0.0009

Step One: Press 0.04 → x^2
Step Two: Press =
Answer: 0.0016

Step One: Press 0.05 → x^2
Step Two: Press =
Answer: 0.0025

Step One: Press 0.07 → x^2
Step Two: Press =
Answer: 0.0049

To find the average variance:

Step One: Press 0.0001 → + → 0.0009 → + → 0.0016 → + → 0.0025 + → 0.0049
Step Two: Press = “0.01”
Step Three: Press ÷ 5
Step Four: Press =
Answer: 0.002

To find the volatility:

Step One: Press 0.07 → √
Step Two: Press =
Answer: 0.0049
APPLICATION 17.4.3A (page 569)

What would be the short position in a four-year zero-coupon bond that would form a duration-neutral hedge with a $2 million long position in a bond with a duration of 2.5?

The duration of the four-year zero-coupon bond is 4.0 (i.e., equal to its maturity). The size of the short position must be $2,000,000 \times (2.5/4.0)$, or $1,250,000.

EXPLANATION

To find the short position in a four-year zero-coupon bond that would form a duration-neutral hedge with a $2 million long position in a bond with a duration of 2.5, we need to divide the long bond position duration by the short bond position duration and multiply by the nominal amount of the position that needs to be hedged. In this application, we need to divide 2.5 (duration of long bond position) by 4.0 (duration of short bond position) and multiply the quotient by $2,000,000$ (size of long position, which is the position being hedged) for a short bond position of $1,250,000.

CALCULATIONS

Step One: Press 2.5 $\rightarrow$ ÷ $\rightarrow$ 4
Step Two: Press x $\rightarrow$ 2000000
Step Three: Press =
Answer: 1250000

WORKOUT AREA: Here are sample problems – cover one of the values and see if you can solve it using the others

<table>
<thead>
<tr>
<th>Short Bond Position (millions)</th>
<th>Short Bond Duration</th>
<th>Long Bond Position (millions)</th>
<th>Long Bond Duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1.25$</td>
<td>4</td>
<td>$2.00$</td>
<td>2.5</td>
</tr>
<tr>
<td>$2.40$</td>
<td>5</td>
<td>$4.00$</td>
<td>3</td>
</tr>
<tr>
<td>$2.86$</td>
<td>3.5</td>
<td>$5.00$</td>
<td>2</td>
</tr>
<tr>
<td>$2.00$</td>
<td>4.5</td>
<td>$6.00$</td>
<td>1.5</td>
</tr>
<tr>
<td>$1.27$</td>
<td>5.5</td>
<td>$7.00$</td>
<td>1</td>
</tr>
<tr>
<td>$2.49$</td>
<td>6.5</td>
<td>$4.50$</td>
<td>3.6</td>
</tr>
<tr>
<td>$5.50$</td>
<td>2.5</td>
<td>$5.50$</td>
<td>2.5</td>
</tr>
</tbody>
</table>
A prepayable mortgage has an option-adjusted spread of 50 basis points. Analysis indicates that 70 basis points of the prepayable mortgage yield is attributable to the prepayment option. If the prepayable mortgage has a yield of 6.00%, what is the yield on a comparable treasury security?

Inserting into Equation 17.10 and solving identifies that the corresponding Treasury yield is 4.80%.

EXPLANATION

To find the yield on a comparable treasury security, we simply have to manipulate Equation 17.10:

\[
\text{Prepayable Mortgage Yield} = \text{Treasury Yield} + \text{OAS} + \text{Spread Due to Prepayment Option}
\]

\[
\text{Treasury Yield} = \text{Prepayable Mortgage Yield} - \text{OAS} - \text{Spread Due to Prepayment Option}
\]

\[
\text{Treasury Yield} = 6.00\% - 0.50\% - 0.70\% = 4.80\%
\]

CALCULATIONS

Step One: Press 0.06 → - → 0.005 → - → 0.007

Step Two: Press =

Answer: 0.048

WORKOUT AREA: Here are sample problems – cover one of the values and see if you can solve it using the others

<table>
<thead>
<tr>
<th>Prepayable Mortgage Yield</th>
<th>Treasury Yield</th>
<th>OAS</th>
<th>Spread Due to Prepayment Option</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.00%</td>
<td>4.80%</td>
<td>0.50%</td>
<td>0.70%</td>
</tr>
<tr>
<td>5.00%</td>
<td>3.60%</td>
<td>0.60%</td>
<td>0.80%</td>
</tr>
<tr>
<td>4.00%</td>
<td>2.60%</td>
<td>0.40%</td>
<td>1.00%</td>
</tr>
<tr>
<td>3.00%</td>
<td>1.40%</td>
<td>1.00%</td>
<td>0.60%</td>
</tr>
<tr>
<td>10.00%</td>
<td>3.00%</td>
<td>5.00%</td>
<td>2.00%</td>
</tr>
<tr>
<td>8.00%</td>
<td>4.00%</td>
<td>3.00%</td>
<td>1.00%</td>
</tr>
</tbody>
</table>
APPLICATION 18.5.1A (page 595)

Suppose that a short seller establishes a short position in one share of XYZ Corporation at $50 per share and that XYZ pays a dividend of $0.30 per share each calendar quarter. The current rebate on XYZ share is 1% per year. What would be the dollar return to the short seller if XYZ rose to $51 at the end of one year? First, the short position loses $1 (a capital loss) when the stock rises from $50 to $51. When the stock pays four quarterly dividends of $0.30, it is up to the short seller to make a cash payment to the securities lender in lieu of dividends, so the short seller loses another $1.20. Finally, an institutional short seller typically receives a short stock rebate; in this case, the rebate would be $0.50 (1% × $50). The total loss is $1.70 ($1.00 + $1.20 – $0.50).

EXPLANATION

There are three components of profit/loss in this short position, capital gain/loss, dividend gain/loss, and rebate gain/loss. When the stock rises to $51 from $50, the short position loses $1 of capital loss. The stock pays a dividend quarterly, which the short seller pays the securities lender, so $0.30 (stock dividend) multiplied by 4 (number of quarters in a year) equals $1.20 of dividends the short seller owes the securities lender. Lastly, the short seller earns a rebate equal to 0.01 multiplied by $50 (the price the security was sold short) or $0.50. Putting it all together, - $1 (capital loss of as share price rises to $51 from $50) – $1.20 (quarterly dividend owed to securities lender from short seller) + $0.50 (short stock rebate) equals -$1.70 or the dollar return to short seller in one year.

CALCULATIONS

Step One: Press 50 → - → 51
Step Two: Press = “-1”
Step Three: Press 4 → x → 0.30
Step Four: Press = “1.20”
Step Five: Press 0.01 → x → 50
Step Six: Press = “0.50”
Step Seven: 1 → +|-1
Step Eight: - → 1.20
Step Nine: + → 0.50
Step Ten: =
Answer: -1.70
WORKOUT AREA: Here are sample problems – cover one of the values and see if you can solve it using the others

<table>
<thead>
<tr>
<th>Dollar Return to Short Seller in One Year</th>
<th>Short Position Shares</th>
<th>Price of XYZ Corp. Per Share</th>
<th>Quarterly Dividend of XYZ Corp. Per Share</th>
<th>Rebate on XYZ Corp.</th>
<th>Price of XYZ Corp. Per Share in One-Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>($1.70)</td>
<td>1</td>
<td>$50.00</td>
<td>$0.30</td>
<td>1.00%</td>
<td>$51.00</td>
</tr>
<tr>
<td>($19.50)</td>
<td>10</td>
<td>$65.00</td>
<td>$0.40</td>
<td>1.00%</td>
<td>$66.00</td>
</tr>
<tr>
<td>($279.38)</td>
<td>25</td>
<td>$55.00</td>
<td>$0.50</td>
<td>1.50%</td>
<td>$65.00</td>
</tr>
<tr>
<td>$305.00</td>
<td>50</td>
<td>$75.00</td>
<td>$0.10</td>
<td>2.00%</td>
<td>$70.00</td>
</tr>
<tr>
<td>$450.00</td>
<td>100</td>
<td>$10.00</td>
<td>$0.15</td>
<td>1.00%</td>
<td>$5.00</td>
</tr>
<tr>
<td>$876.00</td>
<td>200</td>
<td>$15.00</td>
<td>$0.20</td>
<td>1.20%</td>
<td>$10.00</td>
</tr>
<tr>
<td>($810.30)</td>
<td>300</td>
<td>$13.00</td>
<td>$0.25</td>
<td>2.30%</td>
<td>$15.00</td>
</tr>
</tbody>
</table>
APPLICATION 20.3.8A (page 646)

A potential VC investment has a projected EBITDA of $25 million (if successful) and an EBITDA multiple of 8, if the project can be exited in 7 years. Ignoring the percentage of the firm that the investor will not own, the costs of providing oversight and managerial assistance, and any other existing claims to the firm such as indebtedness, what is the estimated enterprise value of the investment if its required IRR is 65%?

\[
Value\ of\ Venture = \frac{\$25\ million \times 8.0}{1.65^7} = \$6.01\ million
\]

EXPLANATION

To solve this problem, we should use Equation 20.1.

\[
Value\ of\ Venture = \frac{EBITDA \times EBITDA\ Multiple}{(1 + IRR)^T}
\]

By multiplying the projected EBITDA by a multiple, the numerator of this equation represents the investment value in seven years, or $200 million. We then have to discount this investment value back to the present value using the investment’s internal rate of return (IRR), which is 65%.

\[
Value\ of\ Venture = \frac{\$25\ million \times 8.0}{1.65^7} = \$6.01\ million
\]

CALCULATIONS

Note: these calculations are in millions.

Step One: Press 25 → x → 8
Step Two: Press = “200”
Step Three: Press ÷ → ( → 1.65
Step Four: Press → x^y → 7 → )
Step Five: Press =
Answer: 6.0067
WORKOUT AREA: \textit{Cover one of the values and see if you can solve for it using the other variables.}

<table>
<thead>
<tr>
<th>Value of Venture (millions)</th>
<th>EBITDA (millions)</th>
<th>EBITDA Multiple</th>
<th>IRR</th>
<th>Time Horizon (years)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$6.01</td>
<td>$25.00</td>
<td>8</td>
<td>65%</td>
<td>7</td>
</tr>
<tr>
<td>$10.82</td>
<td>$30.00</td>
<td>5</td>
<td>55%</td>
<td>6</td>
</tr>
<tr>
<td>$4.34</td>
<td>$25.00</td>
<td>10</td>
<td>50%</td>
<td>10</td>
</tr>
<tr>
<td>$61.73</td>
<td>$40.00</td>
<td>5</td>
<td>80%</td>
<td>2</td>
</tr>
<tr>
<td>$26.83</td>
<td>$30.00</td>
<td>8</td>
<td>55%</td>
<td>5</td>
</tr>
<tr>
<td>$14.09</td>
<td>$20.00</td>
<td>10</td>
<td>70%</td>
<td>5</td>
</tr>
<tr>
<td>$0.11</td>
<td>$25.00</td>
<td>5</td>
<td>60%</td>
<td>15</td>
</tr>
</tbody>
</table>
APPLICATION 20.5.4A (page 651)

A potential growth equity investment has projected annual revenues of $80 million in six years, at which point an IPO is anticipated. Similar publicly traded firms have enterprise-value-to-revenue multiples of 2.25. The firm seeks growth equity financing and offers a 25% stake in the company. Based on Equation 20.2 and ignoring any cash or debt in the existing firm, determine the estimated value of the growth equity if the required IRR is 45%.

\[
Value \text{ of } Enterprise = \frac{\$80 \text{ million} \times 2.25}{1.45^6} = \$19.37 \text{ million}
\]

\[
Value \text{ of } Equity \text{ Growth} = 25\% \times \$19.37 \text{ million} = \$4.84 \text{ million}
\]

EXPLANATION

To solve the first part of this problem, we should use Equation 20.2. Note how similar this equation is to Equation 20.1 in Application 20.5.4A:

\[
Value \text{ of } Enterprise = \frac{\text{Revenue} \times \text{Revenue Multiple}}{(1 + \text{IRR})^t}
\]

By multiplying the projected EBITDA by a multiple, the numerator of this equation represents the investment value in seven years, or $200 million. We then have to discount this investment value back to the present value using the investment’s internal rate of return (IRR), which is 65%.

\[
Value \text{ of } Enterprise = \frac{\$80 \text{ million} \times 2.25}{1.45^6} = \$19.37 \text{ million}
\]

Now that we have determined the total value of the enterprise, all we must do is determine the value of a 25% stake in the firm. This is done by multiplying 25% by $19.37 million, or $4.84 million.

CALCULATIONS

Note: these calculations are in millions.

Step One: Press 80 \(\rightarrow\) \(\times\) \(\rightarrow\) 2.25
Step Two: Press = “180”
Step Three: Press \(+\) \(\rightarrow\) ( \(\rightarrow\) 1.45
Step Four: Press \(\rightarrow\) \(\times\) \(\rightarrow\) 6 \(\rightarrow\) )
Step Five: Press = “19.367”
Step Six: Press \(\times\) \(\rightarrow\) 0.25
Step Seven: Press =
Answer: 4.8418
WORKOUT AREA: Cover one of the values and see if you can solve for it using the other variables.

<table>
<thead>
<tr>
<th>Value of Enterprise (millions)</th>
<th>Revenue (millions)</th>
<th>Revenue Multiple</th>
<th>IRR</th>
<th>Time Horizon (years)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$19.37</td>
<td>$80.00</td>
<td>2.25</td>
<td>45%</td>
<td>6</td>
</tr>
<tr>
<td>$4.37</td>
<td>$70.00</td>
<td>5</td>
<td>55%</td>
<td>10</td>
</tr>
<tr>
<td>$104.86</td>
<td>$80.00</td>
<td>4</td>
<td>25%</td>
<td>5</td>
</tr>
<tr>
<td>$9.12</td>
<td>$76.00</td>
<td>4</td>
<td>55%</td>
<td>8</td>
</tr>
<tr>
<td>$3.71</td>
<td>$10.00</td>
<td>5</td>
<td>45%</td>
<td>7</td>
</tr>
<tr>
<td>$14.09</td>
<td>$20.00</td>
<td>10</td>
<td>70%</td>
<td>5</td>
</tr>
<tr>
<td>$6.56</td>
<td>$2.00</td>
<td>10</td>
<td>45%</td>
<td>3</td>
</tr>
</tbody>
</table>
Returning to the previous example, suppose that all other facts remain the same except that the discount rate used at the end of seven years is 15%. The projected value of the company becomes $120 million/(0.15 - 0.02) = $923 million and the seven-year rate of return becomes ($923 million/$100 million)\(^{1/7} - 1 = 37.4\%.

EXPLANATION

The key issues in this application are that the value of the company at the exit horizon is being changed by the change in the discount factor used in the valuation model. The new exit value generates a new rate of return.

The old valuation was:

$120 million \div (0.12 - 0.02) = $1.2 billion

The new valuation is:

$120 million \div (0.15 - 0.02) = $0.923 billion

The old rate of return was:

($1.2 billion \div $100 million)\(^{1/7} - 1 = 42.6\%

The new rate of return is:

($0.923 billion \div $100 million)\(^{1/7} - 1 = 37.4\%.

CALCULATIONS

Previous market price

Step One: Press 0.120 \div .13 =

Step Two: Press 0.923 \div .1 = y^x .142857 = Step

Three: Press – 1 =

Answer: .374 or 37.4\%
WORKOUT AREA: Here are sample problems – cover the values in the rightmost two columns and see if you can solve for them using the others.

<table>
<thead>
<tr>
<th>Initial Invest.</th>
<th>Valuation Cash</th>
<th>Discount Rate</th>
<th>Growth Rate</th>
<th>Ending Value</th>
<th>Rate of Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>$100</td>
<td>$120</td>
<td>15.00%</td>
<td>2.00%</td>
<td>$923</td>
<td>37.37%</td>
</tr>
<tr>
<td>$100</td>
<td>$120</td>
<td>12.00%</td>
<td>5.00%</td>
<td>$1,714</td>
<td>50.07%</td>
</tr>
<tr>
<td>$100</td>
<td>$120</td>
<td>12.00%</td>
<td>2.00%</td>
<td>$1,200</td>
<td>42.62%</td>
</tr>
<tr>
<td>$150</td>
<td>$150</td>
<td>12.00%</td>
<td>7.00%</td>
<td>$3,000</td>
<td>53.41%</td>
</tr>
<tr>
<td>$150</td>
<td>$120</td>
<td>14.00%</td>
<td>1.00%</td>
<td>$923</td>
<td>29.64%</td>
</tr>
</tbody>
</table>
Returning to the original example, supposed that all other facts remain the same except that the growth rate used at the end of the seven years is 5%. The projected value of the company becomes $120 million \div (0.12 - 0.05) = $1.714 billion, and the seven-year rate of return becomes $(1.714 \text{ billion} / 100 \text{ million})^{1/7} - 1 = 50.1\%$

**EXPLANATION**

The key issues in this application are that the value of the company at the exit horizon is being changed by the change in the growth rate used in the valuation model. The new exit value generates a new rate of return.

The old valuation was:

$120 \text{ million} \div (0.12 - 0.02) = 1.2 \text{ billion}$

The new valuation is:

$120 \text{ million} \div (0.12 - 0.05) = 1.714 \text{ billion}$

The old rate of return was:

$(1.2 \text{ billion} / 100 \text{ million})^{1/7} - 1 = 42.6\%$ The new rate of return is:

$(1.714 \text{ billion} / 100 \text{ million})^{1/7} - 1 = 50.1\%$

**CALCULATIONS**

Previous market price

Step One: Press $0.120 \div .07 =$

Step Two: Press $1.714 \div 0.1 = y^x \times 142857 =$

Step Three: Press $-1 =$

Answer: .501 or 50.1\%
**WORKOUT AREA:** Here are sample problems – cover the values in the two rightmost columns and see if you can solve for them using the others.

<table>
<thead>
<tr>
<th>Initial Invest.</th>
<th>Valuation Cash</th>
<th>Discount Rate</th>
<th>Growth Rate</th>
<th>Ending Value</th>
<th>Rate of Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>$100</td>
<td>$120</td>
<td>15.00%</td>
<td>2.00%</td>
<td>$923</td>
<td>37.37%</td>
</tr>
<tr>
<td>$100</td>
<td>$120</td>
<td>12.00%</td>
<td>5.00%</td>
<td>$1,714</td>
<td>50.07%</td>
</tr>
<tr>
<td>$100</td>
<td>$120</td>
<td>12.00%</td>
<td>2.00%</td>
<td>$1,200</td>
<td>42.62%</td>
</tr>
<tr>
<td>$50</td>
<td>$120</td>
<td>13.00%</td>
<td>2.00%</td>
<td>$1,091</td>
<td>55.33%</td>
</tr>
<tr>
<td>$200</td>
<td>$150</td>
<td>14.00%</td>
<td>4.00%</td>
<td>$1,500</td>
<td>33.35%</td>
</tr>
</tbody>
</table>
Returning to the original example, suppose that all other facts remain the same except that the investment requires eight years to exit. The projected value of the company becomes $120 million / (0.12 − 0.02) = $1.2 billion, and the eight-year rate of return becomes $(1.2 \text{ billion} / 100 \text{ million})^{1/8} − 1 = 36.4\%$.

**EXPLANATION**

The key issues in this application is that the value of the company at the exit horizon is the same, but the time interval changes (from 7 years to 8 years) and the rate of return therefore changes:

The old valuation is still:

\[ \frac{120 \text{ million}}{0.12 − 0.02} = 1.2 \text{ billion} \]

The new rate of return is:

\[ \left( \frac{1.714 \text{ billion}}{100 \text{ million}} \right)^{1/8} − 1 = 36.4\% \]

**CALCULATIONS**

Previous market price

Step One: Press 0.120 ÷ .10 =

Step Two: Press 1.2 ÷ 0.1 = \( y^X \).125 =

Step Three: Press − 1 =

Answer: .364 or 36.4\%
WORKOUT AREA: Here are sample problems – cover the values in the two rightmost columns and see if you can solve for them using the others

<table>
<thead>
<tr>
<th>Initial Invest.</th>
<th>Valuation Cash Flow</th>
<th>Discount Rate</th>
<th>Growth Rate</th>
<th>Ending Value</th>
<th>Rate of Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>$100</td>
<td>$120</td>
<td>15.00%</td>
<td>2.00%</td>
<td>$923</td>
<td>32.02%</td>
</tr>
<tr>
<td>$100</td>
<td>$120</td>
<td>12.00%</td>
<td>5.00%</td>
<td>$1,714</td>
<td>42.65%</td>
</tr>
<tr>
<td>$100</td>
<td>$120</td>
<td>12.00%</td>
<td>2.00%</td>
<td>$1,200</td>
<td>36.43%</td>
</tr>
<tr>
<td>$50</td>
<td>$120</td>
<td>13.00%</td>
<td>2.00%</td>
<td>$1,091</td>
<td>47.01%</td>
</tr>
<tr>
<td>$200</td>
<td>$150</td>
<td>14.00%</td>
<td>4.00%</td>
<td>$1,500</td>
<td>28.64%</td>
</tr>
</tbody>
</table>
Returning to the original example, suppose that all other facts remain the same except that the $120 million cash flow estimate given is a year 7 cashflow that is anticipated to grow by year 8. The $120 million seven-year cash flow is therefore estimated to grow to an eight-year cash flow as $120 million × (1.02) = $122.4 million. The projected value of the company becomes $122.4 million / (0.12 – 0.02) = $1.224 billion, and the seven-year rate of return becomes \((1.224 \text{ billion}/100 \text{ million})^{1/7} - 1 = 43.0\%\)

EXPLANATION

The key issue is that the cash flow being earned at exit is higher. The application attempts to emphasize that when using the perpetual growth model the proper cash flow to use in the numerator is the cash flow at the end of the year following the valuation date.

The old valuation was:
$120 \text{ million} / (0.12 - 0.02) = 1.2 \text{ billion}$

The new valuation is:
$120 \text{ million} \times (1.02) / (0.12 - 0.02) = 1.224 \text{ billion}$

The old rate of return was:
\((1.2 \text{ billion}/100 \text{ million})^{1/7} - 1 = 42.6\%\) The new rate of return is:
\((1.224 \text{ billion}/100 \text{ million})^{1/7} - 1 = 43.0\%\)

CALCULATIONS

Previous market price

Step One: Press 0.120 x 1.02 = \(\rightarrow\) .10 =

Step Two: Press ÷ .1 = \(\text{y}^\text{x}\) .142857 =

Step Three: Press – 1 =

Answer: .430 or 43.0\%
WORKOUT AREA: Here are sample problems – cover the values in the two rightmost columns and see if you can solve for them using the others.

<table>
<thead>
<tr>
<th>Initial Invest.</th>
<th>Valuation Cash</th>
<th>Discount Rate</th>
<th>Growth Rate</th>
<th>Ending Value</th>
<th>Rate of Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>$100</td>
<td>$120.00</td>
<td>12.00%</td>
<td>2.00%</td>
<td>$1,200</td>
<td>42.62%</td>
</tr>
<tr>
<td>$100</td>
<td>$122.40</td>
<td>12.00%</td>
<td>2.00%</td>
<td>$1,224</td>
<td>43.02%</td>
</tr>
<tr>
<td>$100</td>
<td>$124.80</td>
<td>12.00%</td>
<td>2.00%</td>
<td>$1,248</td>
<td>43.42%</td>
</tr>
<tr>
<td>$100</td>
<td>$120.00</td>
<td>15.00%</td>
<td>2.00%</td>
<td>$923</td>
<td>37.37%</td>
</tr>
<tr>
<td>$100</td>
<td>$150.00</td>
<td>12.00%</td>
<td>5.00%</td>
<td>$2,143</td>
<td>54.93%</td>
</tr>
</tbody>
</table>
A VC fund manager raises $100 million in committed capital for his VC fund. The management fee is 2.5%. To date, only $50 million of the raised capital has been called and invested in start-ups. What would be the annual management fee? The annual management fee that the manager collects is $2.5 million (2.5% × $100 million), even though not all of the capital has been invested.

EXPLANATION

The key trait in most VC fund deals is that the management fee is collected on the total capital including the committed capital that has not even been funded (let alone invested in companies).

CALCULATIONS

Step One: Press 100000000 → x → 0.025
Step Two: Press =
Answer: 2,500,000

WORKOUT AREA: Here are sample problems – cover one of the values and see if you can solve it using the others.

<table>
<thead>
<tr>
<th>Committed Cap.</th>
<th>Called Capital</th>
<th>Invested Capital</th>
<th>Mgmt. Fee %</th>
<th>Mgmt. Fee $</th>
</tr>
</thead>
<tbody>
<tr>
<td>$100 m</td>
<td>$70 m</td>
<td>$50 m</td>
<td>1%</td>
<td>$1 m</td>
</tr>
<tr>
<td>$500 m</td>
<td>$400 m</td>
<td>$100 m</td>
<td>2%</td>
<td>$10 m</td>
</tr>
</tbody>
</table>
APPLICATION 21.4.3A (page 683)

Assume a $200 million contribution by the LPs in the first year to fund an investment, a 6% hurdle rate, a 100% catch-up, an 80/20 carry split, and the sale of the investment by the fund in the third year for $300 million. What is the total cash flow to the LPs and the total cash flow to the GPs?

For simplicity, assume that the hurdle rate is not compounded and ignore all other expenses such as management fees. The preferred return to the LPs is $24 million ($200 million × 6% × 2), leaving $76 million after also returning the $200 million to the LPs. The 100% catch-up to the GPs for an 80%/20% split is $6 million ($24 million × 20% / (1 - 20%)). The residual is $70 million, found as $300 million – ($200 million + $24 million + $6 million), and is split 80%/20%.

EXPLANATION

We are told the LPs contribute $200 million at the beginning of the period, which turns into $300 million by the third year. With a 6% hurdle rate, the investment must clear 6% per year. If we ignore compounding in this problem, that amounts to 6% × 2 × $200 or $24 million that the LPs must make prior to GPs collecting any carried interest.

The carried interest split is allocated 20% to the GPs and 80% to the LPs. Since there is a 100% catch-up provision in place for the GPs of the fund, and the fund more than exceeded the $24 million required return, the GPs are entitled to their catch-up provision. This is where the math seem complicated, but think of it like this: because the carried interest split is 80/20, the LPs have effectively already “earned” a portion of their 80%, or $24 million, we must then “catch-up” the GPs so that they can earn their 20%. Mathematically, this looks like this: ($24 million x 20%)/(1 - 20%) = $6 million.

At this point $30 million ($24 + $6) of the $100 million in total profit has been accounted for, which means we have $70 million left to distribute. Now that all hurdle rates and catch-up provisions have been handled, we can simply distribute the carried interest between the LPs and GPs per the 80/20 agreement. This would mean $56 million (80% x 70 million) would be distributed to the LPs and $14 million (20% x $70 million) to the LPs.

In total, $20 million would be distributed to the LPs ($6 million catch-up + $14 residual profits) and $80 million to the LPs ($24 million hurdle rate + $56 residual profits)

CALCULATIONS

To find the hurdle rate in dollar amounts:

Step One: Press 0.06 → x → 200 → x → 2
Step Two: Press =
Answer: 24

To find the GP catch-up dollar amount:

Step One: Press 24 → x → 0.20
Step Two: Press ÷ (→ 1 → - → 0.20 → )
Step Three: Press =
Answer: 6
To find the residual profit distributions to the LPs:

Step One: 100 → - → 24 → - → 6
Step Two: Press “70”
Step Three: Press x → 0.80
Step Four: Press =
Answer: 56

To find the residual profit distributions to the GPs:

Step One: 100 → - → 24 → - → 6
Step Two: Press “70”
Step Three: Press x → 0.20
Step Four: Press =
Answer: 14

To find the total distribution to the LPs:

Step One: Press 56 → + → 14
Step Two: Press =
Answer: 80

To find the total distribution to the GPs:

Step One: Press 6 → + → 14
Step Two: Press =
Answer: 20
APPLICATION 21.8.2A (page 695)

Shares of closed-end fund ABC were selling at a premium of 10% and then fell to $44 per share while ABC’s net asset value held constant at $50 per share. What were the previous market price, subsequent discount, NAV-based return, and market-price return for ABC?

The previous market price was $55 (solved using Equation 21.1, with 0.10 on the left-hand side and $50 for the NAV). The subsequent discount (solved as \(-12\%\) using Equation 21.1, with $44/$50 as the fraction inside the parentheses) was 12%. The NAV-based return was 0%, since the NAV was assumed unchanged and the market-price return was \(-20\%\) (\(-11/55\)), assuming no dividends or other distributions.

EXPLANATION

The previous market price is equal to NAV multiplied by 1 + the premium. In this case, $50 \times (1+0.10) = 55. The subsequent discount is calculated using the same formula, but arranged differently, in this case, $44 / $50 - 1 = -12\%. The NAV return is zero as the NAV is assumed to have stayed constant throughout the example. The market price return is calculated by $(44 - 55) / 55$ or -20%.

CALCULATIONS

Previous market price

Step One: Press 1 → + → 0.10
Step Two: Press x → $50
Step Three: Press = Answer: 55

Subsequent discount

Step One: Press 44 → ÷ → 50
Step Two: Press - → 1
Step Three: Press = Answer: -0.12 or -12% 

Market price return

Step One: Press 44 → - → 55
Step Two: Press ÷ → 55
Step Three: Press =
Answer: -0.20 or -20%
WORKOUT AREA: Here are sample problems – cover the values in the rightmost column and see if you can solve it using the others

<table>
<thead>
<tr>
<th>Previous Market Price</th>
<th>Subsequent Discount</th>
<th>Premium</th>
<th>Subsequent Price Per Share</th>
<th>NAV</th>
<th>Market Price Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>$55.00</td>
<td>12.00%</td>
<td>10.00%</td>
<td>$44.00</td>
<td>$50.00</td>
<td>-20.00%</td>
</tr>
<tr>
<td>$44.80</td>
<td>0.00%</td>
<td>12.00%</td>
<td>$40.00</td>
<td>$40.00</td>
<td>-10.71%</td>
</tr>
<tr>
<td>$22.40</td>
<td>0.00%</td>
<td>12.00%</td>
<td>$20.00</td>
<td>$20.00</td>
<td>-10.71%</td>
</tr>
<tr>
<td>$21.00</td>
<td>20.00%</td>
<td>5.00%</td>
<td>$16.00</td>
<td>$20.00</td>
<td>-23.81%</td>
</tr>
<tr>
<td>$30.30</td>
<td>0.00%</td>
<td>1.00%</td>
<td>$30.00</td>
<td>$30.00</td>
<td>-0.99%</td>
</tr>
<tr>
<td>$20.60</td>
<td>10.00%</td>
<td>3.00%</td>
<td>$18.00</td>
<td>$20.00</td>
<td>-12.62%</td>
</tr>
<tr>
<td>$25.50</td>
<td>12.00%</td>
<td>2.00%</td>
<td>$22.00</td>
<td>$25.00</td>
<td>-13.73%</td>
</tr>
</tbody>
</table>
A convertible preferred stock with a par or face value of $100 per share is convertible into four shares of common stock. What is the conversion ratio, and what is the conversion price? What would be the conversion ratio if the conversion price were $20?

The original example of the preferred stock has a conversion ratio of 4:1. The conversion option may be expressed as a conversion price of $25 (using the face value of the preferred stock to make the purchase). In the second example of a $20 conversion price, the conversion ratio would be 5:1.

**EXPLANATION**

The conversion ratio is 4:1. It is how many shares the preferred stock can be converted into. In this case, one share of preferred stock can be converted into 4 shares. The conversion price is $100 (face value of the preferred stock) divided by 4 equals $25, the conversion price. If the conversion price was $20, then $100 (face value of the preferred stock) divided by $20 equals 5 (5:1), the conversion ratio.

**CALCULATIONS**

Conversion price

Step One: Press 100 → ÷ → 4

Step Two: Press =

Answer: 25

Conversion ratio with conversion price of $20

Step One: Press 100 → ÷ → 20

Step Two: Press = Answer: 5

**WORKOUT AREA:** Here are sample problems – cover one of the values and see if you can solve it using the others

<table>
<thead>
<tr>
<th>Conversion Price</th>
<th>Face value</th>
<th>Conversion ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>$25.00</td>
<td>$100.00</td>
<td>4 :1</td>
</tr>
<tr>
<td>$20.00</td>
<td>$100.00</td>
<td>5 :1</td>
</tr>
<tr>
<td>$50.00</td>
<td>$100.00</td>
<td>2 :1</td>
</tr>
<tr>
<td>$50.00</td>
<td>$350.00</td>
<td>7 :1</td>
</tr>
<tr>
<td>$83.33</td>
<td>$250.00</td>
<td>3 :1</td>
</tr>
<tr>
<td>$90.00</td>
<td>$450.00</td>
<td>5 :1</td>
</tr>
<tr>
<td>$187.50</td>
<td>$750.00</td>
<td>4 :1</td>
</tr>
</tbody>
</table>
APPLICATION 22.2.3A (page 713)

A fixed-rate bond and a floating-rate note each have a five-year maturity. The fixed-rate bond has a duration of 4.5 years. The floating-rate note will reset its coupon to market rates in 0.5 years. Approximating bond price changes as the product of duration and interest rate shifts (times minus one), if continuously compounded interest rates increase by 1.5%, how much will the price of the fixed-rate bond and floating-rate note decline? The price of the fixed-rate bond will drop by approximately 6.75% (4.5 duration × 1.5% rate rise). The price of the floating-rate note will drop by approximately 0.75% (0.5 duration × 1.5% rise).

EXPLANATION

This application is showing the relationships between changes in interest rates and a bond’s duration. A bond’s duration is the sensitivity to the changes in interest rates: the longer the duration, the greater the sensitivity. Remember, fixed rate bond durations tend to be close to their maturity, such as the five-year fixed-rate bond which has a duration of 4.5 years. The floating rate note also has a five-year maturity, but its duration is tied to the period in which the floating interest rate resets, which is 0.5 years. To calculate the change in both bonds’ values when interest rates increase, simply multiply the change in rates by their respective durations:

\[
\text{Change in Bond Price} = \text{Change in Interest Rates} \times -\text{Duration}
\]

In the case of the fixed-rate bond, this equates to \(+0.015 \times 4.5 = -0.0675\). In the case of the floating-rate note, this equates to \(+0.015 \times 0.5 = -0.0075\).

CALCULATIONS

For the fixed-rate bond:
Step One: Press 0.015 → x → 4.5
Step Two: Press =
Answer: 0.0675

For the floating-rate bond:
Step One: Press 0.015 → x → 0.5
Step Two: Press =
Answer: 0.0075
WORKOUT AREA: Here are sample problems – cover one of the values and see if you can solve it using the others

<table>
<thead>
<tr>
<th>Fixed-Rate Bond Duration</th>
<th>Floating-Rate Note Duration</th>
<th>Change in Rates</th>
<th>Price Change in Fixed-Rate Bond Value</th>
<th>Price Change in Floating-Rate Note</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.50</td>
<td>0.50</td>
<td>1.50%</td>
<td>-6.75%</td>
<td>-0.75%</td>
</tr>
<tr>
<td>5.00</td>
<td>1.00</td>
<td>2.00%</td>
<td>-10.00%</td>
<td>-2.00%</td>
</tr>
<tr>
<td>6.00</td>
<td>0.50</td>
<td>-1.00%</td>
<td>6.00%</td>
<td>0.50%</td>
</tr>
<tr>
<td>3.00</td>
<td>0.25</td>
<td>-1.50%</td>
<td>4.50%</td>
<td>0.38%</td>
</tr>
<tr>
<td>3.50</td>
<td>0.10</td>
<td>2.00%</td>
<td>-7.00%</td>
<td>-0.20%</td>
</tr>
<tr>
<td>4.00</td>
<td>0.25</td>
<td>-2.00%</td>
<td>8.00%</td>
<td>0.50%</td>
</tr>
</tbody>
</table>
APPLICATION 22.2.4A (page 715)

A bond has a duration of exactly 5.0 and a stated annual yield of 4.00%. Calculate the bond’s modified duration for each of the following cases: annual compounding, semi-annual compounding, and continuous compounding. Based on Equation 22.1, the modified duration = $\frac{5.0}{[1+(.04/m)]}$. Inserting values form of 1, 2, and infinity solves for the cases of annual compounding, semi-annual compounding, and continuous compounding, respectively, as 4.81, 4.90 and 5.00.

EXPLANATION

To solve this problem, we must use Equation 22.1:

$$
Modified\ Duration = \frac{Duration}{1 + \left(\frac{y}{m}\right)}
$$

For annual compounding:

$$
Modified\ Duration = \frac{5.0}{1 + (0.04)} = 4.81
$$

For semiannual compounding:

$$
Modified\ Duration = \frac{5.0}{1 + (0.04/2)} = 4.90
$$

For continuously compounding:

$$
Modified\ Duration = \frac{5.0}{1 + (0.04/0)} = 5.0
$$

Notice continuously compounding puts “0” in the denominator of the $y/m$ function. This is because $m \rightarrow 0$ when continuously compounded, therefore the denominator is simply 1.0.

CALCULATIONS

For annual compounding:

Step One: Press $5 \rightarrow +$
Step Two: Press $( \rightarrow 1 \rightarrow + \rightarrow$
Step Three: Press $( \rightarrow 0.04 \rightarrow + \rightarrow 1 \rightarrow ) \rightarrow )$
Step Three: Press $=$
Answer: 4.808

For semi-annual compounding:

Step One: Press $5 \rightarrow +$
Step Two: Press $( \rightarrow 1 \rightarrow + \rightarrow$
Step Three: Press $( \rightarrow 0.04 \rightarrow + \rightarrow 2 \rightarrow ) \rightarrow )$
Step Three: Press =
Answer: 4.902

For continuous compounding:
Step One: Press 5 ÷ 1
Step Two: Press =
Answer: 5

WORKOUT AREA: Here are sample problems — cover one of the values and see if you can solve it using the others

<table>
<thead>
<tr>
<th>Modified Duration</th>
<th>Duration</th>
<th>Yield</th>
<th>Compounding Period</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.90</td>
<td>5</td>
<td>4%</td>
<td>2</td>
</tr>
<tr>
<td>5.77</td>
<td>6</td>
<td>4%</td>
<td>1</td>
</tr>
<tr>
<td>3.94</td>
<td>4</td>
<td>3%</td>
<td>2</td>
</tr>
<tr>
<td>5.93</td>
<td>6</td>
<td>5%</td>
<td>4</td>
</tr>
<tr>
<td>7.41</td>
<td>8</td>
<td>8%</td>
<td>1</td>
</tr>
<tr>
<td>4.88</td>
<td>5</td>
<td>5%</td>
<td>2</td>
</tr>
<tr>
<td>9.98</td>
<td>10</td>
<td>2%</td>
<td>12</td>
</tr>
</tbody>
</table>
APPLICATION 22.6.2A (page 731)

Suppose that the structure on the right-hand side of Exhibit 22.4 is changed such that the mezzanine debt rises to being 30% of the capital structure, and the bank debt falls to being 50% of the capital structure. If the costs of bank debt and equity remain the same (8% and 32%, respectively), what must the new cost of mezzanine debt be such that the weighted average cost of capital would be 15.8%?

The answer is found by solving for $x$: $15.8\% = (0.20 \times 32\%) + (0.30x) + (0.50 \times 8\%)$. The solution is that the cost of mezzanine debt, $x$, is 18%.

EXPLANATION

Let's manipulate the WACC equation to solve for the cost of mezzanine debt:

\[
WACC = C_a \times P_a + C_b \times P_b + C_c \times P_c
\]

\[
WACC - C_b \times P_b - C_c \times P_c = C_a \times P_a
\]

\[
\frac{WACC - C_b \times P_b - C_c \times P_c}{P_a} = C_a
\]

After manipulating the equation, we can now solve for the cost of mezzanine debt:

\[
\frac{0.158 - 0.32 \times 0.20 - 0.08 \times 0.50}{0.30} = C_a
\]

\[
\frac{0.158 - 0.064 - 0.04}{0.30} = C_a
\]

\[
\frac{0.054}{0.30} = C_a
\]

\[
0.18 = C_a
\]

The cost of mezzanine debt is 0.18 or 18%.
CALCULATIONS

Step One: Press 0.32 → +|
Step Two: Press x → 0.20
Step Three: Press = “-0.064” Step
Four: Press 0.08 → +|- Step Five:
Press x → 0.50 Step Six: Press =
“0.04”
Step Seven: Press 0.158 → - → 0.064 Step Eight:
Press - → 0.04
Step Nine: Press ÷ → 0.30 Step
Ten: Press = Answer: 0.18

WORKOUT AREA: Here are sample problems – cover one of the values and see if you can solve it using the others

<table>
<thead>
<tr>
<th>WACC</th>
<th>Cost of Bank Debt</th>
<th>Cost of Mezzanine Debt</th>
<th>Cost of Equity</th>
<th>Mezzanine Debt</th>
<th>Bank Debt</th>
<th>Equity</th>
</tr>
</thead>
<tbody>
<tr>
<td>12.40%</td>
<td>8.00%</td>
<td>12.00%</td>
<td>20.00%</td>
<td>20.00%</td>
<td>50.00%</td>
<td>30.00%</td>
</tr>
<tr>
<td>12.50%</td>
<td>8.00%</td>
<td>12.00%</td>
<td>22.00%</td>
<td>25.00%</td>
<td>50.00%</td>
<td>25.00%</td>
</tr>
<tr>
<td>13.25%</td>
<td>8.00%</td>
<td>13.00%</td>
<td>23.00%</td>
<td>15.00%</td>
<td>55.00%</td>
<td>30.00%</td>
</tr>
<tr>
<td>13.00%</td>
<td>8.00%</td>
<td>13.00%</td>
<td>24.00%</td>
<td>20.00%</td>
<td>55.00%</td>
<td>25.00%</td>
</tr>
<tr>
<td>13.15%</td>
<td>8.00%</td>
<td>14.00%</td>
<td>25.00%</td>
<td>15.00%</td>
<td>60.00%</td>
<td>25.00%</td>
</tr>
<tr>
<td>13.20%</td>
<td>8.00%</td>
<td>14.00%</td>
<td>28.00%</td>
<td>20.00%</td>
<td>60.00%</td>
<td>20.00%</td>
</tr>
<tr>
<td>15.80%</td>
<td>8.00%</td>
<td>18.00%</td>
<td>32.00%</td>
<td>30.00%</td>
<td>50.00%</td>
<td>20.00%</td>
</tr>
</tbody>
</table>


APPLICATION 22.7.4A (page 740)

If 20% of the bonds in a portfolio default each year and if 60% of each defaulted bonds’ value is ultimately unrecoverable (i.e., 40% of the bonds’ cost is recovered), what would be the expected annual default losses as a percentage rate relative to the portfolio’s value?

The total annual loss due to default is 12%, found from Equation 22.3: 

\[ 20\% \times (1 - 40\%) = 12\% \]

EXPLANATION

Total Loss due to Default is calculated by multiplying the annual default rate by the loss rate given default. In this application, 20% (annual default rate) is multiplied by 60% (loss rate given default) for a product of 12% (total loss due to default).

CALCULATIONS

Step One: Press 0.20 → x → 0.60

Step Two: Press =

Answer: 0.12

WORKOUT AREA: Here are sample problems – cover one of the values and see if you can solve it using the others

<table>
<thead>
<tr>
<th>Total Loss Due to Default</th>
<th>Annual Default Rate</th>
<th>Loss Rate Given Default</th>
</tr>
</thead>
<tbody>
<tr>
<td>12.00%</td>
<td>20.00%</td>
<td>60.00%</td>
</tr>
<tr>
<td>24.00%</td>
<td>30.00%</td>
<td>80.00%</td>
</tr>
<tr>
<td>6.00%</td>
<td>20.00%</td>
<td>30.00%</td>
</tr>
<tr>
<td>0.75%</td>
<td>15.00%</td>
<td>5.00%</td>
</tr>
<tr>
<td>20.00%</td>
<td>25.00%</td>
<td>80.00%</td>
</tr>
<tr>
<td>21.00%</td>
<td>30.00%</td>
<td>70.00%</td>
</tr>
<tr>
<td>26.00%</td>
<td>40.00%</td>
<td>65.00%</td>
</tr>
</tbody>
</table>
If the expected recovery rate is 50% and the annual default losses as a percentage rate relative to the portfolio’s value is 9%, what would be the minimum credit spread that an investor would require if the investor seeks a 5% premium for bearing the risks associated with the portfolio’s various risks?

The total minimum credit spread is 14% found from Equation 22.4 (Credit Spread ≥ 9% + 5% = 14%). The recovery rate of 50% was already included in the annual default loss rate of 9%.

EXPLANATION

We are trying to find the credit spread an investor would be willing to receive in order to compensate them for the risk of default. Using Equation 22.4:

\[
\text{Credit Spread} \geq \text{Loan Loss Rate} + \text{Required Risk Premiums}
\]

In this problem, the annual default loss rate is given as 9% of the portfolio’s value, and the investor requires a 5% risk premium for bearing the risk of that default. Therefore,

\[
14\% \geq 9\% + 5\%
\]

If the credit spread is lower than 14%, the investor would not be properly compensated for taking on these risks. Anything over 14% would be additional yield in excess of what the investor would require.

CALCULATION

Step One: Press 0.09 → + → 0.05
Step Two: Press =
Answer: 0.14

WORKOUT AREA: Here are sample problems — cover one of the values and see if you can solve it using the others

<table>
<thead>
<tr>
<th>Credit Spread (minimum)</th>
<th>Loan Loss Rate</th>
<th>Required Risk Premium</th>
</tr>
</thead>
<tbody>
<tr>
<td>14%</td>
<td>9%</td>
<td>5%</td>
</tr>
<tr>
<td>15%</td>
<td>9%</td>
<td>6%</td>
</tr>
<tr>
<td>18%</td>
<td>10%</td>
<td>8%</td>
</tr>
<tr>
<td>7%</td>
<td>4%</td>
<td>3%</td>
</tr>
<tr>
<td>12%</td>
<td>8%</td>
<td>4%</td>
</tr>
<tr>
<td>13%</td>
<td>10%</td>
<td>3%</td>
</tr>
</tbody>
</table>
APPLICATION 23.5.3A (page 764)

Consider a firm with $50 million in assets and $25 million in equity value. The firm has one debt issue: a zero-coupon bond maturing in one year with a face value of $30 million. A riskless zero-coupon bond of the same maturity sells for 90% of its face value. What is the value of the firm’s debt? What is the value of a one-year put option on the firm’s assets with a strike price of $30 million?

Since the assets are worth $50 million and the equity is worth $25 million, the firm’s risky debt must be worth $25 million, since Assets = Equity + Risky Debt. Since the riskless bond in Equation 23.3 is worth $27 million, the put option must be worth $2 million.

EXPLANATION

The easiest and most important step is recognizing that the value of the firm’s debt is simply the difference between $50 million (the firm’s asset) and $25 million (the firm’s equity value or the value of the call option) for a difference of $25 million. Next, we break the value of the firm’s debt into its two components: the riskless portion and the short put position. So we rearrange equation 23.3 to solve for the value of the put.

\[
\text{Assets} = [\text{Call}] + [\text{Riskless Bond} - \text{Put}]
\]

\[
\text{Assets} - \text{Call} = \text{Debt} = \text{Riskless Bond} - \text{Put}
\]

After rearranging the equation to solve for the value of the put, we can solve:

\[
$25 \text{ million} = \text{Riskless Bond} - \text{Put}
\]

\[
$25 \text{ million} = $27 \text{ million} - \text{Put}
\]

\[
\text{Put} = $2 \text{ million}
\]

The value of the put option is $2 million.
CALCULATIONS

Step One: Press $0.90 \rightarrow x \rightarrow 30,000,000$

Step Two: Press $+ \rightarrow 25,000,000$

Step Three: Press $+ \rightarrow 50,000,000 \rightarrow +|-$

Step Four: Press $=$

Answer: $2,000,000$

WORKOUT AREA: Here are sample problems – cover one of the values and see if you can solve it using the others, especially the second column

<table>
<thead>
<tr>
<th>Value of the Firms Debt (millions)</th>
<th>Value of a One-Year Put Option on the Firm's Assets (millions)</th>
<th>Equity (millions)</th>
<th>Face Value (millions)</th>
<th>Percent of Face Value a One-Year Riskless Zero Coupon Bond Sell For</th>
</tr>
</thead>
<tbody>
<tr>
<td>$25.00</td>
<td>$2.00</td>
<td>$50.00</td>
<td>$25.00</td>
<td>$30.00</td>
</tr>
<tr>
<td>$20.00</td>
<td>$1.25</td>
<td>$25.00</td>
<td>$5.00</td>
<td>$25.00</td>
</tr>
<tr>
<td>$15.00</td>
<td>$4.00</td>
<td>$25.00</td>
<td>$10.00</td>
<td>$20.00</td>
</tr>
<tr>
<td>$25.00</td>
<td>$3.00</td>
<td>$50.00</td>
<td>$25.00</td>
<td>$35.00</td>
</tr>
<tr>
<td>$45.00</td>
<td>$1.80</td>
<td>$75.00</td>
<td>$30.00</td>
<td>$48.00</td>
</tr>
<tr>
<td>$5.00</td>
<td>$0.25</td>
<td>$55.00</td>
<td>$50.00</td>
<td>$7.00</td>
</tr>
<tr>
<td>$10.00</td>
<td>$3.00</td>
<td>$65.00</td>
<td>$55.00</td>
<td>$20.00</td>
</tr>
</tbody>
</table>
APPLICATION 23.5.4A (page 765)

Consider a firm with $100 million in assets and $60 million in equity value. The firm’s debt has a face value of $50 million and a maturity of one year. The volatility of the firm’s equity is estimated at 40%. How would an analyst estimate the value of the firm’s equity if the volatility of the firm’s assets doubled?

Step 2 is based on Equation 23.4 and unlevers the current equity volatility from 40% to an asset volatility of 24% through multiplying the equity volatility (40%) by the ratio of the value of the equity to the value of the assets ($60 million/$100 million, or 0.60). A doubling in the asset volatility increases the asset volatility to 48%. The value of the firm’s equity can be found using an option pricing model for a call option, with an underlying asset value of $100 million, an underlying asset volatility of 48%, a strike price of $50 million, a time to expiration of one year, and the prevailing riskless rate.

EXPLANATION

The idea behind this application is to understand three things: (1) how the Black-Scholes option pricing model can value a firm’s equity as a call option on the firm’s assets, (2) how the asset volatility required in that model can be inferred from the equity volatility, and (3) how changes in the equity volatility would be transmitted through the model into a high equity price.

Let’s look closer at the second step. We need to unlever the current equity volatility from 40% into the corresponding asset volatility. This is done by multiplying the equity volatility by the equity-to-asset ratio (40% x 0.40 for a product of 24%, the asset volatility). The final step is to note that a doubling of the equity volatility transmits into a doubling of the asset volatility: Multiply 24% (the volatility of the firm’s assets) by 2 to get the expected volatility of the firm’s assets as 48%. The value of the firm’s equity with the higher volatility would be solved using the Black-Scholes option pricing model (which is not required in this application).
CALCULATIONS

Find Volatility of Firms Assets

Step One: Press $60 \rightarrow \div \rightarrow 100$
Step Two: Press $x \rightarrow 0.40$
Step Three: Press $=$

Answer: 0.24

Find the Approximate Equity with Expected Asset Volatility

Step One: Press $0.24 \rightarrow x \rightarrow 2 =$

Answer: 0.48 (ready for use in option pricing model)

WORKOUT AREA: Here are sample problems – cover one of the values and see if you can solve it using the others

<table>
<thead>
<tr>
<th>Assets (millions)</th>
<th>Equity (millions)</th>
<th>Volatility of the Firm’s Equity</th>
<th>Volatility of the Firm’s Assets</th>
</tr>
</thead>
<tbody>
<tr>
<td>$100.00</td>
<td>$60.00</td>
<td>40.00%</td>
<td>24.00%</td>
</tr>
<tr>
<td>$75.00</td>
<td>$45.00</td>
<td>35.00%</td>
<td>21.00%</td>
</tr>
<tr>
<td>$50.00</td>
<td>$20.00</td>
<td>30.00%</td>
<td>12.00%</td>
</tr>
<tr>
<td>$150.00</td>
<td>$75.00</td>
<td>25.00%</td>
<td>12.50%</td>
</tr>
<tr>
<td>$200.00</td>
<td>$125.00</td>
<td>45.00%</td>
<td>28.13%</td>
</tr>
<tr>
<td>$250.00</td>
<td>$200.00</td>
<td>50.00%</td>
<td>40.00%</td>
</tr>
<tr>
<td>$300.00</td>
<td>$275.00</td>
<td>60.00%</td>
<td>55.00%</td>
</tr>
</tbody>
</table>
Firm XYZ buys an interest rate floor from Bank DEF. The floor is for three years, has a strike rate of 7%, is settled quarterly, and has a notional value of $10 million. What are the payments, if any, from Bank DEF to Firm XYZ in the first four quarters if the reference rates for those quarters are, respectively, 4%, 6%, 8%, and 10%?

The solution is found using Equation 23.6, with $m = 4$ and the strike rate equal to 7%. For the first quarter, the formula is $(7\% - 4\%) \times \frac{10,000,000}{4}$, which is equal to $75,000. The four answers are $75,000, $25,000, $0, and $0. Note that the formula for a floor generates no payment when the strike rate is equal to or less than the reference rate.

**EXPLANATION**

When solving for a floor payment, remember that an interest rate floor only generates a positive payment if the reference rate is below the strike rate. Otherwise, the payment is $0. To solve this problem, we must use Equation 23.6:

\[
Floor\ Payment = \max[(Strike\ Rate - Reference\ Rate), 0] \times \frac{Notional\ Value}{m}
\]

From here, we can substitute the values for the four strike rates relative to the reference rate of 7%:

\[
Floor\ Payment = \max[(7\% - 4\%), 0] \times \frac{10,000,000}{4} = 75,000
\]

\[
Floor\ Payment = \max[(7\% - 6\%), 0] \times \frac{10,000,000}{4} = 25,000
\]

\[
Floor\ Payment = \max[7\% - 8\%, 0] \times \frac{10,000,000}{4} = 0
\]

\[
Floor\ Payment = \max[7\% - 10\%, 0] \times \frac{10,000,000}{4} = 0
\]

Notice the last two payments are $0. This is because the reference rates are higher than the strike rates.

**CALCULATIONS**

To solve for the 4% reference rate payment:

Step One: Press 0.07 → - → 0.04
Step Two: Press = “0.03”
Step Three: Press x → 10,000,000 → + 4
Step Four: Press =
Answer: 75,000

To solve for the 6% reference rate payment:

Step One: Press 0.07 → - → 0.06
Step Two: Press = “0.01”
Step Three: Press \( x \to 10,000,000 \to + 4 \)
Step Four: Press =
Answer: 25,000

**WORKOUT AREA:** Cover one of the values and solve for it using the other variables.

<table>
<thead>
<tr>
<th>Strike Rate</th>
<th>Reference Rate</th>
<th>Notional Value</th>
<th>m</th>
<th>Floor Payout</th>
</tr>
</thead>
<tbody>
<tr>
<td>7%</td>
<td>6%</td>
<td>$10,000,000.00</td>
<td>4</td>
<td>$25,000.00</td>
</tr>
<tr>
<td>8%</td>
<td>4%</td>
<td>$10,000,000.00</td>
<td>4</td>
<td>$100,000.00</td>
</tr>
<tr>
<td>9%</td>
<td>10%</td>
<td>$10,000,000.00</td>
<td>4</td>
<td>$-</td>
</tr>
<tr>
<td>9%</td>
<td>8%</td>
<td>$10,000,000.00</td>
<td>2</td>
<td>$50,000.00</td>
</tr>
<tr>
<td>4%</td>
<td>3%</td>
<td>$10,000,000.00</td>
<td>2</td>
<td>$50,000.00</td>
</tr>
<tr>
<td>2%</td>
<td>2%</td>
<td>$10,000,000.00</td>
<td>1</td>
<td>$-</td>
</tr>
<tr>
<td>11%</td>
<td>9%</td>
<td>$10,000,000.00</td>
<td>4</td>
<td>$50,000.00</td>
</tr>
</tbody>
</table>
Suppose that the CDO depicted in Exhibit 23.5 alters its portfolio such that the average coupon on the assets is 6%. Ignoring defaults, fees, and expenses, how much annual income should be available to the equity tranche?

The answer is that $3.3 million would go to the senior and mezzanine tranches, and $2.7 million would be available for the equity tranche.

EXPLANATION

The equity tranche receives all the income from the assets that is left over after paying funds due to all the other tranches. Therefore, we need to determine overall CDO income, income to the senior tranche, and income to the mezzanine tranche. To determine the overall income we need to multiple 6% (average yield on assets) by $100 million (total value of assets) for a product of $6 million (net income from the assets of the CDO). Now, the cash flow to the mezzanine and senior tranche is based on their coupons and principal amounts (because there is enough income for both of the tranches. The income to the senior tranche is 3% (yield on senior tranche) multiplied by $70 million for a product of $2.1 million (income to the senior tranche). Now there is $3.9 million of income left for the mezzanine and equity tranches. The income to the mezzanine tranche is 6% (yield on mezzanine tranche) multiplied by $20 million for a product of $1.2 million (income to the mezzanine tranche). That leaves $2.7 million of income for
the equity tranche, which is calculated by subtracting $6 million (income to the CDO) by the sum of $2.1 million (income to the senior tranche) and $1.2 million (income to the mezzanine tranche).

**CALCULATIONS**

Find income to the Equity Tranche

Step One: Press 100 → x → 0.06
Step Two: Press = “6“
Step Three: Press 70 → x → 0.03
Step Four: Press = “2.1“
Step Five: Press 20 → x → 0.06
Step Six: Press = “1.2“
Step Seven: Press 6 → - → 2.1
Step Eight: Press - → 1.2
Step Nine: Press =
Answer: 2.7
APPLICATION 23.7.2A (page 771)

Suppose that the CDO depicted in Exhibit 23.5 experiences defaults in $50 million of the assets with 30% recovery. What will happen to the tranches?

First, note that the 30% recovery reduces the losses to 70% of $50 million ($35 million). After the equity tranche is eliminated due to the first $10 million in defaults, the mezzanine tranche is eliminated due to the next $20 million in defaults. The remaining $5 million of defaults will bring down the notional value of the senior tranche from $70 million to $65 million. The senior tranche has first priority to the recovered value of the bonds ($15 million), which may be distributed to the senior tranche, further reducing its notional value to $50 million.

EXPLANATION

To calculate the recovered assets multiply $50 million (the combined principal values of the bonds experiencing default) by 1 minus 30% (the 30% is the loss given default rate and the 70% is the recovery rate) for a product of $35 million (the recovery on the defaulted assets). Now the equity tranche is the least protected from defaults (first to bear losses) and is currently valued at $10 million. Therefore, the entire equity tranche is eliminated because $35 million (defaulted assets) is greater than $10 million (value of equity tranche). The next subordinated tranche is the mezzanine tranche, which is valued at $20 million. There is $25 million left of default losses to be covered, as we have allocated $10 million to the equity tranche, which eliminated that tranche. Therefore, the mezzanine tranche is also eliminated because $25 million (remaining defaulted assets after the equity tranche) is greater than $20 million (value of mezzanine tranche). There is now $5 million left of the defaults to be covered, which will impact the senior tranche. The senior tranche is valued at $70 million. Thus, we will subtract the $5 million remaining of the defaulted assets by the $70 million value of the senior tranche reducing the value of the senior tranche to $65 million.
CALCULATIONS

Find value after default of each tranche

Step One: Press 1 → - → 0.30
Step Two: Press x → 50
Step Three: Press = “35”
Step Four: Press 35 → - → 10 (eliminated the equity tranche)
Step Five: Press = “25”
Step Six: Press 25 → - → 20 (eliminated the mezzanine tranche)
Step Seven: Press = “5”
Step Eight: Press 70 → - → 5 (reducing the senior tranche)
Step Nine: Press =

Answer: 65 (remaining value in millions of the senior tranche)

WORKOUT AREA: Here are sample problems—cover the values in the four leftmost columns and see if you can solve it using the other columns

<table>
<thead>
<tr>
<th>Value of Equity Tranche</th>
<th>Value of Senior Tranche After Default</th>
<th>Value of Mezzanine Tranche After Default</th>
<th>Total Value of Assets After Default</th>
<th>Start Value of Equity Tranche</th>
<th>Start Value of Senior Tranche</th>
<th>Start Value of Mezzanine Tranche</th>
<th>Total Value of Assets</th>
<th>Principal Value of Defaults</th>
<th>Recovery Rate</th>
<th>Default Losses</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0.00</td>
<td>$65.00</td>
<td>$0.00</td>
<td>$65.00</td>
<td>$10.00</td>
<td>$70.00</td>
<td>$20.00</td>
<td>$100</td>
<td>$50.00</td>
<td>30.00%</td>
<td>$35.00</td>
</tr>
<tr>
<td>$0.00</td>
<td>$15.00</td>
<td>$66.25</td>
<td>$81.25</td>
<td>$15.00</td>
<td>$15.00</td>
<td>$70.00</td>
<td>$100</td>
<td>$25.00</td>
<td>25.00%</td>
<td>$18.75</td>
</tr>
<tr>
<td>$0.00</td>
<td>$70.25</td>
<td>$0.00</td>
<td>$70.25</td>
<td>$20.00</td>
<td>$75.00</td>
<td>$5.00</td>
<td>$100</td>
<td>$35.00</td>
<td>15.00%</td>
<td>$29.75</td>
</tr>
<tr>
<td>$0.00</td>
<td>$50.00</td>
<td>$9.50</td>
<td>$59.50</td>
<td>$25.00</td>
<td>$50.00</td>
<td>$25.00</td>
<td>$100</td>
<td>$45.00</td>
<td>10.00%</td>
<td>$40.50</td>
</tr>
<tr>
<td>$8.00</td>
<td>$33.00</td>
<td>$34.00</td>
<td>$75.00</td>
<td>$33.00</td>
<td>$33.00</td>
<td>$34.00</td>
<td>$100</td>
<td>$50.00</td>
<td>50.00%</td>
<td>$25.00</td>
</tr>
<tr>
<td>$0.00</td>
<td>$64.25</td>
<td>$0.00</td>
<td>$64.25</td>
<td>$10.00</td>
<td>$80.00</td>
<td>$10.00</td>
<td>$100</td>
<td>$55.00</td>
<td>35.00%</td>
<td>$35.75</td>
</tr>
<tr>
<td>$0.00</td>
<td>$10.00</td>
<td>$54.25</td>
<td>$64.25</td>
<td>$15.00</td>
<td>$10.00</td>
<td>$75.00</td>
<td>$100</td>
<td>$65.00</td>
<td>45.00%</td>
<td>$35.75</td>
</tr>
</tbody>
</table>
A bank has extended a $50 million one-year loan at an interest rate of 14% to a client with a BBB credit rating. Suppose that historical data indicate that the one-year probability of default for firms with a BBB rating is 5% and that investors are typically able to recover 40% of the notional value of an unsecured loan to such firms. What is the expected credit loss?

The expected credit loss of the bank is as follows:

\[ PD = 5\% \]
\[ EAD = \$50 \text{ million } \times (1+0.14) = \$57 \text{ million} \]
\[ RR = 0.40 \text{ so that LGD } = 0.60 \]
\[ \text{Expected Credit Loss } = 0.05 \times \$57 \text{ million } \times (1-0.40) = \$1.71 \text{ million} \]

Note that this calculation is an estimate of the average loss. If a default actually occurs, then the loss in this example is 60% × $57 million = $34.2 million.

**EXPLANATION**

The expected credit loss is found by implementing equation 26.1. PD is 5%. Estimated amount of default is $50 million × (1+0.14) or $57 million. The recovery rate is 40%, so the amount not recovered is 1 minus 40% (the recovery rate). Now we can utilize equation 26.1 and solve for the expected credit loss:

\[
\text{Expected Credit Loss} = \text{Probability of Default} \times \text{Estimated Amount of Default} \times (1 - \text{Recovery Rate})
\]

\[
\text{Expected Credit Loss} = 5\% \times \$57 \text{ million } \times (1 - 40\%)
\]
\[
\text{Expected Credit Loss} = \$2.85 \text{ million } \times (1 - 40\%)
\]
\[
\text{Expected Credit Loss} = \$1.71 \text{ million}
\]

The expected credit loss is $1.71 million.

**CALCULATIONS**

Find the Expected Credit Loss

Step One: Press 1 → - → 0.40
Step Two: Press × → 57
Step Three: Press × → 0.05
Step Four: Press =
Answer: 1.71
WORKOUT AREA: Here are sample problems – cover one of the values and see if you can solve it using the others

<table>
<thead>
<tr>
<th>Expected Credit Loss (millions)</th>
<th>Interest Rate</th>
<th>One-Year Loan Value (millions)</th>
<th>Probability of Default</th>
<th>Recovery Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1.71</td>
<td>14.00%</td>
<td>$50.00</td>
<td>5.00%</td>
<td>40.00%</td>
</tr>
<tr>
<td>$4.03</td>
<td>12.00%</td>
<td>$100.00</td>
<td>6.00%</td>
<td>40.00%</td>
</tr>
<tr>
<td>$2.37</td>
<td>13.00%</td>
<td>$50.00</td>
<td>7.00%</td>
<td>40.00%</td>
</tr>
<tr>
<td>$3.43</td>
<td>10.00%</td>
<td>$65.00</td>
<td>8.00%</td>
<td>40.00%</td>
</tr>
<tr>
<td>$3.50</td>
<td>11.00%</td>
<td>$75.00</td>
<td>7.00%</td>
<td>40.00%</td>
</tr>
<tr>
<td>$3.64</td>
<td>19.00%</td>
<td>$85.00</td>
<td>6.00%</td>
<td>40.00%</td>
</tr>
<tr>
<td>$3.24</td>
<td>20.00%</td>
<td>$90.00</td>
<td>5.00%</td>
<td>40.00%</td>
</tr>
</tbody>
</table>
Suppose that the risk-free rate is 5% per year and that a one-year, zero-coupon corporate bond yields 6% per year. What are the precise and approximate risk neutral probabilities of default?

Assuming a recovery rate of 80% on the corporate bond, the precise risk neutral probability of default can be estimated as shown in Equation 24.4:

\[
\lambda = \frac{1}{1 - 0.80} \left( \frac{0.01}{1 + 0.05 + 0.01} \right) = 4.7\%
\]

If the approximation formula (the approximation in Equation 24.5) is used, the risk-neutral probability of default would be 5%, found as 0.01/0.20.

**EXPLANATION**

The approximate risk neutral probability of default found by subtracting 0.05 (the risk-free rate) from 0.06 (the corporate bond yield) and then divide the result (1%) by the loss given default (1.00 minus the recovery rate of 0.80) for an answer of 5%. This is very intuitive. Ignoring a risk premium, a bond with default risk with a 1% higher yield that will lose 20% if default occurs must have a probability of default of only 5% so that on average the bond will pay the same return as the riskless rate.

The precise risk-neutral probability of default is calculated using equation 24.4.

\[
\lambda = \frac{1}{1 - RR} \left( \frac{s}{1 + r + s} \right)
\]

In this equation, RR is the recovery rate (or 80% in this application). s is the credit spread which is the corporate bond yield (6%) minus the risk free rate (5%). r is the risk free rate (5%). Let's solve for \( \lambda \), the precise risk-neutral probability of default.

\[
\lambda = \frac{1}{1 - 0.80} \left( \frac{0.06 - 0.05}{1 + 0.05 + (0.06 - 0.05)} \right)
\]

\[
\lambda = \frac{1}{1 - 0.80} \left( \frac{0.01}{1 + 0.05 + 0.01} \right)
\]

\[
\lambda = \frac{1}{1 - 0.80} \left( \frac{0.01}{1 + 0.05 + 0.01} \right)
\]

\[
\lambda = \frac{1}{0.20} \left( \frac{0.01}{1.06} \right)
\]

\[
\lambda = 5 \left( \frac{0.01}{1.06} \right)
\]

\[
\lambda = 0.04716
\]
The precise risk-neutral probability of default is 4.72%.

CALCULATIONS

Find the approximate risk neutral probability of default

Step One: Press 0.06 → - → 0.05
Step Two: Press = “0.01”
Step Three: Press 1 → - → 0.80
Step Four: Press = “0.20”
Step Five: Press 0.01 → ÷ → 0.20
Step Six: Press =
Answer: 0.05

Find the precise risk-neutral probability of default

Step One: Press 0.06 → - → 0.05
Step Two: Press = “0.01”
Step Three: Press 1 → + → 0.05
Step Four: Press + → 0.01
Step Five: Press = “1.06”
Step Six: Press 1 → - → 0.80
Step Seven: Press = “0.20”
Step Eight: Press 1 → ÷ → 0.20
Step Nine: Press = “5”
Step Ten: Press 0.01 → ÷ → 1.06
Step Eleven: Press x → 5
Step Twelve: Press =
Answer: 0.04716
WORKOUT AREA: Here are sample problems – cover one of the values and see if you can solve it using the others.

<table>
<thead>
<tr>
<th>Approximate Risk-Neutral Probability of Default</th>
<th>Precise Risk-Neutral Probability of Default</th>
<th>Riskless Interest Rate</th>
<th>1 Year Corporate Bond Yield</th>
<th>Recovery Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.00%</td>
<td>4.72%</td>
<td>5.00%</td>
<td>6.00%</td>
<td>80.00%</td>
</tr>
<tr>
<td>20.00%</td>
<td>18.18%</td>
<td>8.00%</td>
<td>10.00%</td>
<td>90.00%</td>
</tr>
<tr>
<td>18.00%</td>
<td>16.82%</td>
<td>2.50%</td>
<td>7.00%</td>
<td>75.00%</td>
</tr>
<tr>
<td>13.33%</td>
<td>12.40%</td>
<td>5.50%</td>
<td>7.50%</td>
<td>85.00%</td>
</tr>
<tr>
<td>50.00%</td>
<td>47.39%</td>
<td>3.00%</td>
<td>5.50%</td>
<td>95.00%</td>
</tr>
<tr>
<td>4.29%</td>
<td>4.08%</td>
<td>3.50%</td>
<td>5.00%</td>
<td>65.00%</td>
</tr>
<tr>
<td>10.00%</td>
<td>9.26%</td>
<td>4.00%</td>
<td>8.00%</td>
<td>60.00%</td>
</tr>
</tbody>
</table>
APPLICATION 24.2.5B (page 780)

Suppose that the risk-neutral probability of default for a bond is 5% per year and that the recovery rate of the bond is 70%. What is the approximate spread by which the bond should trade relative to the yield of a riskless bond?

The approximate credit spread (from Equation 24.6) is $5\% \times (1 - 0.70)$, or 1.5%.

EXPLANATION

Equation 24.6 addresses this mathematical relationship. To solve for the approximate credit spread, subtract 70% from 1 (1 - 0.70) and multiply that difference of 30% by 5% for a product of 1.5%. 1.5% is the approximate credit spread.

CALCULATIONS

Find the approximate credit spread

Step One: Press $1 \rightarrow - \rightarrow 0.70 = 0.03$
Step Two: Press $x \rightarrow 0.05$
Step Three: Press $=\$

Answer: 0.015

WORKOUT AREA: Here are sample problems — cover one of the values and see if you can solve it using the others

<table>
<thead>
<tr>
<th>Credit Spread</th>
<th>Risk-Neutral Probability of Default</th>
<th>Recovery Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.50%</td>
<td>5.00%</td>
<td>70.00%</td>
</tr>
<tr>
<td>0.55%</td>
<td>5.50%</td>
<td>90.00%</td>
</tr>
<tr>
<td>1.63%</td>
<td>6.50%</td>
<td>75.00%</td>
</tr>
<tr>
<td>0.90%</td>
<td>6.00%</td>
<td>85.00%</td>
</tr>
<tr>
<td>0.35%</td>
<td>7.00%</td>
<td>95.00%</td>
</tr>
<tr>
<td>1.40%</td>
<td>4.00%</td>
<td>65.00%</td>
</tr>
<tr>
<td>1.80%</td>
<td>4.50%</td>
<td>60.00%</td>
</tr>
</tbody>
</table>
Suppose that the junior debt of XYZ Corporation is frequently traded and currently trades at a credit spread of 2.50% over riskless bonds of comparable maturity. The senior debt of the firm has not been regularly traded because it was primarily held by a few institutions, and a new issue of debt that is subordinated to all other debt has been rated as speculative. The expected recovery rate of the senior debt is 80%, the old junior debt is 50%, and the recently issued speculative debt is 20%. Using approximation formulas, what arbitrage-free credit spreads should be expected on the senior and speculative debt issues?

The 2.50% credit spread and 50% recovery rate of the junior debt implies a risk-neutral default probability of 5.0% using Equation 24.5. The same risk neutral default probability (in this case, 5%) is then used with recovery rates of 80% and 20% to find credit spreads on the other debt using Equation 24.6. That process generates a credit spread of 1.0% on the senior debt and 4.0% on the speculative debt.

EXPLANATION

To solve for the arbitrage-free credit spreads for the senior and speculative debt issues we need to use equation 24.6. We know that the junior debt with its 2.50% spread and 50% recovery rate implies a risk-neutral probability of default (λ) of 5%. The default rate of both bonds must be equal because they are in the same corporate structure. Because we know the recovery rate for the senior debt is 80%, we can solve for the arbitrage-free credit spreads using 24.6.

Arbitrage-Free Credit Spread for Senior Debt:

\[
s \approx 0.05 \times (1 - 0.80)
\]
\[
s \approx 0.05 \times 0.20
\]
\[
s \approx 0.01
\]

The arbitrage-free credit spread for the senior debt is 1%.

Arbitrage-Free Credit Spread for Speculative Debt:

\[
s \approx 0.05 \times (1 - 0.20)
\]
\[
s \approx 0.05 \times 0.80
\]
\[
s \approx 0.04
\]

The arbitrage-free credit spread for the speculative debt is 4%. 
CALCULATIONS

Find the approximate credit spread for the Senior debt

Step One: Press 1 → - → 0.80
Step Two: Press x → 0.05
Step Three: Press =
Answer: 0.01

Find the approximate credit spread for the Speculative debt

Step One: Press 1 → - → 0.2
Step Two: Press x → 0.05
Step Three: Press =
Answer: 0.043

WORKOUT AREA: Here are sample problems – cover one of the values and see if you can solve it using the others

<table>
<thead>
<tr>
<th>Senior Debt Credit Spread</th>
<th>Speculative Debt Credit Spread</th>
<th>Junior Debt Credit Spread</th>
<th>Approx. Risk-Neutral Probability of Default</th>
<th>Junior Debt Recovery Rate</th>
<th>Senior Debt Recovery Rate</th>
<th>Speculative Debt Recovery Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00%</td>
<td>4.00%</td>
<td>2.50%</td>
<td>5.00%</td>
<td>50.00%</td>
<td>80.00%</td>
<td>20.00%</td>
</tr>
<tr>
<td>2.19%</td>
<td>7.44%</td>
<td>3.50%</td>
<td>8.75%</td>
<td>60.00%</td>
<td>75.00%</td>
<td>15.00%</td>
</tr>
<tr>
<td>1.09%</td>
<td>2.73%</td>
<td>2.00%</td>
<td>3.64%</td>
<td>45.00%</td>
<td>70.00%</td>
<td>25.00%</td>
</tr>
<tr>
<td>2.33%</td>
<td>4.67%</td>
<td>3.00%</td>
<td>6.67%</td>
<td>55.00%</td>
<td>65.00%</td>
<td>30.00%</td>
</tr>
<tr>
<td>2.91%</td>
<td>4.73%</td>
<td>4.00%</td>
<td>7.27%</td>
<td>45.00%</td>
<td>60.00%</td>
<td>35.00%</td>
</tr>
<tr>
<td>1.13%</td>
<td>5.25%</td>
<td>4.50%</td>
<td>7.50%</td>
<td>40.00%</td>
<td>85.00%</td>
<td>30.00%</td>
</tr>
<tr>
<td>0.50%</td>
<td>2.75%</td>
<td>1.50%</td>
<td>5.00%</td>
<td>70.00%</td>
<td>90.00%</td>
<td>45.00%</td>
</tr>
</tbody>
</table>
APPLICATION 24.4.5A (page 788)

On January 1, ABC pension fund enters a one-year swap, agreeing to pay 4.3464% fixed rate on a notional amount of $10 million and receive a floating payment based on three-month LIBOR. Both the fixed and the floating payments will be made on a quarterly basis. The three-month LIBOR rate on January 1 is observed to be 4%. In addition, the interest rate futures market indicates the following rates for the next three quarters: 4.20%, 4.40%, and 4.80%. Calculate the expected payments for the swap. The expected payments for both fixed and floating payments are displayed in Exhibit 24.2.

EXPLANATION

Exhibit 24.2 is displayed below:

<table>
<thead>
<tr>
<th>Quarter Starts</th>
<th>Quarter Ends</th>
<th>Number of Days in Quarter</th>
<th>Current LIBOR</th>
<th>Future LIBOR Rates Start of Quarter</th>
<th>Quarterly Future LIBOR Start of Quarter</th>
<th>Floating Payment End of Quarter</th>
<th>Fixed Payment End of Quarter</th>
</tr>
</thead>
<tbody>
<tr>
<td>January 1</td>
<td>March 31</td>
<td>90</td>
<td>4.00%</td>
<td>1.00%</td>
<td>100,000</td>
<td>108,660</td>
<td></td>
</tr>
<tr>
<td>April 1</td>
<td>June 30</td>
<td>90</td>
<td>4.20%</td>
<td>1.05%</td>
<td>105,000</td>
<td>108,660</td>
<td></td>
</tr>
<tr>
<td>July 1</td>
<td>September 30</td>
<td>90</td>
<td>4.40%</td>
<td>1.10%</td>
<td>110,000</td>
<td>108,660</td>
<td></td>
</tr>
<tr>
<td>October 1</td>
<td>December 31</td>
<td>90</td>
<td>4.80%</td>
<td>1.20%</td>
<td>120,000</td>
<td>108,660</td>
<td></td>
</tr>
</tbody>
</table>

It is important to remember that the floating-rate payments are determined based on the LIBOR rate at the beginning of the quarter. To use the first payment as an example, the January 1 LIBOR rate is 4%, but the payment is not made until March 31. To calculate the payments, we simply multiply the notional value by the interest rates, adjusted for time. These payments are shown above, but for illustration purposes, here is the equation worked out:

\[
$100,000 = \frac{90}{360} \times 4.0\% \times 10,000,000
\]

The fixed-rate payments are similar, although the fixed rate has been determined to be 4.3464%.

\[
$108,660 = \frac{90}{360} \times 4.3464\% \times 10,000,000
\]

CALCULATIONS

For the 4.0% floating rate payment:

Step One: Press 90 → + → 360
Step Two: Press x → 0.04 → x 10,000,000
Step Three: Press=
Answer: 100,000

For the 4.2% floating rate payment:

Step One: Press 90 → ÷ → 360
Step Two: Press x → 0.042 → x 10,000,000
Step Three: Press=
Answer: 105,000

For the 4.0% floating rate payment:

Step One: Press 90 → ÷ → 360
Step Two: Press x → 0.044 → x 10,000,000
Step Three: Press=
Answer: 110,000

For the 4.0% floating rate payment:

Step One: Press 90 → ÷ → 360
Step Two: Press x → 0.048 → x 10,000,000
Step Three: Press=
Answer: 120,000

For the 4.3464% fixed rate payments (note, all four are the same)

Step One: Press 90 → ÷ → 360
Step Two: Press x → 0.043464 → x 10,000,000
Step Three: Press=
Answer: 108,660
APPLICATION 24.4.5B (page 790)

Given the cash flows and interest rates from Exhibit 24.2, calculate the value of the swap as the discounted values of the expected cash flows. To value the expected future cash flows of the swap, it is necessary to specify the discount rate that needs to be applied to future cash flows. It turns out that the interest rates obtained from the futures contracts can provide us with the information needed to calculate these present values. Exhibit 24.2 is based on the figures displayed in Exhibit 24.1, but three new columns have been added and columns 3–5 have been removed because of space concerns. The exhibit displays all the information needed to calculate the present values of the two streams of cash flows.

Note that the sum of the present values of the floating payments in column 10 is equal to the sum of the present values of the fixed payments in column 11. This demonstrates that the swap has an initial value of zero. Note that a twelfth column of the netted expected cash flows could be formed and used to calculate the same net value. The keys to our calculations are the forward discounts that appear in column 9. They are based on the quarterly three-month LIBOR rates that appear in column 6. For instance,

\[
0.990099 = \frac{1}{1 + 1.00\%}
\]

\[
0.979811 = \frac{1}{(1 + 1.00\%) \times (1 + 1.05\%)}
\]

In other words, the denominator of each discount factor is compounded using the current and previous three-month LIBOR rates. The present values of the two streams are calculated using these discount rates. It can be seen that when using 4.3464% as the swap rate, the present values of the two streams are equal when the swap contract is initiated.

EXPLANATION

While this looks complex, this is simply another present value calculation. The difference is that the interest rates are not constant over each period, and so each floating and fixed-rate payment will have be discounted using multiple rates.

The first step would be to take the annualized LIBOR rates of 4.0%, 4.20%, 4.40%, and 4.80% and make them quarterly: 1.00%, 1.05%, 1.10%, and 1.20%. This is because we are only discounting each payment on a quarterly basis.

Next, we must solve for the discount factor that will multiply against the floating- and fixed-rate payments. Each payment away from time 0 will add another interest rate to the denominator:

\[
March \ Payment \ Discount \ Factor = 0.990099 = \frac{1}{1 + 1.00\%}
\]

\[
June \ Payment \ Discount \ Factor = 0.979811 = \frac{1}{(1 + 1.00\%) \times (1 + 1.05\%)}
\]


\[ September \ Payment \ Discount \ Factor = 0.969150 = \frac{1}{(1 + 1.00\%) \times (1 + 1.05\%) \times (1 + 1.10\%)} \]

\[ December \ Payment \ Discount \ Factor = 0.957658 = \frac{1}{(1 + 1.00\%) \times (1 + 1.05\%) \times (1 + 1.10\%) \times (1 + 1.20\%)} \]

The next step is to apply these discount rates to their relevant floating-rate payments:

\[ PV \ of \ March \ Floating \ Payment = 0.990099 \times 100,000 = 99,010 \]

\[ PV \ of \ June \ Floating \ Payment = 0.979811 \times 105,000 = 102,880 \]

\[ PV \ of \ September \ Floating \ Payment = 0.969150 \times 110,000 = 106,607 \]

\[ PV \ of \ December \ Floating \ Payment = 0.957658 \times 120,000 = 114,919 \]

Finally, we must apply these same discount factors to the fixed-rate payments.

\[ PV \ of \ March \ Fixed \ Payment = 0.990099 \times 108,660 = 107,584 \]

\[ PV \ of \ June \ Fixed \ Payment = 0.979811 \times 108,660 = 106,466 \]

\[ PV \ of \ September \ Fixed \ Payment = 0.969150 \times 108,660 = 105,307 \]

\[ PV \ of \ December \ Fixed \ Payment = 0.957658 \times 108,660 = 104,059 \]

Notice, if you separately add up the present values of the fixed payments and the present values of the floating payments, you will see the same answer: $423,416. This is how it should be at the initiation of a fixed-floating swap contract. The value of the swap upon initiation (represented by the difference between the two PV of payments) should be $0.00.

CALCULATIONS

To calculate the present value factors:

**March**

Step One: Press 1 \( \rightarrow \) + \( \rightarrow \) 0.01

Step Two: Press 1/x \( \rightarrow \) =

Answer: 0.990099

**June**

Step One: Press 1 \( \rightarrow \) + \( \rightarrow \) 0.01 \( \rightarrow \) + \( \rightarrow \) 0.0105

Step Two: Press 1/x \( \rightarrow \) =

Answer: 0.979811

**September**

Step One: Press 1 \( \rightarrow \) + \( \rightarrow \) 0.01 \( \rightarrow \) + \( \rightarrow \) 0.0105 \( \rightarrow \) + \( \rightarrow \) 0.011

Step Two: Press 1/x \( \rightarrow \) =

Answer: 0.969150
December
Step One: Press 1 → + → 0.01 → + → 0.0105 → + → 0.011 → + → 0.012
Step Two: Press 1/x → =
Answer: 0.957658

To calculate the PV of floating rate payments

March
Step One: Press 0.990099 → x → 100,000
Step Two: Press =
Answer: 99,010

June
Step One: Press 0.979811 → x → 105,000
Step Two: Press =
Answer: 102,880

September
Step One: Press 0.969150 → x → 110,000
Step Two: Press =
Answer: 106,607

December
Step One: Press 0.957658 → x → 120,000
Step Two: Press =
Answer: 114,919

To calculate the PV of fixed rate payments

March
Step One: Press 0.990099 → x → 108,660
Step Two: Press =
Answer: 107,584

June
Step One: Press 0.979811 → x → 108,660
Step Two: Press =
Answer: 106,466

September
Step One: Press 0.969150 → x → 108,660
Step Two: Press =
Answer: 105,307

December
Step One: Press 0.957658 → x → 108,660
Step Two: Press =
Answer: 104,059
APPLICATION 24.5.2A (page 797)

In this example, a hypothetical transaction takes place between a hedge fund (the Fund) as a credit protection seller and a commercial bank (the Bank) as a credit protection buyer. The reference entity is an airline company (the Firm). The referenced asset is $20 million of face value debt. The term of the transaction is seven years. In exchange for the protection provided over the next seven years, the Fund receives 2% of the notional amount per year, payable quarterly. The contract will be settled physically. This means that if a credit event takes place, the Bank will deliver $20 million in face value of any qualifying senior unsecured paper issued by the Firm in return for a $20 million payment by the Fund. Further, the contract will be terminated, and no further payments will be made by the Bank. Let’s assume that default takes place after exactly three years. What cash flows and exchanges take place?

Each quarter for 12 quarters, the Bank pays the Fund $100,000. This value is found by multiplying the notional amount ($20 million) by the quarterly rate of 0.5% (i.e., 2%/4). When the default occurs, the Bank delivers $20 million in face value of the referenced bond to the Fund in exchange for $20 million in cash. The CDS terminates immediately after these exchanges.

EXPLANATION

The bank pays the Fund $20 million multiplied by 2% (notional amount per year). The value is paid quarterly, so in such an instance we must divide $400,000 (the amount per year) by 4 (number of quarters in a year) for a result of $100,000 per quarter. In this particular scenario since the default takes place in exactly three years after a total of $1.2 million in payments.

Now since there was a default in year three, the bank delivers the defaulted debt to the fund and receives a lump sum payment of $20 million. In total, by year three the fund will have received $1.2 million plus the defaulted debt from the bank. The bank will have paid $1.2 million for protection, will have received a high interest rate while holding the risky debt before it defaulted and then will be able to get rid of the defaulted bond at its face value by delivering the bond to the fund as the provider of the credit protection.
CALCULATIONS

Find the total bank payments over three years

Step One: Press 20 → x → 0.02
Step Two: Press x → 3 (years)
Step Three: Press =
Answer: 1.2

WORKOUT AREA: Here are sample problems—cover one of the values and see if you can solve it using the others

<table>
<thead>
<tr>
<th>Cash Flow From the Bank in First 3 Years Assuming no Default</th>
<th>Face Value of Debt (millions)</th>
<th>Annual Premium for Protection (payable quarterly)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1.20</td>
<td>$20.00</td>
<td>2.00%</td>
</tr>
<tr>
<td>$1.88</td>
<td>$25.00</td>
<td>2.50%</td>
</tr>
<tr>
<td>$2.70</td>
<td>$30.00</td>
<td>3.00%</td>
</tr>
<tr>
<td>$4.73</td>
<td>$45.00</td>
<td>3.50%</td>
</tr>
<tr>
<td>$6.00</td>
<td>$50.00</td>
<td>4.00%</td>
</tr>
<tr>
<td>$8.78</td>
<td>$65.00</td>
<td>4.50%</td>
</tr>
<tr>
<td>$3.38</td>
<td>$75.00</td>
<td>1.50%</td>
</tr>
</tbody>
</table>
APPLICATION 25.4.1A (page 818)

Consider a bank with a $500 million loan portfolio that it wishes to sell. It must hold risk-based capital equal to 8% to support these loans. If the bank sponsors a CDO trust in which the trust purchases the $500 million loan portfolio from the bank for cash, how much reduction in risk-based capital will the bank receive if it finds outside investors to purchase all of the CDO securities? Since the bank no longer has any exposure to the basket of commercial loans, it has now freed $40 million of regulatory capital (8% × $500 million = $40 million) from needing to be held to support these loans.

EXPLANATION

Since the bank sold the entire $500 million loan portfolio to the CDO trust, which is a separate entity, the bank can make available all of the 8% of the risk capital set aside. In that case, we need to multiply 8% by $500 million for a $40 million reduction in risk-based capital.

CALCULATIONS

Find the reduction of risk-based capital

Step One: Press 0.08 → × → 500

Step Two: Press =

Answer: 40

WORKOUT AREA: Here are sample problems — cover one of the values and see if you can solve it using the others

<table>
<thead>
<tr>
<th>Reduction in Risk-Based Capital (millions)</th>
<th>Value of Loan Portfolio (millions)</th>
<th>Percent of Risk Based Capital</th>
</tr>
</thead>
<tbody>
<tr>
<td>$40.00</td>
<td>$500.00</td>
<td>8.00%</td>
</tr>
<tr>
<td>$337.50</td>
<td>$450.00</td>
<td>75.00%</td>
</tr>
<tr>
<td>$36.00</td>
<td>$400.00</td>
<td>9.00%</td>
</tr>
<tr>
<td>$60.00</td>
<td>$600.00</td>
<td>10.00%</td>
</tr>
<tr>
<td>$71.50</td>
<td>$650.00</td>
<td>11.00%</td>
</tr>
<tr>
<td>$42.00</td>
<td>$350.00</td>
<td>12.00%</td>
</tr>
<tr>
<td>$60.00</td>
<td>$300.00</td>
<td>20.00%</td>
</tr>
</tbody>
</table>
APPLICATION 25.4.1B (page 818)

Consider a bank with a $400 million loan portfolio that it wishes to sell. It must hold risk-based capital equal to 8% to support these loans. If the sponsoring bank has to retain a $10 million equity piece in the CDO trust to attract other investors, how much reduction in regulatory capital will result?

Since the bank must take a one-for-one regulatory capital charge ($10 million) for this first-loss position, only $22 million ($32 million — $10 million) of regulatory capital is freed by the CDO trust.

EXPLANATION

The sponsoring bank needs to hold $10 million in equity in order to sell the loan portfolio. Therefore, the bank will be able to free $400 million (loan portfolio) multiplied by 8% (required risk-based capital) minus the $10 million (the required equity piece to sell the loan portfolio), which is equal to $22 million (risk-based capital freed by sale of loan portfolio).

CALCULATIONS

Find the reduction of risk-based capital

Step One: Press 400 → x → 0.08
Step Two: Press - → 10
Step Three: Press =
Answer: 22

WORKOUT AREA: Here are sample problems – cover one of the values and see if you can solve it using the others

<table>
<thead>
<tr>
<th>Reduction in Regulatory Capital (millions)</th>
<th>Value of Loan Portfolio (millions)</th>
<th>Percent of Risk Based Capital</th>
<th>Equity in CDO (millions)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$22.00</td>
<td>$400.00</td>
<td>8.00%</td>
<td>$10.00</td>
</tr>
<tr>
<td>$14.75</td>
<td>$350.00</td>
<td>8.50%</td>
<td>$15.00</td>
</tr>
<tr>
<td>$20.50</td>
<td>$450.00</td>
<td>9.00%</td>
<td>$20.00</td>
</tr>
<tr>
<td>$22.50</td>
<td>$500.00</td>
<td>9.50%</td>
<td>$25.00</td>
</tr>
<tr>
<td>$46.00</td>
<td>$600.00</td>
<td>10.00%</td>
<td>$14.00</td>
</tr>
<tr>
<td>$35.75</td>
<td>$650.00</td>
<td>7.50%</td>
<td>$13.00</td>
</tr>
<tr>
<td>$32.00</td>
<td>$700.00</td>
<td>7.00%</td>
<td>$17.00</td>
</tr>
</tbody>
</table>
APPLICATION 26.2.2A (page 833)

An investor in a 40% tax bracket earns an after-tax return of 9%. What must be the investor’s pre-tax return?

Rearranging Equation 26.1 generates an answer of 15%, found as 9% divided by 0.6.

EXPLANATION

Let’s use equation 26.1 to solve for the investors pre-tax return. First we need to rearrange the formula, since we know the after-tax return, but not the pre-tax return.

\[ r^* = r(1 - T) \]

\[ \frac{r^*}{1 - T} = r \]

Now, we need to plug in the variables: \( T = 0.40 \), and \( r^* = 0.09 \):

\[ \frac{0.09}{1 - 0.40} = r \]

\[ \frac{0.09}{0.60} = r = 0.15 \]

The pre-tax return for the investor is 15%.

CALCULATIONS

Find the pre-tax return to the investor Step

One: Press 1 → - → 0.4
Step Two: Press “0.6”
Step Three: Press 0.09 → ÷ → 0.6 Step
Four: Press =
Answer: 0.15
WORKOUT AREA: Here are sample problems – cover one of the values and see if you can solve it using the others.

<table>
<thead>
<tr>
<th>Pre-Tax Return</th>
<th>Tax Bracket</th>
<th>After-Tax Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>12.40%</td>
<td>40.00%</td>
<td>7.44%</td>
</tr>
<tr>
<td>15.83%</td>
<td>45.00%</td>
<td>8.71%</td>
</tr>
<tr>
<td>18.86%</td>
<td>38.00%</td>
<td>11.69%</td>
</tr>
<tr>
<td>6.21%</td>
<td>35.00%</td>
<td>4.04%</td>
</tr>
<tr>
<td>33.69%</td>
<td>42.00%</td>
<td>19.54%</td>
</tr>
<tr>
<td>27.69%</td>
<td>43.00%</td>
<td>15.79%</td>
</tr>
<tr>
<td>39.90%</td>
<td>50.00%</td>
<td>19.95%</td>
</tr>
</tbody>
</table>
APPLICATION 26.2.2B (page 834)

An investor in a 40% tax bracket on ordinary income invests in a product that earns a pre-tax return of 10%. Sixty percent of the income is distributed as a capital gain that is taxed at 40% of the ordinary income tax rate. What is the investor’s total after-tax return?

The investor’s total after-tax return is the weighted average of the after-tax returns of the return components. Sixty percent of the total return (i.e., 6%) is taxed at a capital gains rate of 16% (found as 40%×40%), leaving an after-tax capital gain return of 5.04%. Forty percent of the total return (i.e., 4%) is taxed at the ordinary rate of 40%, leaving an after-tax ordinary income return of 2.40%. The total weighted average is 7.44%, found as the sum of the two components (5.04% + 2.40%). This can also be found as the pre-tax return of 10% reduced by the weighted average tax rate of 25.6%. The average tax rate of 25.6% reflects the weighted average of 60% of the income being taxed as capital gains at 16%, and 40% of the income being taxed at the ordinary rate of 40%.

EXPLANATION

We need to find the investor’s total after tax return. We are provided with the tax rates, pre-tax returns, and the capital gains proportion. Let’s begin with the capital gains return 60% (pre-tax return proportion of capital gains) of the 10% pre-tax return is from capital gains that needs to be taxed at 16%. Therefore, 60% multiplied by 10% multiplied by 1 minus 16% equals 5.04% of after-tax gains. Now, let’s tackle the non-capital gains portion. Now, 1 minus 60% (pre-tax return proportion of capital gains) or 40% is the proportion of pre-tax returns that is taxed as regular income. Therefore, we need to multiply 40% by 10% (pre-tax return) for a product of 4% multiplied by 1 minus 40% equals 2.4% of after-tax gains. The sum of 2.4% and 5.04% is the total after-tax gains of 7.44%
CALCULATIONS

Find the investor’s total after tax return

Step One: Press $1 \rightarrow - \rightarrow 0.16$
Step Two: Press $\times \rightarrow 0.6$
Step Three: Press $\times \rightarrow 0.1$
Step Four: Press “0.0504”
Step Five: Press $1 \rightarrow - \rightarrow 0.60$
Step Six: Press $\times \rightarrow 0.1$
Step Seven: Press “0.04”
Step Eight: Press $1 \rightarrow - \rightarrow 0.40$
Step Nine: Press “0.60”
Step Ten: Press $0.04 \rightarrow \times \rightarrow 0.60$
Step Eleven: Press “0.024”
Step Twelve: Press $+ \rightarrow 0.0504$
Step Thirteen: Press =

Answer: 0.0744

WORKOUT AREA: Here are sample problems — cover one of the values and see if you can solve it using the others

<table>
<thead>
<tr>
<th>After-Tax Return</th>
<th>Pre-Tax Return</th>
<th>Ordinary Tax Bracket</th>
<th>Capital Gain Proportion Gains</th>
<th>Capital Gain Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.44%</td>
<td>10.00%</td>
<td>40.00%</td>
<td>16.00%</td>
<td>60.00%</td>
</tr>
<tr>
<td>8.71%</td>
<td>12.00%</td>
<td>45.00%</td>
<td>18.00%</td>
<td>65.00%</td>
</tr>
<tr>
<td>11.69%</td>
<td>15.00%</td>
<td>38.00%</td>
<td>15.20%</td>
<td>70.00%</td>
</tr>
<tr>
<td>4.04%</td>
<td>5.00%</td>
<td>35.00%</td>
<td>14.00%</td>
<td>75.00%</td>
</tr>
<tr>
<td>19.54%</td>
<td>25.00%</td>
<td>42.00%</td>
<td>16.80%</td>
<td>80.00%</td>
</tr>
<tr>
<td>15.79%</td>
<td>20.00%</td>
<td>43.00%</td>
<td>17.20%</td>
<td>85.00%</td>
</tr>
<tr>
<td>19.95%</td>
<td>30.00%</td>
<td>50.00%</td>
<td>20.00%</td>
<td>55.00%</td>
</tr>
</tbody>
</table>
APPLICATION 26.2.3A (page 834)

Consider an investor with a current and anticipated tax rate of 30% who anticipates withdrawing funds in 20 years. If the investor places money into a wrap-per that offers tax deferment, how much will the after-tax annual rate of return improve through the use of the wrapper if the pre-tax rate is 8% and the time horizon is 20 years?

The answer is found as follows: \((1 + r)^N\) is the pre-tax future value, 4.661, which generates a taxable income of \(4.661 - 1 = 3.661\). The 3.661 is taxed at 30%, leaving 2.563. Re-adding the principal (1.0) gives an after-tax future value of 3.563 (the value inside the outermost brackets). The 20th root of 3.563 generates 1.0656, from which 1 is subtracted to yield 0.0656. The answer (6.56% interest) improves by 0.96% the 5.60% after-tax return found using the same inputted values in Equation 26.1.

EXPLANATION

There are two components we need to calculate in order to determine the rate of return improvement achieved by using the tax deferment wrapper. First, let’s calculate the simple after-tax rate of return without using the tax deferment wrapper, which uses equation 26.1.

\[
r^* = r(1 - T)
\]

\[
r^* = 0.08(1 - 0.30)
\]

\[
r^* = 0.08(0.70)
\]

\[
r^* = 0.056
\]

The after-tax rate of return without using the tax deferment wrapper is 5.6%. To calculate the after-tax rate of return with the tax wrapper, we need to turn to equation 26.2.

\[
r^* = \left\{1 + [(1 + r)^N - 1](1 - T)\right\}^{\frac{1}{N}} - 1
\]

\[
r^* = \left\{1 + [(1 + 0.08)^{20} - 1](1 - 0.30)\right\}^{\frac{1}{20}} - 1
\]

\[
r^* = \left\{1 + [(1.08)^{20} - 1](0.70)\right\}^{\frac{1}{20}} - 1
\]

\[
r^* = \left\{1 + [4.661 - 1](0.70)\right\}^{\frac{1}{20}} - 1
\]

\[
r^* = \left\{1 + 3.661(0.70)\right\}^{\frac{1}{20}} - 1
\]

\[
r^* = \left\{1 + 2.563\right\}^{\frac{1}{20}} - 1
\]

\[
r^* = \left\{3.563\right\}^{\frac{1}{20}} - 1
\]

295
The after-tax return with the tax wrapper is 6.56%. The improvement is 6.56% minus 5.60% which is a difference of 0.96%.

**CALCULATIONS**

Find the investor’s after tax return improvement using a tax deferment wrapper

Step One: Press $1 \rightarrow - \rightarrow 0.30$

Step Two: Press “0.70“

Step Three: Press $1 \rightarrow + \rightarrow 0.08$

Step Four: Press $y^x \rightarrow 20$

Step Five: Press $- \rightarrow 1$

Step Six: Press $x \rightarrow 0.70$

Step Seven: Press $+ \rightarrow 1$

Step Eight: Press “3.563“

Step Nine: Press $1 \rightarrow + \rightarrow 20$

Step Ten: Press “0.05“

Step Eleven: Press $3.563 \rightarrow y^x \rightarrow 0.05$

Step Twelve: Press $- \rightarrow 1$

Step Thirteen: Press $=$

Answer: 0.0656

**WORKOUT AREA:** Here are sample problems — cover one of the values and see if you can solve it using the others

<table>
<thead>
<tr>
<th>After Tax Rate of Return Improvement</th>
<th>After Tax Return</th>
<th>After-tax Rate of Return with Tax Wrapper</th>
<th>After-Tax Future Value</th>
<th>Pre-Tax Rate</th>
<th>Tax Bracket</th>
<th>Term (years)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.96%</td>
<td>5.60%</td>
<td>6.56%</td>
<td>3.56</td>
<td>8.00%</td>
<td>30.00%</td>
<td>20</td>
</tr>
<tr>
<td>0.81%</td>
<td>6.75%</td>
<td>7.56%</td>
<td>2.98</td>
<td>9.00%</td>
<td>25.00%</td>
<td>15</td>
</tr>
<tr>
<td>2.09%</td>
<td>9.75%</td>
<td>11.84%</td>
<td>3.83</td>
<td>15.00%</td>
<td>35.00%</td>
<td>12</td>
</tr>
<tr>
<td>3.91%</td>
<td>12.00%</td>
<td>15.91%</td>
<td>6.82</td>
<td>20.00%</td>
<td>40.00%</td>
<td>13</td>
</tr>
<tr>
<td>8.31%</td>
<td>13.75%</td>
<td>22.06%</td>
<td>146.03</td>
<td>25.00%</td>
<td>45.00%</td>
<td>25</td>
</tr>
<tr>
<td>0.57%</td>
<td>8.00%</td>
<td>8.57%</td>
<td>2.27</td>
<td>10.00%</td>
<td>20.00%</td>
<td>10</td>
</tr>
<tr>
<td>5.86%</td>
<td>8.50%</td>
<td>14.36%</td>
<td>56.03</td>
<td>17.00%</td>
<td>50.00%</td>
<td>30</td>
</tr>
</tbody>
</table>
APPLICATION 26.2.4A (page 835)

Consider an investor in a current tax rate of 35% who anticipates a reduced tax rate of 20% in 10 years (after retirement). If the investor places money into a wrapper that offers tax deduction and tax deferment, what will the investor’s after-tax rate of annual return be if the pre-tax rate is 6% and the time horizon is 10 years?

The future value (1.791) is multiplied by the after-tax ratio \((1 - 0.20) / (1 - 0.35) = 1.231\) to generate 2.2041. The 10th root of 2.2041 followed with the subtraction of 1 generates the answer that the after-tax rate is 8.22%. Note the dramatic magnitude of the after-tax yield (8.22%), which exceeds the pre-tax yield (6%).

EXPLANATION

We need to solve equation 26.3 to solve for the after-tax rate of annual return with the tax deferment wrapper.

\[
r^* = \left(1 + r\right)^N \left[\frac{1 - T_N}{1 - T_0}\right]^{\frac{1}{N}} - 1
\]

\[
r^* = \left(1 + 0.06\right)^{10} \left[\frac{1 - 0.20}{1 - 0.35}\right]^{\frac{1}{10}} - 1
\]

\[
r^* = \left(1 + 0.06\right)^{10} \left[\frac{0.80}{0.65}\right]^{\frac{1}{10}} - 1
\]

\[
r^* = \{(1.791) [1.231]\}^{\frac{1}{10}} - 1
\]

\[
r^* = \{2.2041\}^{\frac{1}{10}} - 1
\]

\[
r^* = 1.0822 - 1 = 0.0822
\]

The after-tax rate of return with the tax deferment wrapper is 8.22%

CALCULATIONS

Find the investor’s after tax rate of annual return with the tax deferment wrapper Step One: Press \(1 \rightarrow + \rightarrow 0.06\)

Step Two: Press \(y^x \rightarrow 10\)
Step Three: Press = “1.791”
Step Four: Press $y^x \rightarrow 20$

Step Five: Press $1 \rightarrow - \rightarrow 0.20$
Step Six: Press “0.80”
Step Seven: Press $+ \rightarrow 1$
Step Eight: Press $1 \rightarrow - \rightarrow 0.35$
Step Nine: Press “0.65”
Step Ten: Press $0.80 \rightarrow ÷ \rightarrow 0.65$
Step Eleven: Press “1.231”
Step Twelve: Press $1.231 \rightarrow x \rightarrow 1.791$
Step Thirteen: Press “2.2041”
Step Fourteen: Press $1 \rightarrow ÷ \rightarrow 10$
Step Fifteen: Press “0.10”

Step Sixteen: Press $2.2041 \rightarrow y^x \rightarrow 0.10$
Step Seventeen: Press $- \rightarrow 1$
Answer: 0.0822

WORKOUT AREA: Here are sample problems – cover one of the values and see if you can solve it using the others

<table>
<thead>
<tr>
<th>After-Tax Rate of Annual Return</th>
<th>Pre-Tax Future Value</th>
<th>Pre-Tax Rate</th>
<th>Tax Bracket</th>
<th>Term (years)</th>
<th>Reduced Tax Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.22%</td>
<td>1.791</td>
<td>6.00%</td>
<td>35.00%</td>
<td>10</td>
<td>20.00%</td>
</tr>
<tr>
<td>9.40%</td>
<td>2.518</td>
<td>8.00%</td>
<td>40.00%</td>
<td>12</td>
<td>30.00%</td>
</tr>
<tr>
<td>7.03%</td>
<td>2.415</td>
<td>6.50%</td>
<td>30.00%</td>
<td>14</td>
<td>25.00%</td>
</tr>
<tr>
<td>4.87%</td>
<td>2.191</td>
<td>4.00%</td>
<td>45.00%</td>
<td>20</td>
<td>35.00%</td>
</tr>
<tr>
<td>4.07%</td>
<td>3.386</td>
<td>5.00%</td>
<td>25.00%</td>
<td>25</td>
<td>40.00%</td>
</tr>
<tr>
<td>10.31%</td>
<td>1.307</td>
<td>5.50%</td>
<td>32.00%</td>
<td>5</td>
<td>15.00%</td>
</tr>
<tr>
<td>9.41%</td>
<td>1.783</td>
<td>5.40%</td>
<td>37.00%</td>
<td>11</td>
<td>5.00%</td>
</tr>
</tbody>
</table>
APPLICATION 26.3.1A (page 838)

Consider a five-year zero-coupon cash-and-call position on the S&P 500 Index that has an initial cost of $1,000 and offers $1,000 principal protection (ignoring counterparty risk). The product’s payout will be the greater of $1,000 and $1,000*(1 + r), where r is the total return (non-annualized) of the underlying index over the five-year life of the product. If the riskless market interest is 5% (compounded annually), what is the value of the call option and the cash that replicates this product as a cash-and-call strategy (ignoring dividends)?

Assuming that the position is efficiently priced and that the riskless market interest rate is 5% (compounded annually), the present value of the minimum $1,000 payout is $783.53. Thus, the cash position at the start of the investment is $783.53. The remaining value of the structured product ($216.47) is attributable to the call option with a strike price of $1,000.

EXPLANATION

To solve this application, we need to calculate the present value of $1,000 the minimum payout of this strategy. 1 plus 0.05 to the fifth power will be the divisor to the $1,000 numerator for a quotient of $783.53, which is the cash position at the start of the investment or the present value of $1,000 (the minimum payout of this strategy). The remaining value, that is $1,000 minus $783.53 or $216.47, is the value of the call option with a strike price of $1,000.

CALCULATIONS

Step One: Press 1 → + → 0.05

Step Two: Press $y^X$ → 5

Step Three: Press = “1.2763”

Step Four: Press 1000 → + → 1.2763

Step Five: Press = “783.53” (the cash position at the start of the investment)

Step Six: Press 1000 → - → 783.53

Step Seven: Press =

Step Eight: Press “216.47” (the value of the call option with a strike price of $1,000)
**WORKOUT AREA:** Here are sample problems – cover one of the values and see if you can solve it using the others.

<table>
<thead>
<tr>
<th>Value of the Call Option</th>
<th>Value of the Cash</th>
<th>Risk-Less</th>
<th>Principal Protection</th>
</tr>
</thead>
<tbody>
<tr>
<td>$216.47</td>
<td>$783.53</td>
<td>5.0%</td>
<td>$1,000.00</td>
</tr>
<tr>
<td>$469.73</td>
<td>$1,530.27</td>
<td>5.5%</td>
<td>$2,000.00</td>
</tr>
<tr>
<td>$758.23</td>
<td>$2,241.77</td>
<td>6.0%</td>
<td>$3,000.00</td>
</tr>
<tr>
<td>$135.06</td>
<td>$364.94</td>
<td>6.5%</td>
<td>$500.00</td>
</tr>
<tr>
<td>$256.81</td>
<td>$1,043.19</td>
<td>4.5%</td>
<td>$1,300.00</td>
</tr>
<tr>
<td>$169.17</td>
<td>$780.83</td>
<td>4.0%</td>
<td>$950.00</td>
</tr>
<tr>
<td>$241.67</td>
<td>$600.33</td>
<td>7.0%</td>
<td>$842.00</td>
</tr>
</tbody>
</table>
APPLICATION 26.3.5A (page 841)

An asset sells for $100. A European knock-in call option on that asset has a strike price of $110 and a barrier of $90. Describe the option using the terms in Exhibit 26.2 and describe that payoff under each of the following scenarios: (a) the asset moves monotonically to $120; (b) the asset declines monotonically to $89 before rising monotonically to $110 at expiration.

Answer: The option is a down-and-in call option. It pays nothing under scenario (a) because the option never knocks in; it pays nothing under scenario (b) because although the option becomes active, it does not finish in-the-money.

EXPLANATION

For a barrier call option to work it must first reach the barrier and then reach the strike price, then the difference between the current price and the strike price is the payoff. In scenario A, the call option surpassed the strike price, but never declined to $90 to hit the barrier. In scenario B, the option reached the barrier, but expired at-the-money, which implies a $0 payoff.

WORKOUT AREA: Here are sample problems – cover one of the values and see if you can solve it using the others

<table>
<thead>
<tr>
<th>Option Type</th>
<th>Maximum Asset Price</th>
<th>Minimum Asset Price</th>
<th>Final Asset Price</th>
<th>Strike Price</th>
<th>Barrier</th>
<th>Option Payoff</th>
</tr>
</thead>
<tbody>
<tr>
<td>Knock-In call</td>
<td>$120.00</td>
<td>$100.00</td>
<td>$120.00</td>
<td>$110.00</td>
<td>$90.00</td>
<td>$0.00</td>
</tr>
<tr>
<td>Knock-Out call</td>
<td>$120.00</td>
<td>$100.00</td>
<td>$120.00</td>
<td>$110.00</td>
<td>$115.00</td>
<td>$0.00</td>
</tr>
<tr>
<td>Knock-In put</td>
<td>$120.00</td>
<td>$100.00</td>
<td>$80.00</td>
<td>$95.00</td>
<td>$90.00</td>
<td>$15.00</td>
</tr>
<tr>
<td>Knock-Out put</td>
<td>$120.00</td>
<td>$100.00</td>
<td>$80.00</td>
<td>$95.00</td>
<td>$85.00</td>
<td>$0.00</td>
</tr>
<tr>
<td>Knock-In call</td>
<td>$120.00</td>
<td>$100.00</td>
<td>$120.00</td>
<td>$110.00</td>
<td>$115.00</td>
<td>$10.00</td>
</tr>
<tr>
<td>Knock-Out call</td>
<td>$105.00</td>
<td>$85.00</td>
<td>$80.00</td>
<td>$110.00</td>
<td>$75.00</td>
<td>$0.00</td>
</tr>
<tr>
<td>Knock-In put</td>
<td>$105.00</td>
<td>$70.00</td>
<td>$70.00</td>
<td>$95.00</td>
<td>$85.00</td>
<td>$25.00</td>
</tr>
<tr>
<td>Knock-Out put</td>
<td>$110.00</td>
<td>$70.00</td>
<td>$70.00</td>
<td>$95.00</td>
<td>$105.00</td>
<td>$0.00</td>
</tr>
</tbody>
</table>
APPLICATION 26.3.6A (page 842)

Consider two indices: a gold index and a copper index. Consider a European option that pays 0% if the gold index has performance equal to or better than −2% relative to the copper index. For each percentage point that the gold index return is worse than 2% below the copper index, the option pays 1% of its notional value. Describe the type of option and its strike price in terms of both calls and puts.

Answer: The option is a spread option. In the case of a spread put, the strike price of the put is −2%, and the spread is defined as the performance of the gold index less the performance of the copper index. In the case of a spread call, the strike price of the call is +2%, and the spread is defined as the performance of the copper index less the performance of the gold index.

EXPLANATION

This is a spread option because it is based on the relative performance between the copper and gold indices. In the case of a spread put, the strike price of the put is −2%, and the spread is defined as the performance of the gold index less the performance of the copper index. In the case of a spread call, the strike price of the call is +2%, and the spread is defined as the performance of the copper index less the performance of the gold index.
<table>
<thead>
<tr>
<th>Page Number</th>
<th>Section</th>
<th>Keyword</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>1.1</td>
<td>alternative investments</td>
<td><strong>Alternative investments</strong> are sometimes viewed as including any investment that is not simply a long position in traditional investments.</td>
</tr>
<tr>
<td>3</td>
<td>1.1</td>
<td>institutional-quality investment</td>
<td>An <strong>institutional-quality investment</strong> is the type of investment that financial institutions such as pension funds or endowments might include in their holdings because they are expected to deliver reasonable returns at an acceptable level of risk.</td>
</tr>
<tr>
<td>3</td>
<td>1.1</td>
<td>investment</td>
<td>An <strong>investment</strong> is that it is deferred consumption. Any net outlay of cash made with the prospect of receiving future benefits might be considered an investment.</td>
</tr>
<tr>
<td>3</td>
<td>1.1</td>
<td>traditional investments</td>
<td><strong>Traditional investments</strong> include publicly traded equities, fixed-income securities, and cash.</td>
</tr>
<tr>
<td>4</td>
<td>1.2.1</td>
<td>real assets</td>
<td><strong>Real assets</strong> are investments in which the underlying assets involve direct ownership of nonfinancial assets rather than ownership through financial assets, such as the securities of manufacturing or service enterprises.</td>
</tr>
<tr>
<td>5</td>
<td>1.2.1</td>
<td>commodities</td>
<td><strong>Commodities</strong> are homogeneous goods available in large quantities, such as energy products, agricultural products, metals, and building materials.</td>
</tr>
<tr>
<td>5</td>
<td>1.2.1</td>
<td>operationally focused real assets</td>
<td><strong>Operationally focused real assets</strong> include real estate, land, infrastructure, and intellectual property.</td>
</tr>
<tr>
<td>5</td>
<td>1.2.1</td>
<td>real estate</td>
<td><strong>Real estate</strong> focuses on land and improvements that are permanently affixed, like buildings.</td>
</tr>
<tr>
<td>6</td>
<td>1.2.1</td>
<td>farmland</td>
<td><strong>Farmland</strong> consists of land cultivated for row crops (e.g., vegetables and grains) and permanent crops (e.g., orchards and vineyards).</td>
</tr>
<tr>
<td>6</td>
<td>1.2.1</td>
<td>financial asset</td>
<td>A <strong>financial asset</strong> is not a real asset—it is a claim on cash flows, such as a share of stock or a bond.</td>
</tr>
</tbody>
</table>
Infrastructure investments are claims on the income of toll roads, regulated utilities, ports, airports, and other real assets that are traditionally held and controlled by the public sector (i.e., various levels of government).

Land comprises a variety of forms, including undeveloped land, timberland, and farmland.

Timberland includes both the land and the timber of forests of tree species typically used in the forest products industry.

A hedge fund as a privately organized investment vehicle that uses its less regulated nature to generate investment opportunities that are substantially distinct from those offered by traditional investment vehicles, which are subject to regulations such as those restricting their use of derivatives and leverage.

The term private equity is used in the CAIA curriculum to include both equity and debt positions that, among other things, are not publicly traded.

Distressed debt refers to the debt of companies that have filed or are likely to file in the near future for bankruptcy protection.

Mezzanine debt derives its name from its position in the capital structure of a firm: between the ceilings of senior secured debt and the floor of equity.

Structured products are instruments created to exhibit particular return, risk, taxation, or other attributes.

Absolute return products are investment products viewed as having little or no return correlation with traditional assets, and have investment performance that is often analyzed on an absolute basis rather than relative to the performance of traditional investments.

A diversifier is an investment with a primary purpose of contributing diversification benefits to its owner.
Illiquidity means that the investment trades infrequently or with low volume (i.e., thinly).

Lumpy assets are assets that can be bought and sold only in specific quantities, such as a large real estate project.

Efficiency refers to the tendency of market prices to reflect all available information.

Inefficiency refers to the deviation of actual prices from valuations that would be anticipated in an efficient market.

Compensation structure refers to the ways that organizational issues, especially compensation schemes, influence particular investments.

Structuring refers to the partitioning of claims to cash flows through leverage and securitization.

Regulatory factors in the context of investing refer to the role of government, including both regulation and taxation, in influencing the nature of an investment.

Trading strategies refer to the role of an investment vehicle’s investment managers in developing and implementing trading strategies that alter the nature of an investment.

Institutional factors refer to the financial markets (and their policies, such as restrictions on short selling, leverage, and trading) and financial institutions related to a particular investment, such as whether the investment is publicly traded.
Incomplete markets refer to markets with insufficient distinct investment opportunities.

Information asymmetries refer to the extent to which market participants possess different data and knowledge.

Moral hazard is risk that the behavior of one or more parties will change after entering into a contract.

Active management refers to efforts of buying and selling securities in pursuit of superior combinations of risk and return.

Passive investing tends to focus on buying and holding securities in an effort to match the risk and return of a target, such as a highly diversified index.

Innovation is the application of creativity.

Active return is the difference between the return of a portfolio and its benchmark that is due to active management.

Active risk is that risk that causes a portfolio’s return to deviate from the return of a benchmark due to active management.

A benchmark is a performance standard for a portfolio that reflects the preferences of an investor with regard to risk and return.

A benchmark return is the return of the benchmark index or benchmark portfolio.

An absolute return standard means that returns are to be evaluated relative to zero, a fixed rate, or relative to the riskless rate, and therefore independently of performance in equity markets, debt markets, or any other markets.

A relative return standard means that returns are to be evaluated relative to a benchmark.
1.8.3 pure arbitrage

**Pure arbitrage** is the attempt to earn risk-free profits through the simultaneous purchase and sale of identical positions trading at different prices in different markets.

20 1.8.3 return diversifier

If the primary objective of including the product is the reduction in the portfolio’s risk that it is believed to offer through its lack of correlation with the portfolio’s other assets, then that product is often referred to as a **return diversifier**.

20 1.8.3 return enhancer

If the primary objective of including an investment product in a portfolio is the superior average returns that it is believed to offer, then that product is often referred to as a **return enhancer**.

25 2.1.1 buy side

**Buy side** refers to the institutions and entities that buy large quantities of securities for the portfolios they manage.

25 2.1.1 plan sponsor

A **plan sponsor** is a designated party, such as a company or an employer, that establishes a health care or retirement plan (pension) that has special legal or taxation status, such as a 401(k) retirement plan in the United States for employees.

26 2.1.1 endowment

An **endowment** is a fund bestowed on an individual or institution (e.g., a museum, university, hospital, or foundation) to be used by that entity for specific purposes and with principal preservation in mind.

26 2.1.1 family office

A **family office** is a group of investors joined by familial or other ties who manage their personal investments as a single entity, usually hiring professionals to manage money for members of the office.

26 2.1.1 foundation

A **foundation** is a non-for-profit organization that donates funds and support to other organizations for its own charitable purposes.

26 2.1.1 private limited partnerships

**Private limited partnerships** are a form of business organization that potentially offers the benefit of limited liability to the organization’s limited partners (similar to that enjoyed by shareholders of corporations) but not to its general partner.

26 2.1.1 sovereign wealth funds

**Sovereign wealth funds** are state-owned investment funds held by that state’s central bank for the purpose of future generations and/or to stabilize the state currency.

26 2.1.1 separately managed accounts

**Separately managed accounts (SMAs)** are individual investment accounts offered by a brokerage firm and managed by independent investment management firms.

27 2.1.1 40 Act funds

**Mutual funds**, or ‘**40 Act funds**’, are registered investment pools offering their shareholders pro rata claims on the fund’s portfolio of assets.

27 2.1.1 master limited partnerships (MLPs)

**Master limited partnerships (MLPs)** are publicly traded investment pools that are structured as limited partnerships and that offer their owners pro rata claims.
2.1.1 mutual funds

**Mutual funds**, or '40 Act funds, are registered investment pools offering their shareholders pro rata claims on the fund's portfolio of assets.

2.1.2 sell side

**Sell-side** institutions, such as large dealer banks, act as agents for investors when they trade securities.

2.1.2 large dealer banks

**Large dealer banks** are major financial institutions, such as Goldman Sachs, Deutsche Bank, and the Barclays Group, that deal in securities and derivatives.

2.1.2 proprietary trading

**Proprietary trading** occurs when a firm trades securities with its own money in order to make a profit.

2.1.2 back office operations

**Back office operations** play a supportive role in the maintenance of accounts and information systems used to transmit important market and trader information in all trading transactions, as well as in the clearance and settlement of the trades.

2.1.2 front office operations

**Front office operations** involve investment decision-making and, in the case of brokerage firms, contact with clients.

2.1.2 middle office operations

**Middle office operations** form the interface between the front office and the back office, with a focus on risk management.

2.1.3 prime broker

The **prime broker** has the following primary functions: clearing and financing trades for its client, providing research, arranging financing, and producing portfolio accounting.

2.1.3 fund administrator

The **fund administrator** maintains a general ledger account, marks the fund’s books, maintains its records, carries out monthly accounting, supplies its monthly profit and loss (P&L) statements, calculates its returns, verifies asset existence, independently calculates fees, and provides an unbiased, third-party resource for price confirmation on security positions.

2.1.3 financial data providers

**Financial data providers** supply funds primarily with raw financial market data, including security prices, trading information, and indices.
<table>
<thead>
<tr>
<th>Page</th>
<th>Section</th>
<th>Text</th>
</tr>
</thead>
<tbody>
<tr>
<td>31</td>
<td>2.1.3</td>
<td>Financial platforms are systems that provide access to financial markets, portfolio management systems, accounting and reporting systems, and risk management systems.</td>
</tr>
<tr>
<td>31</td>
<td>2.1.3</td>
<td>Financial software may consist of prepackaged software programs and computer languages tailored to the needs of financial organizations. Some funds use open-source software, and others pay licensing fees for proprietary software.</td>
</tr>
<tr>
<td>31</td>
<td>2.1.3</td>
<td>The hedge fund infrastructure may have three main financial components: (1) platforms, (2) software, and (3) data providers.</td>
</tr>
<tr>
<td>31</td>
<td>2.1.3</td>
<td>Consulting conflicts of interest can emerge when consultants are compensated by money managers because this form of payment can detract from the ability to offer independent advice to clients and encourage the consultant to favor the money managers offering compensation.</td>
</tr>
<tr>
<td>32</td>
<td>2.1.3</td>
<td>A commercial bank focuses on the business of accepting deposits and making loans, with modest investment-related services.</td>
</tr>
<tr>
<td>32</td>
<td>2.1.3</td>
<td>Depositories and custodians are very similar entities that are responsible for holding their clients’ cash and securities and settling clients’ trades, both of which maintain the integrity of clients’ assets while ensuring that trades are settled quickly.</td>
</tr>
<tr>
<td>32</td>
<td>2.1.3</td>
<td>Depositories and custodians are very similar entities that are responsible for holding their clients’ cash and securities and settling clients’ trades, both of which maintain the integrity of clients’ assets while ensuring that trades are settled quickly.</td>
</tr>
<tr>
<td>32</td>
<td>2.1.3</td>
<td>The Depository Trust Company (DTC) is the principal holding body of securities for traders all over the world and is part of the Depository Trust and Clearing Corporation (DTCC), which provides clearing, settlement, and information services.</td>
</tr>
<tr>
<td>32</td>
<td>2.1.3</td>
<td>An investment bank focuses on providing sophisticated investment services, including underwriting and raising capital, as well as other activities such as brokerage services, mergers, and acquisitions.</td>
</tr>
<tr>
<td>32</td>
<td>2.1.3</td>
<td>Germany uses universal banking, which means that German banks can engage in both commercial and investment banking.</td>
</tr>
<tr>
<td>33</td>
<td>2.2.1</td>
<td>Limited liability is the protection of investors from losses that exceed their investment.</td>
</tr>
<tr>
<td>33</td>
<td>2.2.1</td>
<td>In the context of limiting liability, passive investments are positions in entities (such as operating firms or investment firms) over which the owner of the position does not exert substantial control and therefore may receive reduced liability exposures and/or passive investment tax treatments.</td>
</tr>
</tbody>
</table>
2.2.1 Probitiy

Probitiy is the quality of exercising strong principles such as honesty, decency, and integrity.

2.2.2 limited liability company (LLC)

A limited liability company (LLC) is a distinct entity: (1) designed to offer its investors (“members”) protection from losses exceeding their investments absent fraud or other activities that could “pierce the veil” between the member’s ownership interest in the LLC and the member’s other holdings, and (2) that does not require that distributions and any other advantages of ownership be made in proportion to each member’s capital contribution to the firm.

2.2.4 special purpose vehicle (SPV)

A special purpose vehicle (SPV) is a legal entity at the heart of a CDO structure that is established to accomplish a specific transaction, such as holding the collateral portfolio.

2.2.5 feeder fund

A feeder fund is a legal structure through which investors have access to the investment performance of the master trust.

2.2.5 master trust

The master trust is the legal structure used to invest the assets of both onshore investors and offshore investors in a consistent if not identical manner, so that both funds share the benefit of the fund manager’s insights.

2.2.4 special purpose entity (SPE)

A special purpose vehicle (SPV) or special purpose entity (SPE) is a legal entity such as an LLC that serves a specific function (such as holding assets), often with the goal of being bankruptcy remote. Master-feeder funds are designed to provide efficient access to investors who are subject to different taxation but wish to invest in the same portfolio.

2.3.1 management company operating agreement

A management company operating agreement is an agreement between members related to a limited liability company and the conduct of its business as it pertains to the law.

2.3.1 partnership agreement

A partnership agreement is a formal written contract creating a partnership.

2.3.1 private-placement memoranda

Private-placement memoranda (a.k.a. offering documents) are formal descriptions of an investment opportunity that comply with federal securities regulations.

2.3.1 subscription agreement

A subscription agreement is an application submitted by an investor who desires to join a limited partnership.

2.3.1 adverse selection

Adverse selection takes place before a transaction is completed, when the decisions made by one party cause less desirable parties to be attracted to the transaction.
The **limited partnership agreement (LPA)** defines its legal framework and its terms and conditions.

A **qualified majority** is generally more than 75% of LPs in contrast to the over 50% required for a simple majority.

The **LP advisory committee (LPAC)**'s responsibilities are defined in the LPA and normally relate to dealing with conflicts of interest, reviewing valuation methods, and any other consents predefined in the LPA.

A **primary market** refers to the methods, institutions, and mechanisms involved in the placement of new securities to investors.

A **secondary market** facilitates trading among investors of previously existing securities.

In major markets, **limit orders** can be placed by market participants to buy securities at specified maximum prices or sell securities at specified minimum prices.

**Securitization** involves bundling assets, especially unlisted assets, and issuing claims on the bundled assets.

The price difference between the highest bid price (the best bid price) and the lowest offer (the best ask price) is the **bid-ask spread**.

**Market making** is a practice whereby an investment bank or another market participant deals securities by regularly offering to buy securities and sell securities.

Market participants that wish to have transactions executed without delay may place **market orders**, which cause immediate execution at the best available price.

Participants that place market orders are **market takers**, which buy at ask prices and sell at bid prices, generally paying the bid-ask spread for taking liquidity.

**Third markets** are regional exchanges where stocks listed in primary secondary markets can also be traded. In the United States, third markets allow brokers and dealers to set up trades away from an exchange by listing their prices on the NASDAQ Intermarket.
Systemic risk is the potential for economy-wide losses attributable to failures or concerns over potential failures in financial markets, financial institutions, or major participants.

Fourth markets are electronic exchanges that allow traders to quickly buy and sell exchange-listed stocks via the electronic

Liquid alternatives are investment vehicles that offer alternative strategies in a form that provides investors with liquidity through opportunities to sell their positions in a market.

Hedge fund replication is the attempt to mimic the returns of an illiquid or highly sophisticated hedge fund strategy using liquid assets and simplified trading rules.

Closed-end mutual fund structures provide investors with relatively liquid access to the returns of underlying assets even when the underlying assets are illiquid.

Progressive taxation places higher-percentage taxation on individuals and corporations with higher incomes.

Section 1256 contracts include many futures and options contracts; have potentially enormous tax advantages in the United States, including having their income treated as 60% long-term capital gain and 40% short-term capital gain regardless of holding period.

Short selling financial assets is the process of borrowing securities from a securities lender, selling the securities at their market price, and eventually purchasing identical securities in the market to extinguish the loan from the securities lender.
Street name refers to the brokerage practice of having the direct legal ownership of customer securities held in the name of the brokerage firm on behalf of the customers rather than having the legal ownership of the shares reside directly with the economic owners of the securities (i.e., the customers of the brokerage firm).

A rebate is a payment of interest to the securities’ borrower on the collateral posted.

Substitute dividends are cash flows paid by share borrowers to share lenders to compensate the lenders for the distributions paid by the corporation while the loan of stock is outstanding.

Dividend irrelevancy is the proposition that, in the absence of imperfections such as income taxation that penalized dividends, the distribution of corporate dividends does not alter shareholder wealth.

A special stock is a stock for which higher net fees are demanded when it is borrowed.

General collateral stocks, which are stocks not facing heavy borrowing demand, may earn a 2% rebate when Treasury bill rates are at 2%, whereas stocks on special may earn zero rebates or even negative rebates, wherein borrowers must pay the lenders money in addition to the interest that the lender is earning on the collateral.

When the inventory of stock available to borrowers becomes extremely tight, short sellers may find their position bought in, meaning the broker revokes the borrowing privilege for that specific stock and requires the trader to cover the short position.

A short squeeze occurs when holders of short positions are compelled to purchase shares at increasing prices to cover their positions due to limited liquidity.
Continuous compounding assumes that earnings can be instantaneously reinvested to generate additional earnings.

Discrete compounding includes any compounding interval other than continuous compounding such as daily, monthly, or annual.

Simple interest is an interest rate computation approach that does not incorporate compounding.

A log return is a continuously compounded return that can be formed by taking the natural logarithm of a wealth ratio: $R_m = \ln(1 + R)$, where $\ln( )$ is the natural logarithm function, $R_m$ is the log return, or continuously compounded return, and $m$ is the number of compounding intervals per year.

The return computation interval for a particular analysis is the smallest time interval for which returns are calculated, such as daily, monthly, or even annually.

Notional principal or notional value of a contract is the value of the asset underlying, or used as a reference to, the contract or derivative position.

The return on notional principal divides economic gain or loss by the notional principal of the contract.

Fully collateralized means that a position (such as a forward contract) is assumed to be paired with a quantity of capital equal in value to the notional principal of the contract.
3.2.3 partially collateralized position

A partially collateralized position has collateral lower in value than the notional value.

3.3.1 internal rate of return (IRR)

The internal rate of return (IRR) can be defined as the discount rate that equates the present value of the costs (cash outflows) of an investment with the present value of the benefits (cash inflows) from the investment.

3.3.3 interim IRR

The interim IRR is a computation of IRR based on realized cash flows from an investment and its current estimated residual value.

3.3.3 lifetime IRR

A lifetime IRR contains all of the cash flows, realized or anticipated, occurring over the investment's entire life, from period 0 to period T.

3.3.3 since-inception IRR

A since-inception IRR is commonly used as a measure of fund performance rather than the performance of an individual investment.

3.4.1 borrowing type cash flow pattern

A borrowing type cash flow pattern begins with one or more cash inflows and is followed only by cash outflows.

3.4.1 complex cash flow pattern

A complex cash flow pattern is an investment involving either borrowing or multiple sign changes.

3.4.1 multiple sign change cash flow pattern

A multiple sign change cash flow pattern is an investment where the cash flows switch over time from inflows to outflows, or from outflows to inflows, more than once.

3.4.2 scale differences

Scale differences are when investments have unequal sizes and/or timing of their cash flows.

3.4.3 aggregation of IRRs

Aggregation of IRRs refers to the relationship between the IRRs of individual investments and the IRR of the combined cash flows of the investments.

3.4.5 modified IRR

The modified IRR approach discounts all cash outflows into a present value using a financing rate, compounds all cash inflows into a future value using an assumed reinvestment rate, and calculates the modified IRR as the discount rate that sets the absolute values of the future value and the present value equal to each other.

3.4.4 reinvestment rate assumption

The reinvestment rate assumption refers to the assumption of the rate at which any cash flows not invested in
Dollar-weighted returns are averaged returns that are adjusted for and therefore reflect when cash has been contributed or withdrawn during the averaging period.

Time-weighted returns are averaged returns that assume that no cash was contributed or withdrawn during the averaging period, meaning after the initial investment.

The distribution to paid-in (DPI) ratio, or realized return, is the ratio of the cumulative distribution to investors to the total capital drawn from investors, and can loosely viewed as a non-annualized measure of income (actually, distributions) in the numerator to total investment in the denominator.

The total value to paid-in (TVPI) ratio, or total return, is a measure of the cumulative distribution to investors plus the total value of the unrealized investments relative to the total capital drawn from investors, and is the sum of the income (DPI) and capital gain or loss (RVPI).

The residual value to paid-in (RVPI) ratio, or unrealized return, at time T is the ratio of the total value of the unrealized investments at time T to the total capital drawn from investors during the previous time periods, and can be loosely viewed as a measure of capital gain or loss, with a ratio of one indicating that, ignoring prior distributions, the investment has neither gained or lost value relative to the total contributions.

The Public Market Equivalent (PME) method uses a publicly traded securities index that is believed to have a similar risk exposure to private equity as a return target and requires or finds the corresponding premium over public equity (e.g., 300 to 500 basis points) for a private equity investment using the investment’s cash contributions (calls), distributions, and terminal value.

The accounting convention of conservatism holds that it is prudent to recognize potential expenses and liabilities as soon as possible but not to similarly anticipate potential revenues or gains, often resulting in an understatement of income and assets in the short run.

The J-curve is the classic illustration of the early losses and later likely profitability of venture capital.

Financial Accounting Standard (FAS) 157, which was introduced in 2006, seeks to require asset managers to regularly value their investments at fair value, even when the valuation is not immediately observable from market prices.

The waterfall is a provision of the limited partnership agreement that specifies how distributions from a fund will be split and how the payouts will be prioritized.
Carried interest is synonymous with an incentive fee or a performance-based fee and is the portion of the profit paid to the GPs as compensation for their services, above and beyond management fees.

A catch-up provision permits the fund manager to receive a large share of profits once the hurdle rate of return has been achieved and passed.

A hurdle rate specifies a return level that LPs must receive before GPs begin to receive incentive fees.

Carried interest is synonymous with an incentive fee or a performance-based fee and is the portion of the profit paid to the GPs as compensation for their services, above and beyond management fees.

Carried interest is synonymous with an incentive fee or a performance-based fee and is the portion of the profit paid to the GPs as compensation for their services, above and beyond management fees.

The term preferred return is often used synonymously with hurdle rate—a return level that LPs must receive before GPs begin to receive incentive fees.

Vesting is the process of granting full ownership of conferred rights, such as incentive fees.

A clawback clause, claw back provision, or claw back option is designed to return incentive fees to LPs when early profits are followed by subsequent losses.

The compensation scheme is the set of provisions and procedures governing management fees, general partner investment in the fund, carried-interest allocations, vesting, and distribution.

A catch-up provision contains a catch-up rate, which is the percentage of the profits used to catch up the incentive fee once the hurdle is met.

Management fees are regular fees that are paid from the fund to the fund managers based on the size of the fund rather than the profitability of the fund.

Management fee offsets occur when all fees earned by general partners would reduce the management fee owed to the GP by the LPs.
Deal-by-deal carried interest is when incentive fees are awarded separately based on the performance of each individual investment.

Carried interest can be fund-as-a-whole carried interest, which is carried interest based on aggregated profits and losses across all the investments, or can be structured as deal-by-deal carried interest.

A hard hurdle rate limits incentive fees to profits in excess of the hurdle rate.

A soft hurdle rate allows fund managers to earn an incentive fee on all profits, given that the hurdle rate has been achieved.

Future possible returns and their probabilities are referred to as expectational or ex ante returns.

Ex post returns are realized outcomes rather than anticipated outcomes.

The normal distribution is the familiar bell-shaped distribution, also known as the Gaussian distribution.

The formal statistical explanation for the idea that a variable will tend toward a normal distribution as the number of independent influences becomes larger is known as the central limit theorem.

A variable has a lognormal distribution if the distribution of the logarithm of the variable is normally distributed.

The most common raw moment is the first raw moment and is known as the mean, or expected value, and is an indication of the central tendency of the variable.

The variance is the second central moment and is the expected value of the deviations squared,

The square root of the variance is a popular and useful measure of dispersion known as the standard deviation: Standard Deviation = \sqrt{\sigma^2} = \sigma

In investment terminology, volatility is a popular term that is used synonymously with the standard deviation of returns.

The skewness is equal to the third central moment divided by the standard deviation of the variable cubed and serves as a measure of asymmetry: Skewness = \frac{E[(R - \mu)^3]}{\sigma^3}
4.2.4 kurtosis

Kurtosis serves as an indicator of the peaks and tails of a distribution. Kurtosis = \( \frac{E[(R - \mu)^4]}{\sigma^4} \)

4.2.4 excess kurtosis

Excess kurtosis provides a more intuitive measure of kurtosis relative to the normal distribution because it has a value of zero in the case of the normal distribution: Excess Kurtosis = \( \{E[(R - \mu)^4]/\sigma^4 \} - 3 \)

4.2.5 leptokurtosis

If a return distribution has positive excess kurtosis, meaning it has more kurtosis than the normal distribution, it is said to be leptokurtic, leptokurtotic, or fat tailed, and to exhibit leptokurtosis.

4.2.5 mesokurtosis

If a return distribution has no excess kurtosis, meaning it has the same kurtosis as the normal distribution, it is said to be mesokurtic, mesokurtotic, or normal tailed, and to exhibit mesokurtosis.

4.2.5 platykurtosis

If a return distribution has negative excess kurtosis, meaning less kurtosis than the normal distribution, it is said to be platykurtic, platykurtotic, or thin tailed, and to exhibit platykurtosis.

4.3.1 covariance

The covariance of the return of two assets is a measure of the degree or tendency of two variables to move in relationship with each other.

4.3.2 correlation coefficient

The correlation coefficient (also called the Pearson correlation coefficient) measures the degree of association between two variables, but unlike the covariance, the correlation coefficient can be easily interpreted.

4.3.2 perfect linear negative correlation

A correlation coefficient of -1 indicates that the two assets move in the exact opposite direction and in the same proportion, a result known as perfect linear negative correlation.

4.3.2 perfect linear positive correlation

A correlation coefficient of +1 indicates that the two assets move in the exact same direction and in the same proportion, a result known as perfect linear positive correlation.

4.3.3 Spearman rank correlation

The Spearman rank correlation is a correlation designed to adjust for outliers by measuring the relationship between variable ranks rather than variable values.

4.3.5 beta

The beta of an asset is defined as the covariance between the asset’s returns and a return such as the market index, divided by the variance of the index’s return, or, equivalently, as the correlation coefficient multiplied by the ratio of the asset volatility to market volatility: \( \beta_i = \frac{\text{Cov}(R_i,R_m)}{\text{Var}(R_m)} = \frac{\sigma_{i,m}}{\sigma_m} \) where \( \beta_i \) is the beta of the returns of asset \( i \) (\( R_i \)) with respect to a market index of returns, \( R_m \).
The autocorrelation of a time series of returns from an investment refers to the possible correlation of the returns with one another through time.

First-order autocorrelation refers to the correlation between the return in time period t and the return in the immediately previous time period, \( t - 1 \).

A partial autocorrelation coefficient adjusts autocorrelation coefficients to isolate the portion of the correlation in a time series attributable directly to a particular higher-order relation.

The Jarque-Bera test involves a statistic that is a function of the skewness and excess kurtosis of the sample: 
\[
JB = \left(\frac{n}{6}\right)\left[S^2 + \left(K^2 - 3\right)\right]
\]
where JB is the Jarque-Bera test statistic, \( n \) is the number of observations, \( S \) is the skewness of the sample, and \( K \) is the excess kurtosis of the sample.

GARCH (generalized autoregressive conditional heteroskedasticity) is an example of a time-series method that adjusts for varying volatility.

Heteroskedasticity is when the variance of a variable changes with respect to a variable, such as itself or time.

Homoskedasticity is when the variance of a variable is constant.

ARCH (autoregressive conditional heteroskedasticity) is a special case of GARCH that allows future variances to rely only on past disturbances, whereas GARCH allows future variances to depend on past variances as well.

Autoregressive refers to when subsequent values to a variable are explained by past values of the same variable.

Conditionally heteroskedastic financial market prices have different levels of return variation even when specified conditions are similar (e.g., when they are viewed at similar price levels).

Informational market efficiency refers to the extent to which asset prices reflect available information.
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<td>semistrong form informational market efficiency</td>
<td>The concept of <strong>semistrong form informational market efficiency</strong> (or semistrong level) refers to market prices reflecting all publicly available information (including not only past prices and volumes but also any publicly available information such as financial statements and other underlying economic data).</td>
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<td>strong form informational market efficiency</td>
<td>The concept of <strong>strong form informational market efficiency</strong> (or strong level) refers to market prices reflecting all publicly and privately available information.</td>
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<td>inflation</td>
<td><strong>Inflation</strong> is the decline in the value of money relative to the value of a general bundle of goods and services.</td>
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<td>5.2.4 / 17.4.2</td>
<td>term structure of interest rates</td>
<td>Sometimes the <strong>term structure of interest rates</strong> is distinguished from the yield curve because the yield curve plots yields to maturity of coupon bonds, whereas the term structure of interest rates plots actual or hypothetical yields of zero-coupon bonds.</td>
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<td>124</td>
<td>5.2.3</td>
<td>anticipated inflation rate</td>
<td>The <strong>anticipated inflation rate</strong> ($i_t$) is generally defined as a measure of the expected rate of change in the value of a currency measured through changes in overall price levels.</td>
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<td>124</td>
<td>5.2.3</td>
<td>real interest rate</td>
<td>The <strong>real interest rate</strong> is the annualized rate earned on default-free fixed-income investments, after adjusting the nominal rate downward for the effect of inflation.</td>
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<td>124</td>
<td>5.2.3</td>
<td>modified Fisher equation</td>
<td>The <strong>modified Fisher equation</strong> expresses the nominal interest rate as the combination of the after-tax real interest rate, $r$, and the anticipated rate of inflation ($i_t$), with an adjustment for the income tax rate, $T$.</td>
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<td>Fisher effect or Fisher equation</td>
<td>The <strong>Fisher effect (or Fisher equation)</strong> states that the nominal interest rate ($r$) is equal to the sum of the real interest rate ($i$) and the expected inflation rate ($i_t$), when interest rates are expressed as continuously compounded rates.</td>
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<td>124</td>
<td>5.2.3</td>
<td>nominal interest rate</td>
<td>A <strong>nominal interest rate</strong> is the rate of return measured in terms of a given currency without a downward adjustment for the potential effects of positive inflation.</td>
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<td>5.2.4</td>
<td>yield to maturity</td>
<td>The <strong>yield to maturity</strong> of a fixed income instrument is the rate that discounts all of the promised cash flows of the instrument into a summed present value that equals the instrument's market price.</td>
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<td>127</td>
<td>5.3.1</td>
<td>unbiased expectations theory</td>
<td>The unbiased expectations theory hypothesizes that all fixed-income securities offer the same expected return over the same time interval (i.e., there are no risk premiums), therefore serving as a useful tool in risk-neutral modeling in which all interest rates are formed purely on interest rate expectations.</td>
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<td>liquidity preference theory</td>
<td>The liquidity preference theory hypothesizes that longer-term fixed-income securities offer higher expected returns over the same time interval as shorter-term bonds, that risk premiums are positive and increasing in the bond’s longevity, that all interest rates are formed based on both interest rate expectations and risk premiums, and that fixed-income management reflects a trade-off between risk and return.</td>
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<td>128</td>
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<td>market segmentation theory</td>
<td>The market segmentation theory hypothesizes that the preferred habitats of borrowers and lenders influence the expected returns of each maturity range, resulting in varying risk premiums and varying expected returns across maturity ranges that form humps and other non-monotonic shapes that are not eliminated by arbitrageurs (because the market is segmented).</td>
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<td>129</td>
<td>5.4.1</td>
<td>implied forward rate</td>
<td>An implied forward rate, ( F(t, T) ), is the annual return between time ( t ) and ( T ) (with ( T &gt; t )) inferred from the term structure of interest rates.</td>
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<td>131</td>
<td>5.5</td>
<td>arbitrage</td>
<td>Arbitrage is the attempt to earn riskless profits (in excess of the risk-free rate) by identifying and trading relatively mispriced assets.</td>
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<td>131</td>
<td>5.4.3</td>
<td>term structure of implied forward rates</td>
<td>The term structure of implied forward rates is the relationship between implied forward rates and the starting point of each rate and is often superimposed on the term structure of spot rates.</td>
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<td>132</td>
<td>5.5.1</td>
<td>arbitrage-free model</td>
<td>An arbitrage-free model is a financial model with relationships derived by the assumption that arbitrage opportunities do not exist, or at least do not persist.</td>
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<td>132</td>
<td>5.5.2</td>
<td>relative pricing model</td>
<td>A relative pricing model prescribes the relationship between two prices.</td>
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<tr>
<td>132</td>
<td>5.5.1</td>
<td>key externality of arbitrage activities</td>
<td>A key externality of arbitrage activities is that they tend to drive similar assets toward similar prices which, in turn, improves global economic decisions.</td>
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<td>133</td>
<td>5.5.2</td>
<td>absolute pricing model</td>
<td>An absolute pricing model attempts to describe a price level based on its underlying economic factors.</td>
</tr>
</tbody>
</table>
The spot market or **cash market** is any market in which transactions involve immediate payment and delivery: The buyer immediately pays the price, and the seller immediately delivers the product.

The **spot market** or cash market is any market in which transactions involve immediate payment and delivery: The buyer immediately pays the price, and the seller immediately delivers the product.

A **binomial tree** projects possible outcomes in a variable such as a security price or interest rate by modeling uncertainty as two movements: an upward movement and a downward movement.

A **recombining binomial tree** has $n + 1$ possible final outcomes for an $n$ period tree, rather than $2^n$ outcomes.

A **risk-neutral model** is a framework for valuing financial derivatives in which risk preferences and probabilities of price changes do not alter the solution and are therefore irrelevant, and in which the analyst selects risk-neutrality as the model’s underlying assumption with regard to risk preferences.

The general definition of **duration** is the elasticity of a bond price with respect to a shift in its yield (or a uniform shift in the spot rates, corresponding to each prospective cash flow).

**Interest rate immunization** is the process of eliminating all interest rate risk exposures.

The **duration of a fixed-coupon bond** is the weighted average of the longevities of the cash flows to a coupon bond where the weight of each of the bond’s cash flows is the proportion of the bond’s total value attributable to that cash flow.

An **asset pricing model** is a framework for specifying the return or price of an asset based on its risk, as well as future cash flows and payoffs.

The **capital asset pricing model (CAPM)** provides one of the easiest and most widely understood examples of single-factor asset pricing by demonstrating that the risk of the overall market index is the only risk that offers a risk premium.

The **market portfolio** is a hypothetical portfolio containing all tradable assets in the world.

The **market weight** of an asset is the proportion of the total value of that asset to the total value of all assets in the market portfolio.

A **single-factor asset pricing model** explains returns and systematic risk using a single risk factor.
Ex ante models, such as ex ante asset pricing models, explain expected relationships, such as expected returns. Ex ante means “from before.”

Idiosyncratic return is the portion of an asset’s return that is unique to an investment and not driven by a common association.

Idiosyncratic risk is the dispersion in economic outcomes caused by investment-specific effects. This section focuses on realized returns and the modeling of risk.

Systematic return is the portion of an asset’s return driven by a common association.

Systematic risk is the dispersion in economic outcomes caused by variation in systematic return.

An ex post model describes realized returns and provides an understanding of risk and how it relates to the deviations of realized returns from expected returns.

The excess return of an asset refers to the excess or deficiency of the asset’s return relative to the periodic risk-free rate.

A forward contract is simply an agreement calling for deferred delivery of an asset or a payoff.

A reference rate is a market rate specified in contracts such as a forward contract that fluctuates with market conditions and drives the magnitude and direction of cash settlements.

A forward rate agreement (FRA) is a cash-settled contract in which one party agrees to offer a specified or fixed rate (the FRA rate), such as an interest rate on a specified principal amount and over a specified time in the future (or a currency exchange rate at a specified time in the future) while the other party agrees to provide that rate.

A swap is a string of forward contracts grouped together that vary by time to settlement.
Financed positions enable economic ownership of an asset without the posting of the purchase price.

The carrying cost is the cost of maintaining a position through time and includes direct costs, such as storage or custody costs, as well as opportunity costs, such as forgone cash flows.

A cost-of-carry model specifies a relationship between two positions that must exist if the only difference between the positions involves the expense of maintaining the positions.

In the context of futures and forward contracts, a cost of carry (or carrying cost) is any financial difference between maintaining a position in the cash market and maintaining a position in the forward market.

Convenience yield, \( y \), is the economic benefit that the holder of an inventory in the commodity receives from directly holding the inventory rather than having a long position in a forward contract on the commodity.

Storage costs of physical commodities involve such expenditures as warehouse fees, insurance, transportation, and spoilage.

The marginal market participant to a derivative contract is any entity with individual costs and benefits that make the entity indifferent between physical positions and synthetic positions.

The outstanding quantity of unclosed contracts is known as open interest.

The term marked-to-market means that the side of a futures contract that benefits from a price change receives cash from the other side of the contract (and vice versa) throughout the contract’s life.

A crisis at maturity is when the party owing a payment is forced at the last moment to reveal that it cannot afford to make the payment or when the party obligated to deliver the asset at the original price is forced to reveal that it cannot deliver the asset.

The collateral deposit made at the initiation of a long or short futures position is called the initial margin.

A maintenance margin requirement is a minimum collateral requirement imposed on an ongoing basis until a position is closed.

A margin call is a demand for the posting of additional collateral to meet the initial margin requirement.

Rolling contracts refers to the process of closing positions in short-term futures contracts and simultaneously replacing the exposure by establishing similar positions with longer terms.
Contracts with longer times to settlement are often called **distant contracts**, deferred contracts, or back contracts.

On an exchange, the futures contract with the shortest time to settlement is often referred to as the **front month contract**.

A short option position that is unhedged is often referred to as a **naked option**.

A **covered call** combines being long an asset with being short a call option on the same asset.

A **protective put** combines being long an asset with a long position in a put option on the same asset.

An option combination in which the long option position is at the higher of two strike prices is a **bear spread**, which offers bearish exposure to the underlying asset that begins at the higher strike price and ends at the lower strike price.

An option combination in which the long option position is at the lower of two strike prices is a **bull spread**, which offers bullish exposure to the underlying asset that begins at the lower strike price and ends at the higher strike price.

An option combination (1) contains either call options or put options (not both), and (2) contains both long and short positions in options with the same underlying asset.

An option straddle is a position in a call and put with the same sign (i.e., long or short), the same underlying asset, the same expiration date, and the same strike price.

An option strangle is a position in a call and put with the same sign, the same underlying asset, the same expiration date, but different strike prices.

Spread positions termed **ratio spreads** can be formed in which the number of options in each position differ.

An option collar generally refers only to the long position in a put and a short position in a call.

A long out-of-the-money call combined with a short out-of-the-money put on the same asset and with the same expiration date is termed a **risk reversal**.

**Put-call parity** is an arbitrage-free relationship among the values of an asset, a riskless bond, a call option, and a put option.
Black-Scholes call option formula expresses the price of a call option as a function of five variables: the price of the underlying asset, the strike price, the return volatility of the underlying asset, the time to the option’s expiration, and the riskless rate.

**Rho** is the sensitivity of an option price with respect to changes in the riskless interest rate.

An **elasticity** is the percentage change in a value with respect to a percentage change in another value.

**Omicron** is the partial derivative of an option or a position containing an option to a change in the credit spread and is useful for analyzing option positions on credit-risky assets.

**Lambda** or **omega** for a call option is the elasticity of an option price with respect to the price of the underlying asset and is equal to delta multiplied times the quantity \( \frac{S}{c} \).

**Target semistandard deviation** (TSSD) is simply the square root of the target semivariance.

**Target semivariance** is similar to semivariance except that target semivariance substitutes the investor’s target rate of return in place of the mean return.
7.1.5 tracking error

**Tracking error** indicates the dispersion of the returns of an investment relative to a benchmark return, where a benchmark return is the contemporaneous realized return on an index or peer group of comparable risk.

7.1.6 drawdown

**Drawdown** is defined as the maximum loss in the value of an asset over a specified time interval and is usually expressed in percentage-return form rather than currency.

7.1.6 maximum drawdown

**Maximum drawdown** is defined as the largest decline over any time interval within the entire observation period.

7.1.7 value at risk

**Value at risk (VaR)** is the loss figure associated with a particular percentile of a cumulative loss function.

7.1.7 conditional value-at-risk

**Conditional value-at-risk (CVaR)**, also known as expected tail loss, is the expected loss of the investor given that the VaR has been equaled or exceeded.

7.2.1 parametric VaR

A VaR computation assuming normality and using the statistics of the normal distribution is known as **parametric VaR**.

7.2.6 Monte Carlo analysis

**Monte Carlo analysis** is a type of simulation in which many potential paths of the future are projected using an assumed model, the results of which are analyzed as an approximation to the future probability distributions.

7.3.1 benchmarking

**Benchmarking**, often referred to as performance benchmarking, is the process of selecting an investment index, an investment portfolio, or any other source of return as a standard (or benchmark) for comparison during performance analysis.

7.3.2 peer group

The **peer group** is typically a group of funds with similar objectives, strategies, or portfolio holdings.

7.3.3 performance attribution

**Performance attribution**, also known as return attribution, is the process of identifying the components of an asset’s return or performance.

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**Performance attribution**, also known as return attribution, is the process of identifying the components of an asset’s return or performance.

7.4.1 Sharpe ratio

The **Sharpe ratio** has excess return as its numerator and volatility as its denominator: \( SR = \frac{E(R_p) - R_f}{\sigma_p} \) where \( SR \) is the Sharpe ratio for portfolio \( p \), \( E(R_p) \) is the expected return for portfolio \( p \), \( R_f \) is the riskless rate, and \( \sigma_p \) is the standard deviation of the returns of portfolio \( p \).

7.4.1 well-diversified portfolio

In the field of investments, the term **well-diversified portfolio** is traditionally interpreted as any portfolio containing only trivial amounts of diversifiable risk.

7.4.3 Treynor ratio

The **Treynor ratio** has excess return as its numerator and beta as the measure of risk as its denominator: \( TR = \frac{E(R_p) - R_f}{\beta_p} \) where \( TR \) is the Treynor ratio for portfolio \( p \), \( E(R_p) \) is
the expected return, or mean return, for portfolio \( p \); \( R_f \) is the riskless rate; and \( \beta_p \) is the beta of the returns of portfolio \( p \).

**7.4.5 Sortino ratio**

The Sortino ratio subtracts a benchmark return, rather than the riskless rate, from the asset’s return in its numerator and uses downside standard deviation as the measure of risk in its denominator: 

\[
\text{Sortino Ratio} = \frac{E(R_p) - R_{Target}}{\text{TSSD}}
\]

where \( E(R_p) \) is the expected return, or mean return in practice, for portfolio \( p \); \( R_{Target} \) is the user’s target rate of return; and TSSD is the target semistandard deviation (or downside deviation).

**7.4.6 Information ratio**

The information ratio has a numerator formed by the difference between the average return of a portfolio (or other asset) and its benchmark, and a denominator equal to its tracking error: 

\[
\text{Information Ratio} = \frac{E(R_p) - R_{Benchmark}}{TE}
\]

where \( E(R_p) \) is the expected or mean return for portfolio \( p \), \( R_{Benchmark} \) is the expected or mean return of the benchmark, and TE is the tracking error of the portfolio relative to its benchmark return.

**7.4.7 Return on VaR (RoVaR)**

Return on VaR (RoVaR) is simply the expected or average return of an asset divided by a specified VaR (expressing VaR as a positive number): 

\[
R_{VaR} = \frac{E(R_p)}{VaR}
\]

**7.5.1 Jensen's alpha**

Jensen's alpha may be expressed as the difference between its expected return and the expected return of efficiently priced assets of similar risk.

**7.5.2 M2 approach**

The M2 approach, or M-squared approach, expresses the excess return of an investment after its risk has been normalized to equal the risk of the market portfolio.

**7.5.3 Average tracking error**

Average tracking error refers to the excess of an investment’s return relative to its benchmark. In other words, it is the numerator of the information ratio.

**8.1.2 Alpha**

Alpha refers to any excess or deficient investment return after the return has been adjusted for the time value of money (the risk-free rate) and for the effects of bearing systematic risk (beta).

**8.2.1 Ex ante alpha**

Ex ante alpha is the expected superior return if positive (or inferior return if negative) offered by an investment on a forward-looking basis after adjusting for the riskless rate and for the effects of systematic risks (beta) on expected returns.

**8.2.2 Ex post alpha**

Ex post alpha is the return, observed or estimated in retrospect, of an investment above or below the risk-free rate and after adjusting for the effects of beta (systematic risks).

**8.3.1 Dependent variable**

The dependent variable is the variable supplied by the researcher that is the focus of the analysis and is determined at least in part by other (independent or explanatory) variables.
Independent variables are those explanatory variables that are inputs to the regression and are viewed as causing the observed values of the dependent variable.

A simple linear regression is a linear regression in which the model has only one independent variable.

A regression is a statistical analysis of the relationship that explains the values of a dependent variable as a function of the values of one or more independent variables based on a specified model.

The intercept is the value of the dependent variable when all independent variables are zero.

The residuals of the regression, \( e_{it} \), reflect the regression’s estimate of the idiosyncratic portion of asset \( i \)'s realized returns above or below its mean idiosyncratic return (i.e., the regression’s estimates of the error term).

The slope coefficient is a measure of the change in a dependent variable with respect to a change in an independent variable.

The goodness of fit of a regression is the extent to which the model appears to explain the variation in the dependent variable.

The \( r \)-squared value of the regression, which is also called the coefficient of determination, is often used to assess goodness of fit, especially when comparing models. In a simple linear regression, the \( r \)-squared is simply the squared value of the estimated correlation coefficient between the dependent variable and the independent variable.

The \( t \)-statistic of a parameter is formed by taking the estimated absolute value of the parameter and dividing by its standard error.

A \( t \)-test is a statistical test that rejects or fails to reject a hypothesis by comparing a \( t \)-statistic to a critical value.

Model misspecification is any error in the identification of the variables in a model or any error in identification of the relationships between the variables.

Omitted (or misidentified) systematic return factors is the failure to include relevant factors in an analysis of returns such as momentum or size.
8.5.2 Misestimated betas

**Missetimated betas** is estimation error due to randomness or econometric errors such as failure to correct for heteroskedasticity.

8.5.2 Nonlinear risk-return relation error

**Nonlinear risk-return relation error** is the failure to model nonlinearity such as quadratic or cubic effects.

8.5.3 beta creep

**Beta creep** is when hedge fund strategies pick up more systematic market risk over time.

8.5.3 beta expansion

**Beta expansion** is the perceived tendency of the systematic risk exposures of a fund or asset to increase due to changes in general economic conditions.

8.5.3 beta nonstationarity

**Beta nonstationarity** is a general term that refers to the tendency of the systematic risk of a security, strategy, or fund to shift through time.

8.5.3 full market cycle

A **full market cycle** is a period of time containing a large representation of market conditions, especially up (bull) markets and down (bear) markets.

8.6 abnormal return persistence

**Abnormal return persistence** is the tendency of idiosyncratic performance in one time period to be correlated with idiosyncratic performance in a subsequent time period.

8.7 return driver

The term **return driver** represents the investments, the investment products, the investment strategies, or the underlying factors that generate the risk and return of a portfolio.

8.7 alpha driver

An investment that seeks high returns independent of the market is an **alpha driver**.

8.7 beta driver

An investment that moves in tandem with the overall market or a particular risk factor is a **beta driver**.

8.7.1 equity risk premium

The **equity risk premium** (ERP) is the expected return of the equity market in excess of the risk-free rate.

8.7.1 equity risk premium puzzle

The **equity risk premium puzzle** is the enigma that equities have historically performed much better than can be explained purely by risk aversion, yet many investors continue to invest heavily in low-risk assets.

8.7.2 linear risk exposure

A **linear risk exposure** means that when the returns to such a strategy are graphed against the returns of the market index or another appropriate standard, the result tends to be a straight line.
Asset gatherers are managers striving to deliver beta as cheaply and efficiently as possible, and include the large-scale index trackers that produce passive products tied to well-recognized financial market benchmarks.

A passive beta driver strategy generates returns that follow the up-and-down movement of the market on a one-to-one basis.

Process drivers are beta drivers that focus on providing beta that is fine-tuned or differentiated.

At one end of the spectrum are product innovators, which are alpha drivers that seek new investment strategies offering superior rates of risk-adjusted return.

Hypotheses are propositions that underlie the analysis of an issue.

The alternative hypothesis is the behavior that the analyst assumes would be true if the null hypothesis were rejected.

The null hypothesis is usually a statement that the analyst is attempting to reject, typically that a particular variable has no effect or that a parameter's true value is equal to zero.

A confidence interval is a range of values within which a parameter estimate is expected to lie with a given probability.

The p-value is a result generated by the statistical test that indicates the probability of obtaining a test statistic by chance that is equal to or more extreme than the one that was actually observed (under the condition that the null hypothesis is true).

The term significance level is used in hypothesis testing to denote a small number, such as 1%, 5%, or 10%, that reflects the probability that a researcher will tolerate of the null hypothesis being rejected when in fact it is true.

The test statistic is the variable that is analyzed to make an inference with regard to rejecting or failing to reject a null hypothesis.

Economic significance describes the extent to which a variable in an economic model has a meaningful impact on another variable in a practical sense.
A **type I error**, also known as a false positive, is when an analyst makes the mistake of falsely rejecting a true null hypothesis.

A **type II error**, also known as a false negative, is failing to reject the null hypothesis when it is false.

**Selection bias** is a distortion in relevant sample characteristics from the characteristics of the population, caused by the sampling method of selection or inclusion.

If the selection bias originates from the decision of fund managers to report or not to report their returns, then the bias is referred to as a **self-selection bias**.

**Survivorship bias** is a common problem in investment databases in which the sample is limited to those observations that continue to exist through the end of the period of study.

**Data dredging**, or data snooping, refers to the overuse and misuse of statistical tests to identify historical patterns.

**Data mining** typically refers to the vigorous use of data to uncover valid relationships.

**Backfill bias fun**, or instant history bias, is when the funds, returns, and strategies being added to a data set are not representative of the universe of fund managers, fund returns, and fund strategies.

**Backfilling** typically refers to the insertion of an actual trading record of an investment into a database when that trading record predates the entry of the investment into the database.

**Backtesting** is the use of historical data to test a strategy that was developed subsequent to the observation of the data.

**Overfitting** is using too many parameters to fit a model very closely to data over some past time frame.

**Cherry-picking** is the concept of extracting or publicizing only those results that support a particular viewpoint.

**Chumming** is a fishing term used to describe scattering pieces of cheap fish into the water as bait to attract larger fish to catch.
An outlier is an observation that is markedly further from the mean than almost all other observations.

The difference between true correlation and causality is that causality reflects when one variable’s correlation with another variable is determined by or due to the value or change in value of the other variable.

The difference between spurious correlation and true correlation is that spurious correlation is idiosyncratic in nature, coincidental, and limited to a specific set of observations.

Natural resources are real assets that have received no or almost no human alteration.

A split estate is when surface rights and mineral rights are separately owned.

A pure play on an investment is an investment vehicle that offers direct exposure to the risks and returns of a specific type of investment without the inclusion of other exposures.

An exchange option is an option to exchange one risky asset for another rather than to buy or sell one asset at a fixed exercise or strike price.

A perpetual option is an option with no expiration date.

The low-hanging-fruit principle states that the first action that should be taken is the one that reaps the highest benefits over costs.

An intrinsic option value is the greater of $0 and the value of an option if exercised immediately.

The time value of an option is the excess of an option’s price above its intrinsic value.

Land banking is the practice of buying vacant lots for the purpose of development or disposition at a future date.

Blue top lots are at an interim stage of lot completion. In this case, the owner has completed the rough grading of the property and the lots, including the undercutting of the street section, interim drainage, and erosion control facilities, and has paid all applicable fees required.
Finished lots are fully completed and ready for home construction and occupancy.

Paper lots refer to sites that are vacant and approved for development by the local zoning authority but for which construction on streets, utilities, and other infrastructure has not yet commenced.

Binomial option pricing is a technique for pricing options that assumes that the price of the underlying asset can experience only a specified upward movement or downward movement during each period.

A risk-neutral probability is a probability that values assets correctly if, everything else being equal, all market participants were risk neutral.

A negative survivorship bias is a downward bias caused by excluding the positive returns of the properties or other assets that successfully left the database.

Timberland investment management organizations (TIMOs) provide management services to timberland owned by institutional investors, such as pension plans, endowments, foundations, and insurance companies.

Rotation is the length of time from the start of the timber (typically the planting) until the harvest of the timber.

Reduced integration in the forest products industry refers to the increased separation of ownership of trees, pulp mills, and sawmills and is a key reason for changes in timberland ownership.

In real estate, the cap rate (capitalization rate) or yield is a common term for the return on assets (7.33% in this example).

Agency risk is the economic dispersion resulting from the consequences of having another party (the agent) making decisions contrary to the preferences of the owner (the principal).

Political risk is economic uncertainty caused by changes in government policy that may affect returns, perhaps dramatically.

Row cropland is annual cropland that produces row crops, such as corn, cotton, carrots, or potatoes from annual seeds.

Permanent cropland refers to land with long-term vines or trees that produce crops, such as grapes, cocoa, nuts, or fruit.
Agronomy is the science of soil management, cultivation, crop production, and crop utilization.

Smoothing is reduction in the reported dispersion in a price or return series.

A favorable mark is a biased indication of the value of a position that is intentionally provided by a subjective source.

Managed returns are returns based on values that are reported with an element of managerial discretion.

Model manipulation is the process of altering model assumptions and inputs to generate desired values and returns.

Selective appraisals refers to the opportunity for investment managers to choose how many, and which, illiquid assets should have their values appraised during a given quarter or some other reporting period.

Market manipulation refers to engaging in trading activity designed to cause the markets to produce favorable prices for thinly traded listed securities.

Stale prices are indications of value derived from data that no longer represent current market conditions.

Contagion is the general term used in finance to indicate any tendency of major market movements—especially declines in prices or increases in volatility—to be transmitted from one financial market to other financial markets.

Hotelling's theory states that prices of exhaustible commodities, such as various forms of energy and metals, should increase by the prevailing nominal interest rate—perhaps with a risk premium.

A commodity-linked note (CLN) is an intermediate-term debt instrument whose value at maturity is a function of the value of an underlying commodity or basket of commodities.

Spoilage cost is the loss of value that may naturally occur through time during storage due to physical deterioration.

Inventory shrinkage is loss of inventory through time due to theft, decline in moisture content, and so forth.

With regard to supply, on one end of the spectrum is a perfectly elastic supply, in which any quantity demanded of a commodity can be instantaneously and limitlessly supplied without changes in the market price.

Inelastic supply is when supplies change slowly in response to market prices or when large changes in market prices are necessary to effect supply changes.
When the slope of the term structure of forward prices is negative, the market is in **backwardation**, or is backwardated.

When the term structure of forward prices is upward sloping (i.e., when more distant forward contracts have higher prices than contracts that are nearby), the market is said to be in **contango**.

**Inelastic demand** is a market condition in which the demand for a good does not increase or decrease substantially due to changes in price and therefore is a potential cause of higher price volatility and higher convenience yield.

The **basis** in a forward contract is the difference between the spot (or cash) price of the referenced asset, S, and the price (F) of a forward contract with delivery T.

**Basis risk** is the dispersion in economic returns associated with changes in the relationship between spot prices and futures prices.

A **calendar spread** can be viewed as the difference between futures or forward prices on the same underlying asset but with different settlement dates.

The return generated exclusively from changes in futures prices is known as the **excess return of a futures contract**.

A **fully collateralized position** is a position in which the cash necessary to settle the contract has been posted in the form of short-term, riskless bonds.

**Collateral yield** is the interest earned from the riskless bonds or other money market assets used to collateralize the futures contract.

**Roll yield** or **roll return** is properly defined as the portion of the return of a futures position from the change in the contract's basis through time.

**Roll yield** or **roll return** is properly defined as the portion of the return of a futures position from the change in the contract's basis through time.

**Spot return** is the return on the underlying asset in the spot market.
A heterogeneous value differs across one or more dimensions.

In normal backwardation, the forward price is believed to be below the expected spot price.

In normal contango, the forward price is believed to be above the expected spot price.

Stock-out occurs when storage effectively drops to zero, resulting in consumption being entirely dependent on production and transportation networks, and typically occurs in markets with peak seasonal demand, such as natural gas or heating oil, or with annual crop cycles, such as grains.

The theory of storage is illustrated in the Working curve positively relates the slope of the forward curve to current levels of inventory such that low inventory levels tend to be associated with a negative and nonlinear forward curve slope.

The crude oil futures market has often exhibited a humped curve, which in the typical case of commodity futures means that the market is in contango in the short term, but gives way to backwardation for longer-maturity contracts.

A nominal price refers to the stated price of an asset measured using the contemporaneous values of a currency.

A real price refers to the price of an asset that is adjusted for inflation through being expressed in the value of currency from a different time period.

A volatility asymmetry is a difference in values between two analogous volatilities, such as is the case with commodities, in which volatility tends to be higher when prices are rising than when they are falling.

Inflation risk is the dispersion in economic outcomes caused by uncertainty regarding the value of a currency.

An investable index has returns that an investor can match in practice by maintaining the same positions that constitute the index.

A production-weighted index weights each underlying commodity using estimates of the quantity of each commodity produced.

The Standard & Poor’s GSCI (S&P GSCI) is a long-only index of physical commodity futures.
The Bloomberg Commodity Index (BCOM), formerly the Dow Jones-UBS Commodity Index, is a long-only index composed of futures contracts on 22 physical commodities.

The Thomson Reuters CoreCommodity Research Bureau (CRB) Index is the oldest major commodity index and is currently made up of 19 commodities traded on various exchanges.

Downstream operations focus on refining, distributing, and marketing the oil and gas.

Upstream operations focus on exploration and production; midstream operations focus on storing and transporting the oil and gas.

Midstream operations and midstream MLPs—the largest of the three segments—process, store, and transport energy and tend to have little or no commodity price risk.

Double taxation is the application of income taxes twice: taxation of profits at the corporate income tax level and taxation of distributions at the individual income tax level.

Unrelated business income tax (UBIT) is the income generated through businesses unrelated to the tax-exempt purpose of the organization.

Investable infrastructure is typically differentiated from other assets with seven primary characteristics: (1) public use, (2) monopolistic power, (3) government related, (4) essential, (5) cash generating, (6) conducive to privatization of control, and (7) capital intensive with long-term horizons.

Investable infrastructure can also be an existing project, or brownfield project, that has a history of operations and may have converted from a government asset into something privately investable.

Investable infrastructure can originate as a new, yet-to-be-constructed project, referred to as a greenfield project, which was designed to be investable.

The critical property of infrastructure (i.e., the most important distinction between investable infrastructure and traditional investments) is in the nature of the revenues, with investable infrastructure generating a cash flow stream in a monopolistic environment rather than in a competitive environment.
When a governmental entity sells a public asset to a private operator, this is termed **privatization**.

A **public-private partnership (PPP)** occurs when a private sector party is retained to design, build, operate, or maintain a public building (e.g., a hospital), often for a lease payment for a prespecified period of time.

**Social infrastructure** assets are assets that have end users who are unable to pay for the services or that are used in such a way that it is difficult to determine how many services were used by each person; examples include schools, public roads, prisons, administrative offices, and other government buildings. **Regulatory risk** is the economic dispersion to an investor from uncertainty regarding governmental regulatory actions.

**Regulated pricing** occurs in most countries and the pricing for goods and services deemed essential is largely determined by price changes that must be approved by public entities and are most common in the energy sector.

The **greenfield phase** covers the initial stages of infrastructure, from (1) building and development (including the design), to (2) the construction of the project itself, and (3) the project’s ramp-up period (i.e., its start-up).

The **brownfield phase** involves operations and takes place when assets are already constructed.

**Political infrastructure risk** includes regulatory risk and nonregulatory risks, such as the risk of expropriation.

Unlisted open-end funds, also called **evergreen funds**, allow investors to subscribe to or redeem from these funds on a regular basis.

**Closed-end infrastructure funds** are typically structured like private equity funds, have a life of typically 10 to 15 years, and draw down investor capital commitments over a stated investment period of four to five years.

**Gates** are fund restrictions on investor withdrawals.

An **excludable good** is a good others can be prevented from enjoying.
Intangible assets are economic resources that do not have a physical form.

Intellectual property (IP) is an intangible asset that can be owned, such as copyrighted artwork.

Unbundled intellectual property is IP that may be owned or traded on a standalone basis.

Mature intellectual property is IP that has developed and established a reliable usefulness and will have a more certain valuation and a more clear ability to generate licensing, royalty, or other income associated with its use.

Negative costs refer not to the sign of the values but to the fact that these are costs required to produce what was, in the predigital era, the film’s negative image.

Visual works of art include paintings and have a rich history of prices and returns, but they do not represent a major component of institutional portfolios.

The aesthetic benefit is the nonfinancial benefit to owning art and includes the joy of viewing and otherwise controlling the art.

A mortgage loan can be simply defined as a loan secured by property.

Seven challenges to international real estate investing include: (1) a lack of knowledge and experience regarding foreign real estate markets, (2) a lack of relationships with foreign real estate managers, (3) the time and expense of travel for due diligence, (4) liquidity concerns, (5) political risk (particularly in emerging markets), (6) risk management of foreign currency exposures, and (7) taxation differences.

Commercial real estate properties include the following property sectors: office buildings, industrial centers, data centers, retail (malls and shopping centers, also referred to as “strips”), apartments, health-care facilities (medical office buildings and assisted-living centers), self-storage facilities, and hotels.

Residential real estate or housing real estate includes many property types, such as single-family homes, townhouses, condominiums, and manufactured housing.

Private real estate equity investment involves the direct or indirect acquisition and management of actual physical properties that are not traded on an exchange.

Private real estate is also known as physical, direct, or non-exchange-traded real estate, and may take the form of equity through direct ownership of the property or debt via mortgage claims on the property.

Public real estate investment entails the buying of shares of real estate investment companies and investing in other indirect exchange-traded forms of real estate (including futures...
and options on real estate indices and exchange-traded funds linked to real estate).

The **private real estate market** comprises several segments: housing or residential real estate properties, commercial real estate properties, farmland, and timberland.

Real estate assets are said to trade in a **primary real estate market** if the geographic location of the real estate is in a major metropolitan area of the world, with numerous large real estate properties or a healthy growth rate in real estate projects with easily recognizable names.

**Lumpiness** describes when assets cannot be easily and inexpensively bought and sold in sizes or quantities that meet the preferences of the buyers and sellers.

**Core real estate** includes assets that achieve a relatively high percentage of their returns from income and are expected to have low volatility.

**Styles of real estate investing** refer to the categorization of real estate property characteristics into core, value added, and opportunistic.

**Opportunistic real estate** properties are expected to derive most or all of their returns from property appreciation and may exhibit substantial volatility in value and returns.

**Value-added real estate** includes assets that exhibit one or more of the following characteristics: (1) achieving a substantial portion of their anticipated returns from appreciation in value, (2) exhibiting moderate volatility, and (3) not having the financial reliability of core properties.

**Real estate style boxes** use two categorizations of real estate to generate a box or matrix that can be used to characterize properties or portfolios.

**Fixed-rate mortgage** has interest charges and interest payments based on a single rate established at the initiation of the mortgage.

**Residential mortgage loans** are typically taken out by individual households on properties that generate no explicit rental income, since the houses are usually owner occupied.

**Variable-rate mortgage** has interest charges and interest payments based on a rate that is allowed to vary over the life of the mortgage based on terms established at the initiation of the mortgage.

**Amortization**. Reduction in principal due to payments is known as **amortization**.
An asset is **fully amortized** when its principal is reduced to zero.

If the borrower makes **unscheduled principal payments**, which are payments above and beyond the scheduled mortgage payments, the mortgage’s balance will decline more quickly than illustrated in Exhibit 14.1, and the mortgage will terminate early. In traditional mortgages, payments that exceed the required payment reduce the principal payment but do not lower required subsequent payments until the mortgage is paid off.

The ability of the borrower to make or not make unscheduled principal payments is an option to the borrower: the borrower’s **prepayment option**.

An **index rate** is a variable interest rate used in the determination of the mortgage’s stated interest rate.

A **margin rate** is the spread by which the stated mortgage rate is set above the index rate. (This should not be confused with the same term used to describe a rate associated with margin debt in a brokerage account.)

An **interest rate cap** is a limit on interest rate adjustments used in mortgages and derivatives with variable interest rates.

A **balloon payment** is a large scheduled future payment.

**Negative amortization** occurs when the interest owed is greater than the payments being made such that the deficit is added to the principal balance on the loan, causing the principal balance to increase through time.

**Option adjustable-rate mortgage (option ARM)** is an adjustable-rate mortgage that provides borrowers with the flexibility to make one of several possible payments on their mortgage every month.

Uninsured mortgages with borrowers of relatively high credit risk are generally known as **subprime mortgages**.

**Prime mortgages** are offered to borrowers with lower levels of credit risk and higher levels of creditworthiness.

**Commercial mortgage loans** are loans backed by commercial real estate (multifamily apartments, hotels, offices, retail and industrial properties) rather than owner-occupied residential properties.
The **loan-to-value ratio (LTV ratio)** is the ratio of the amount of the loan to the value (either market or appraised) of the property.

**Covenants** are promises made by the borrower to the lender, such as requirements that the borrower maintain the property in good repair and continue to meet specified financial conditions.

In order to mitigate the risk to which they are exposed, lenders commonly use a **cross-collateral provision**, wherein the collateral for one loan is used as collateral for another loan.

**Recourse** is the set of rights or means that an entity such as a lender has in order to protect its investment.

A related measure is the **debt service coverage ratio (DSCR)**, which is the ratio of the property’s net operating income to all loan payments, including the amortization of the loan.

The **fixed charges ratio** is the ratio of the property’s net operating income to all fixed charges that the borrower pays annually.

Lenders typically examine the **interest coverage ratio**, which can be defined as the property’s net operating income divided by the loan’s interest payments.

**Mortgage-backed securities (MBS)** are a type of asset-backed security that is secured by a mortgage or pool of mortgages.

**Collateralized mortgage obligations (CMOs)** extend this MBS mechanism to create different security classes, called tranches, which have different priorities to receiving cash flows and therefore different risks.

A **pass-through MBS** is perhaps the simplest MBS and consists of the issuance of a homogeneous class of securities with pro rata rights to the cash flows of the underlying pool of mortgage loans.

The **residential mortgage-backed securities (RMBS)** market is backed by residential mortgage loans.

The annualized percentage of a mortgage’s remaining principal value that is prepaid in a particular month is known as the **conditional prepayment rate (CPR)**.

The Public Securities Association (PSA) established the **PSA benchmark**, a benchmark of prepayment speed that is based on the CPR and that has become the standard approach used by market participants.
Factors affecting prepayment decisions other than interest rates or other systematic factors are known as **idiosyncratic prepayment factors**.

Reduced refinancing speeds due to high levels of previous refinancing activity is known as **refinancing burnout**.

**Commercial mortgage-backed securities (CMBS)** are mortgage-backed securities with underlying collateral pools of commercial property loans.

Equity **REITs** invest predominantly in equity ownership within the private real estate market.

Mortgage **REITs** invest predominantly in real estate–based debt.

**A real estate investment trust (REIT)** is an entity structured much like a traditional operating corporation, except that the assets of the entity are almost entirely real estate.

**Real estate development projects** can include one or more stages of creating or improving a real estate project, including the acquisition of raw land, the construction of improvements, and the renovation of existing facilities.

**A real option** is an option on a real asset rather than a financial security.

**A decision tree** shows the various pathways that a decision maker can select as well as the points at which uncertainty is resolved.

An **information node** denotes a point in a decision tree at which new information arrives.

**Backward induction** is the process of solving a decision tree by working from the final nodes toward the first node, based on valuation analysis at each node.

**A decision node** is a point in a decision tree at which the holder of the option must make a decision.
Real estate valuation is the process of estimating the market value of a property and should be reflective of the price at which informed investors would be willing to both buy and sell that property.

The comparable sale prices approach values real estate based on transaction values of similar real estate, with adjustments made for differences in characteristics.

A real estate appraisal is generally viewed as a formal opinion of a value provided by an appraiser and often used in financial reports and in decision-making including lending.

The profit approach to real estate valuation is typically used for properties with a value driven by the actual business use of the premises; it is effectively a valuation of the business rather than a valuation of the property itself.

The cost approach assumes that a buyer will not pay more for a property than it would cost to build an equivalent one.

The income approach values real estate by projecting expected income or cash flows, discounting for time and risk, and summing them to form the total value.

Transaction-based real estate valuation methods are based on relatively large data sets of actual transaction prices of properties within a specified time period and include the repeat-sales and hedonic methods.

A stale appraisal effect comes from the use of dated appraisals.

The NCREIF Property Index (NPI) is the primary example of an appraisal-based real estate index in the United States and is published by the National Council of Real Estate Investment Fiduciaries (NCREIF), a not-for-profit industry association that collects data regarding property values from its members.

The calculation of the returns to the NPI is performed on a before-tax basis (and therefore do not include income tax expense) and is performed for each individual property and then value-weighted in the index calculation.

The income approach is also known as the discounted cash flow (DCF) method when cash flows are discounted rather than accounting estimates of income.

Net operating income (NOI) is a measure of periodic earnings that is calculated as the property’s rental income minus all expenses associated with maintaining and operating the property.

The net sale proceeds (NSP) is the expected selling price minus any expected selling expenses arising from the sale of the property at time T.

The effective gross income is the potential gross income reduced for the vacancy loss rate.
Fixed expenses, examples of which are property taxes and property insurance, do not change directly with the level of occupancy of the property.

Operating expenses are non-capital outlays that support rental of the property and can be classified as fixed or variable.

The potential gross income is the gross income that could potentially be received if all offices in the building were occupied.

The vacancy loss rate is the observed or anticipated rate at which potential gross income is reduced for space that is not generating rental income.

Variable expenses, examples of which are maintenance, repairs, utilities, garbage removal, and supplies, change as the level of occupancy of the property varies.

The risk premium approach to estimation of a discount rate for an investment uses the sum of a riskless interest rate and one or more expected rewards—expressed as rates—for bearing the risks of the investment.

In a net lease, the tenant is responsible for almost all of the operating expenses.

Depreciation is a noncash expense that is deducted from revenues in computing accounting income to indicate the decline of an asset’s value.

In an after-tax discounting approach, the estimated after-tax cash flows (e.g., after-tax bond payments) are discounted using a rate that has been reduced to reflect the net rate received by an investor with a specified marginal tax rate.

The pre-tax discounting approach is commonly used in finance, where pre-tax cash flows are used in the numerator of the present value analysis (as the cash flows to be received), and the pre-tax discount rate is used in the denominator.
An alternative approach, often termed the **equity residual approach**, focuses on the perspective of the equity investor by subtracting the interest expense and other cash outflows due to mortgage holders (in the numerator) and by discounting the remaining cash flows using an interest rate reflective of the required rate of return on the equity of a leveraged real estate investment (in the denominator). **Private equity real estate funds** are privately organized funds that are similar to other alternative investment funds, such as private equity funds and hedge funds, yet have real estate as their underlying asset.

**Commingled real estate funds** (CREFs) are a type of private equity real estate fund that is a pool of investment capital raised from private placements that are commingled to purchase commercial properties. **Syndications** are private equity real estate funds formed by a group of investors who retain a real estate expert with the intention of undertaking a particular real estate project.

**Real estate joint ventures** are private equity real estate funds that consist of the combination of two or more parties, typically represented by a small number of individual or institutional investors, embarking on a business enterprise such as the development of real estate properties. **Gearing** is the use of leverage.

**Open-end real estate mutual funds** are public investments that offer a non-exchange traded means of obtaining access to the private real estate market.

The use of prices that lag changes in true market prices is known as **stale pricing**.

**Exchange-traded funds (ETFs)** represent a tradable investment vehicle that tracks a particular index or portfolio by holding its constituent assets or a subsample of them. **Closed-end real estate mutual funds** are an investment pool that has real estate as its underlying asset and a relatively fixed number of outstanding shares.

The **FTSE NAREIT US Real Estate Index Series** is a family of REIT-based performance indices that covers the different sectors of the U.S. commercial real estate space.

In investments, a **safe harbor** denotes an area that is explicitly protected by one set of regulations from another set of regulations.
Consolidation is an increase in the proportion of a market represented by a relatively small number of participants (i.e., the industry concentration).

The high-water mark (HWM) is the highest NAV of the fund on which an incentive fee has been paid.

Asymmetric incentive fees, in which managers earn a portion of investment gains without compensating investors for investment losses, are generally prohibited for stock and bond funds offered as ‘40 Act mutual funds in the United States.

Managerial coinvesting in this context is an agreement between fund managers and fund investors that the managers will invest their own money in the fund.

Optimal contracting between investors and hedge fund managers attempts to align the interests of both parties to the extent that the interests can be aligned cost-effectively, with marginal benefits that exceed marginal costs.

Excessive conservatism is inappropriately high risk aversion by the manager, since the manager’s total income and total wealth may be highly sensitive to fund performance.

A perverse incentive is an incentive that motivates the receiver of the incentive to work in opposition to the interests of the provider of the incentive.

The annuity view of hedge fund fees represents the prospective stream of cash flows from fees available to a hedge fund manager.

The option view of incentive fees uses option theory to demonstrate the ability of managers to increase the present value of their fees by increasing the volatility of the fund’s assets.

The incentive fee option value is the risk-adjusted present value of the incentive fees to a manager that have been adjusted for its optionality.

The at-the-money incentive fee approximation expresses the value of a managerial incentive fee as the product of 40%, the fund’s NAV, the incentive fee percentage, and the volatility of the assets ($\sigma_f$) over the option’s life.

A closet indexer is a manager who attempts to generate returns that mimic an index while claiming to be an active manager.
A pure asset gatherer is a manager focused primarily on increasing the AUM of the fund. A pure asset gatherer is likely to take very little risk in a portfolio and, like mutual fund managers, become a closet indexer.

The lock-in effect in this context refers to the pressure exerted on managers to avoid further risks once high profitability and a high incentive fee have been achieved.

The terms managing returns and massaging returns refer to efforts by managers to alter reported investment returns toward preferred targets through accounting decisions or investment changes.

A classification of hedge fund strategies is an organized grouping and labeling of hedge fund strategies.

A fund of funds in this context is a hedge fund with underlying investments that are predominantly investments in other hedge funds.

A multistrategy fund deploys its underlying investments with a variety of strategies and sub-managers, much as a corporation would use its divisions.

A single-manager hedge fund, or single hedge fund, has underlying investments that are not allocations to other hedge funds.

Fund mortality, the liquidation or cessation of operations of funds, illustrates the risk of individual hedge funds and is an important issue in hedge fund analysis.

A hedge fund program refers to the processes and procedures for the construction, monitoring, and maintenance of a portfolio of hedge funds.

Absolute return strategies are hedge fund strategies that seek to minimize market risk and total risk.

Diversified strategies are hedge fund strategies that seek to diversify across a number of different investment themes.

Equity strategies are hedge fund strategies that exhibit substantial market risk.
**Event-driven strategies** are hedge fund strategies that seek to earn returns by taking on event risk, such as failed mergers, that other investors are not willing or prepared to take.

**Relative value strategies** are hedge fund strategies that seek to earn returns by taking risks regarding the convergence of values between securities.

Event risk is effectively an off-balance-sheet risk—that is, a risk exposure that is not explicitly reflected in the statement of financial positions.

**Short volatility exposure** is any risk exposure that causes losses when underlying asset return volatilities increase.

**Convergent strategies** profit when relative value spreads move tighter, meaning that two securities move toward relative values that are perceived to be more appropriate.

A relative return product is an investment with returns that are substantially driven by broad market returns and that should therefore be evaluated on the basis of how the investment’s return compares with broad market returns.

An investment strategy is referred to as opportunistic when a major goal is to seek attractive returns through locating superior underlying investments.

**Headline risk** is dispersion in economic value from events so important, unexpected, or controversial that they are the center of major news stories.

Fee bias is when index returns overstate what a new investor can obtain in the hedge fund marketplace because the fees used to estimate index returns are lower than the typical fees that a new investor would pay. The representativeness of a sample is the extent to which the characteristics of that sample are similar to the characteristics of the universe.

Instant history bias or backfill bias occurs when an index contains histories of returns that predate the entry date of the corresponding funds into a database and thereby cause the index to disproportionately reflect the characteristics of funds that are added to a database.
Liquidation bias occurs when an index disproportionately reflects the characteristics of funds that are not near liquidation.

Participation bias may occur for a successful hedge fund manager who closes a fund and stops reporting results because the fund no longer needs to attract new capital.

Strategy definitions, the method of grouping similar funds, raise two problems: (1) definitions of strategies can be very difficult for index providers to establish and specify, and (2) some funds can be difficult to classify in the process of applying the definition.

Style drift (or strategy drift) is the change through time of a fund's investment strategy based on purposeful decisions by the fund manager in an attempt to improve risk-adjusted performance in light of changing market conditions.

Synthetic hedge funds attempt to mimic hedge fund returns using listed securities and mathematical models.

The investability of an index is the extent to which market participants can invest to actually achieve the returns of the index.

Capacity is the limit on the quantity of capital that can be deployed without substantially diminished performance.

Counterparty risk is the uncertainty associated with the economic outcomes of one party to a contract due to potential failure of the other side of the contract to fulfill its obligations, presumably due to insolvency or illiquidity. Systematic fund trading, often referred to as black-box model trading because the details are hidden in complex software, occurs when the ongoing trading decisions of the investment process are automatically generated by computer programs.

Discretionary fund trading occurs when the decisions of the investment process are made according to the judgment of human traders.

Systematic fund trading, often referred to as black-box model trading because the details are hidden in complex software, occurs when the ongoing trading decisions of the investment process are automatically generated by computer programs.

Fundamental analysis uses underlying financial and economic information to ascertain intrinsic values based on economic modeling.

Technical analysis relies on data from trading activity, including past prices and volume data.

Global macro funds have the broadest investment universe: They are not limited by market segment, industry sector,
geographic region, financial market, or currency, and therefore tend to offer high diversification.

**Market microstructure** is the study of how transactions take place, including the costs involved and the behavior of bid and ask prices.

**Thematic investing** is a trading strategy that is not based on a particular instrument or market; rather, it is based on secular and long-term changes in some fundamental economic variables or relationships—for example, trends in population, the need for alternative sources of energy, or changes in a particular region of the world economy.

**Event risk** refers to sudden and unexpected changes in market conditions resulting from a specific event (e.g., Lehman Brothers bankruptcy).

**Leverage** refers to the use of financing to acquire and maintain market positions larger than the assets under management (AUM) of the fund.

**Market risk** refers to exposure to directional moves in general market price levels.

The term **managed futures** refers to the active trading of futures and forward contracts on physical commodities, financial assets, and exchange rates.

Investors can access the managed futures industry either by investing in a futures trading fund (via a managed account or a commingled fund) or through a commodity pool—a commingled investment vehicle that resembles a fund of funds and is managed by a **commodity pool operator (CPO)**, who invests in a number of underlying CTAs.

In the United States, the **Commodity Futures Trading Commission (CFTC)** was initiated in 1974 as a federal regulatory agency for all futures and derivatives trading.

The **National Futures Association (NFA)** is an independent, industry-supported, self-regulatory body created in 1982.

**Futurization** is the movement from traditional OTC contracts to multilateral cleared contracts in the futures market.

**Commodity pools** are investment funds that combine the money of several investors for the purpose of investing in the futures markets.

**Private commodity pools** are funds that invest in the futures markets and are sold privately to high-net-worth investors and institutional investors.
15.3.5 Public commodity pools

Public commodity pools are open to the general public for investing in much the same way that a mutual fund sells its shares to the public.

15.3.5 Commodity trading advisers (CTAs)

Commodity trading advisers (CTAs) are professional money managers who specialize in the futures markets.

15.3.5 Managed account

A managed account (or separately managed account) is created when money is placed directly with a CTA in an individual account rather than being pooled with other investors.

15.4.1 Slippage

Slippage is the unfavorable difference between assumed entry and exit prices and the entry and exit prices experienced in practice.

15.4.3 In-sample data

In-sample data are those observations directly used in the backtesting process.

15.4.3 Out-of-sample data

Out-of-sample data are observations that were not directly used to develop a trading rule or even indirectly used as a basis for knowledge in the research.

15.4.3 Robustness

Robustness refers to the reliability with which a model or system developed for a particular application or with a particular data set can be successfully extended into other applications or data sets.

15.4.3 Validation

Validation of a trading rule refers to the use of new data or new methodologies to test a trading rule developed on another set of data or with another methodology.

15.4.3 Degradation

Degradation is the tendency and process through time by which a trading rule or trading system declines in effectiveness.

15.4.4 Mean-reverting

Mean-reverting refers to the situation in which returns show negative autocorrelation—the opposite tendency of momentum or trending.

15.4.4 Momentum

Momentum is the extent to which a movement in a security price tends to be followed by subsequent movements of the same security price in the same direction.
A **moving average** is a series of averages that is recalculated through time based on a window of observations.

A price series with changes in its prices that are independent from current and past prices is a **random walk**.

**Trend-following strategies** are designed to identify and take advantage of momentum in price direction (i.e., trends in prices).

**Systematic trading strategies** are generally categorized into three groups: trend-following, non-trend-following, and relative value.

The most basic approach uses a **simple moving average**, a simple arithmetic average of previous prices.

A **weighted moving average** is usually formed as an unequal average, with weights arithmetically declining from most recent to most distant prices.

The **exponential moving average** is a geometrically declining moving average based on a weighted parameter, $\lambda$, with $0 < \lambda < 1$.

**Whipsawing** is when a trader alternates between establishing long positions immediately before price declines and establishing short positions immediately before price increases and, in so doing, experiences a sequence of losses. In trend following strategies, whipsawing results from a sideways market.

A **sideways market** exhibits volatility without a persistent direction.

**Breakout strategies** focus on identifying the commencement of a new trend by observing the range of recent market prices (e.g., looking back at the range of prices over a specific time period).

**Countertrend strategies** use various statistical measures, such as price oscillation or a relative strength index, to identify range-trading opportunities rather than price-trending opportunities.

The **relative strength index (RSI)**, sometimes called the relative strength indicator, is a signal that examines average up
A pattern recognition system looks to capture non-trend-based predictable abnormal market behavior in prices or volatilities.

Multistrategy CTAs combine a variety of strategy focuses to provide a diversified set of potential return sources and risk-reward profiles.

Volatility targeting is where the size of the position is determined by the trader’s conviction in the signal, the volatility of the particular futures market, and a volatility target that is determined by the trader.

The risk loading times the equity or capital is sometimes termed the capital at risk.

The point value is the gain or loss in the contract from a one-point change (e.g., $1) in the futures prices.

The futures contract dollar risk is a measure of the riskiness of the underlying asset of the futures contract during the most recent K trading periods.

Equal dollar risk allocation is a strategy that allocates the same amount of dollar risk to each market.

Equal risk contribution is a strategy that allocates risk based on the risk contribution of each market, taking correlation into account.

Market capacity weighting is an approach in which capital is allocated as a function of individual market capacity.

Alpha decay is the speed with which performance degrades as execution is delayed.

The Mount Lucas Management (MLM) Index is a passive, transparent, and investable index designed to capture the returns to active futures investing.
A **natural hedger** is a market participant who seeks to hedge a risk that springs from its fundamental business activities.

**Capacity risk** arises when a managed futures trader concentrates trades in a market that lacks sufficient depth (i.e., liquidity).

**Model risk** is economic dispersion caused by the failure of models to perform as intended.

**Transparency** is the ability to understand the detail within an investment strategy or portfolio.

**Transparency risk** is dispersion in economic outcomes caused by the lack of detailed information regarding an investment portfolio or strategy.

**Lack of trends risk**, which comes into play when the trader continues allocating capital to trendless markets, leading to substantial losses.

**Liquidity risk**, is somewhat related to capacity risk in that it refers to how a large fund that is trading in a thinly traded market will affect the price should it decide to increase or decrease its allocation.

The **event-driven** category of hedge funds includes activist hedge funds, merger arbitrage funds, and distressed securities funds, as well as special situation funds and multistrategy funds that combine a variety of event-driven strategies.

**Corporate event risk** is dispersion in economic outcomes due to uncertainty regarding corporate events.

**Selling insurance** in this context refers to the economic process of earning relatively small returns for providing protection against risks, not the literal process of offering traditional insurance policies.

A **long binary call option** makes one payout when the referenced price exceeds the strike price at expiration and a lower payout or no payout in all other cases.

A **long binary put option** makes one payout when the referenced price is lower than the strike price at expiration and a lower payout or no payout in all other cases.

**The activist investment strategy** involves efforts by shareholders to use their rights, such as voting power or the threat of such power, to influence corporate governance to their financial benefit as shareholders.
Corporate governance describes the processes and people that control the decisions of a corporation.

Shareholder activism refers to efforts by one or more shareholders to influence the decisions of a firm in a direction contrary to the initial recommendations of the firm’s senior management.

A proxy battle is a fight between the firm’s current management and one or more shareholder activists to obtain proxies (i.e., favorable votes) from shareholders.

A free rider is a person or entity that allows others to pay initial costs and then benefits from those expenditures.

Agency theory studies the relationship between principals and agents.

A principal-agent relationship is any relationship in which one person or group, the principal(s), hires another person or group, the agent(s), to perform decision-making tasks.

Agency costs are any costs, explicit (e.g., monitoring and auditing costs) or implicit (e.g., excessive corporate perks), resulting from inherent conflicts of interest between shareholders as principals and managers as agents.

An agent compensation scheme is all agreements and procedures specifying payments to an agent for services, or any other treatment of an agent with regard to employment.

In the United States, Form 13D is required to be filed with the Securities and Exchange Commission (SEC) within 10 days, publicizing an activist’s stake in a firm once the activist owns more than 5% of the firm and has a strategic plan for the firm.

A toehold is a stake in a potential merger target that is accumulated by a potential acquirer prior to the news of the merger attempt becoming widely known.

In the United States, Form 13F is a required quarterly filing of all long positions by all U.S. asset managers with over $100 million in assets under management, including hedge funds and mutual funds, among other investors.

In the United States, Form 13G is required of passive shareholders who buy a 5% stake in a firm, but this filing may be delayed until 45 days after year-end.

A wolf pack is a group of investors who may take similar positions to benefit from an activists’ engagement with corporate management.
Staggered board seats exist when instead of having all members of a board elected at a single point in time, portions of the board are elected at regular intervals.

Interlocking boards occur when board members from multiple firms—especially managers—simultaneously serve on each other’s boards and may lead to a reduced responsiveness to the interests of shareholders.

A spin-off occurs when a publicly traded firm splits into two publicly traded firms, with shareholders in the original firm becoming shareholders in both firms.

A split-off occurs when investors have a choice to own Company A or B, as they are required to exchange their shares in the parent firm if they would like to own shares in the newly created firm.

Merger arbitrage attempts to benefit from merger activity with minimal risk and is perhaps the best-known event-driven strategy.

Stock-for-stock mergers acquire stock in the target firm using the stock of the acquirer and typically generate large initial increases in the share price of the target firm.

Traditional merger arbitrage generally uses leverage to buy the stock of the firm that is to be acquired and to sell short the stock of the firm that is the acquirer.

Cash-for-stock mergers occur wherein the acquirer pays cash for the shares of the firm being acquired.

A bidding contest or bidding war is when two or more firms compete to acquire the same target.

An antitrust review is a government analysis of whether a corporate merger or some other action is in violation of regulations through its potential to reduce competition.

Financing risk is the economic dispersion caused by failure or potential failure of an entity, such as an acquiring firm, to secure the funding necessary to consummate a plan.
Distressed debt hedge funds invest in the securities of a corporation that is in bankruptcy or is likely to fall into bankruptcy.

The bankruptcy process is the series of actions taken from the filing for bankruptcy through its resolution.

In a liquidation process (chapter 7 in U.S. bankruptcy laws), all of the assets of the firm are sold, and the cash proceeds are distributed to creditors.

In a reorganization process (chapter 11 in U.S. bankruptcy laws), the firm’s activities are preserved.

A one-off transaction has one or more unique characteristics that cause the transaction to require specialized skill, knowledge, or effort.

The recovery value of the firm and its securities is the value of each security in the firm and is based on the time it will take the firm to emerge from the bankruptcy process and the condition in which it will emerge.

Capital structure arbitrage involves offsetting positions within a company’s capital structure with the goal of being long relatively underpriced securities, being short overpriced securities, and being hedged against risk.

Financial market segmentation occurs when two or more markets use different valuations for similar assets due to the lack of participants who trade in both markets or who perform arbitrage between the markets.

Event-driven multistrategy funds diversify across a wide variety of event-driven strategies, participating in opportunities in both corporate debt and equity securities.

Special situation funds also invest across a number of event styles are typically focused on equity securities, especially those with a spin-off or recent emergence from bankruptcy.

The classic relative value strategy trade is based on the premise that a particular relationship or spread between two prices or rates has reached an abnormal level and will therefore tend to return to its normal level.

Convergence is the return of prices or rates to relative values that are deemed normal.
The classic convertible bond arbitrage trade is to purchase a convertible bond that is believed to be undervalued and to hedge its risk using a short position in the underlying equity.

Convertible bonds are hybrid corporate securities, mixing fixed-income and equity characteristics into one security.

Bonds with very high conversion premiums are often called busted convertibles, as the embedded stock options are far out-of-the-money.

An equity-like convertible is a convertible bond that is far in-the-money and therefore has a price that tracks its underlying equity very closely.

Convertible bonds with moderately sized conversion ratios have stock options closer to being at-the-money and are called hybrid convertibles.

Moneyness is the extent to which an option is in-the-money, at-the-money, or out-of-the-money.

Delta is the change in the value of an option (or a security with an implicit option) with respect to a change in the value of the underlying asset (i.e., it measures the sensitivity of the option price to small changes in the price of its underlying asset).

Gamma is the second derivative of an option's price with respect to the price of the underlying asset—or, equivalently, the first derivative of delta with respect to the price of the underlying asset.

Theta is the first derivative of an option's price with respect to the time to expiration of the option.

A delta-neutral position is a position in which the value-weighted sum of all deltas of all positions equals zero.

The implied volatility of an option or an option-like position—in this case, the implied volatility of a convertible bond—is the standard deviation of returns that is viewed as being consistent with an observed market price for the option.
realized volatility

**Realized volatility** is the actual observed volatility (i.e., the standard deviation of returns) experienced by an asset—in this case, the underlying stock.

complexity premium

A **complexity premium** is a higher expected return offered by a security to an investor to compensate for analyzing and managing a position that requires added time and expertise.

dilution

**Dilution** takes place when additional equity is issued at below-market values, and the per-share value of the holdings of existing shareholders is diminished.

components of convertible arbitrage returns

The **components of convertible arbitrage returns** include interest, dividends, rebates, and capital gains and losses.

dynamic delta hedging

**Dynamic delta hedging** is the process of frequently adjusting positions in order to maintain a target exposure to delta, often delta neutrality.

net delta

The **net delta** of a position is the delta of long positions minus the delta of short positions.

volatility arbitrage

**Volatility arbitrage** is any strategy that attempts to earn a superior and riskless profit based on prices that explicitly depend on volatility.

anticipated volatility

**Anticipated volatility** is the future level of volatility expected by a market participant.

vega

**Vega** is a measure of the risk of a position or an asset due to changes in the volatility of a price or rate that helps determine the value of that position or asset.

vega risk

**Vega risk** is the economic dispersion caused by changes in the volatility of a price, return, or rate.

variance swaps

**Variance swaps** are forward contracts in which one party agrees to make a cash payment to the other party based on the realized variance of a price or rate in exchange for receiving a predetermined cash flow.

variance notional value

The **variance notional value** of the contract simply scales the size of the cash flows in a variance swap.
The **vega notional value** of a contract serves to scale the contract and determine the size of the payoff in a volatility swap.

A **volatility swap** mirrors a variance swap except that the payoff of the contract is linearly based on the standard deviation of a return series rather than the variance.

**Marking-to-market** refers to the use of current market prices to value instruments, positions, portfolios, and even the balance sheets of firms.

**Marking-to-model** refers to valuation based on prices generated by pricing models. The pricing models generally involve two components.

**Price transparency** is information on the prices and quantities at which participants are offering to buy (bid) and sell (offer) an instrument.

**Pricing risk** is the economic uncertainty caused by actual or potential mispricing of positions.

**Correlation risk** is dispersion in economic outcomes attributable to changes in realized or anticipated levels of correlation between market prices or rates.

**Volatility risk** is dispersion in economic outcomes attributable to changes in realized or anticipated levels of volatility in a market price or rate.

**Tail risk** is the potential for very large loss exposures due to very unusual events, especially those associated with widespread market price declines.

**Portfolio insurance** is any financial method, arrangement, or program for limiting losses from large adverse price movements.

The term **correlations go to one** means that during periods of enormous stress, stocks and bonds with credit risk decline simultaneously and with somewhat similar magnitudes.

The **classic dispersion trade** is a market-neutral short correlation trade, popular among volatility arbitrage practitioners, that typically takes long positions in options listed on the equities of single companies and short positions in a related index option.
The classic dispersion trade is referred to as a **short correlation** trade because the trade generates profits from low levels of realized correlation and losses from high levels of realized correlation.

**Fixed-income arbitrage** involves simultaneous long and short positions in fixed income securities with the expectation that over the investment holding period, the security prices will converge toward a similar valuation standard.

**Duration** is a measure of the sensitivity of a fixed-income security to a change in the general level of interest rates.

These are examples of **intracurve arbitrage positions** because they are based on hedged positions within the same yield curve.

A **yield curve** is the relationship between the yields of various securities, usually depicted on the vertical axis, and the term to maturity, usually depicted on the horizontal axis.

**Carry trades** attempt to earn profits from carrying or maintaining long positions in higher-yielding assets and short positions in lower-yielding assets without suffering from adverse price movements.

There are also **intercurve arbitrage positions**, which means arbitrage (hedged positions) using securities related to different yield curves.

**Sovereign debt** is debt issued by national governments.

A **duration-neutral** position is a portfolio in which the aggregated durations of the short positions equal the aggregated durations of the long positions weighted by value.

A **parallel shift** in the yield curve happens when yields of all maturities shift up or down by equal (additive) amounts.

The process of holding a bond as its yield moves up or down the yield curve due to the passage of time is known as **riding the yield curve**.

**Rolling down** the yield curve is the process of experiencing decreasing yields to maturity as an asset’s maturity declines through time in an upward-sloping yield curve environment.

**Modified duration** is equal to traditional duration divided by the quantity \( \frac{1}{1 + \left( \frac{y}{m} \right)} \), where \( y \) is the stated annual yield, \( m \) is the number of compounding periods per year, and \( y/m \) is the periodic yield.
Still another subset of fixed-income arbitrage trades is **asset-backed securities** (ABS), which are securitized products created from pools of underlying loans or other assets.

**Effective duration** is a measure of the interest rate sensitivity of a position that includes the effects of embedded option characteristics.

**Mortgage-backed securities arbitrage** attempts to generate low-risk profits through the relative mispricing among MBS or between MBS and other fixed-income securities.

A key concept in pricing fixed income securities with embedded prepayment options is the **option-adjusted spread** (OAS), which is a measure of the excess of the return of a fixed-income security containing an option over the yield of an otherwise comparable fixed-income security without an option after the return of the fixed-income security containing the option has been adjusted to remove the effects of the option.

**Equity long/short funds** tend to have net positive systematic risk exposure from taking a net long position, with the long positions being larger than the short positions.

**Equity market-neutral funds** attempt to balance short and long positions, ideally matching the beta exposure of the long and short positions and leaving the fund relatively insensitive to changes in the underlying stock market index.

**Short-bias funds** have larger short positions than long positions, leaving a persistent net short position relative to the market index that allows these funds to profit during times of declining equity prices.

**Liquidity** in this context is the extent to which transactions can be executed with minimal disruption to prices.

More generally, **taking liquidity** refers to the execution of market orders by a market participant to meet portfolio preferences that cause a decrease in the supply of limit orders immediately near the current best bid and offer prices.

**Providing liquidity** refers to the placement of limit orders or other actions that increase the number of shares available to be bought or sold near the current best bid and offer prices.
A **market maker** is a market participant that offers liquidity, typically both on the buy side by placing bid orders and on the sell side by placing offer orders.

**Asynchronous trading** is an example of market inefficiency in which news affecting more than one stock may be assimilated into the price of the stocks at different speeds.

Markets are said to be **informationally efficient** when security prices reflect available information.

Another potential source of abnormal profits for hedge funds is **overreacting** in which short-term price changes are too large relative to the value changes that should occur in a market with perfect informational efficiency.

**Short interest** is the percentage of outstanding shares that are currently held short.

Another potential source of abnormal profits for hedge funds is **underreacting** in which short-term price changes are too small relative to the value changes that should occur in a market with perfect informational efficiency.

**Speculation** is defined as bearing abnormal risk in anticipation of abnormally high expected returns.

Investment strategies that can be identified based on available information and that offer higher expected returns after adjustment for risk are known as **market anomalies**, and they are violations of informational market efficiency.

An empirical test of market efficiency is a **test of joint hypotheses**, because the test assumes the validity of a model of the risk-return relationship to test whether a given trading strategy earns consistent risk-adjusted profits.

**Accounting accrual** is the recognition of a value based on anticipation of a transaction.

**Price momentum** is trending in prices such that an upward price movement indicates a higher expected price and a downward price movement indicates a lower expected price.

**Earnings momentum** is the tendency of earnings changes to be positively correlated.
Earnings surprise is the concept and measure of the unexpectedness of an earnings announcement.

Standardized unexpected earnings (SUE) is a measure of earnings surprise.

A post-earnings-announcement drift anomaly has been documented, in which investors can profit from positive surprises by buying immediately after the earnings announcement or selling short immediately after a negative earnings surprise.

When a company chooses to reduce its shares outstanding, a share buyback program is initiated, and the company purchases its own shares from investors in the open market or through a tender offer.

Issuance of new stock is a firm's creation of new shares of common stock in that firm and may occur as a result of a stock-for-stock merger transaction or through a secondary offering.

Net stock issuance is issuance of new stock minus share repurchases.

Illegal insider trading varies by jurisdiction but may involve using material nonpublic information, such as an impending merger, for trading without required disclosure.

Trading by insiders can be legal insider trading when it is performed subject to legal restrictions.

Multiple-factor scoring models combine the factor scores of a number of independent anomaly signals into a single trading signal.

Pairs trading is a strategy of constructing a portfolio with matching stocks in terms of systematic risks but with a long position in the stock perceived to be relatively underpriced and a short position in the stock perceived to be relatively overpriced.

The limits to arbitrage refer to the potential inability or unwillingness of speculators, such as equity hedge fund managers, to hold their positions without time constraints or to increase their positions without size constraints.

Market impact is the degree of the short-term effect of trades on the sizes and levels of bid prices and offer prices.

An uptick rule permits short sellers to enter a short sale only at a price that is equal to or higher than the previous transaction price of the stock.
Mean neutrality is when a fund is shown to have zero beta exposure or correlation to the underlying market index.

Variance neutrality is when fund returns are uncorrelated to changes in market risk, including extreme risks in crisis market scenarios.

Operational due diligence is the process of evaluating the policies, procedures, and internal controls of an asset management organization.

Fee netting in the case of a multistrategy fund is when the investor pays incentive fees based only on net profits of the combined strategies, rather than on all profitable strategies.

Access is an investor’s ability to place new or increased money in a particular fund.

A liquidity facility is a standby agreement with a major bank to provide temporary cash for specified needs with pre-specified conditions.

Seeding funds, or seeders, are funds of funds that invest in newly created individual hedge funds, often taking an equity stake in the management companies of the newly minted hedge funds.

Nontraditional or unconstrained bond funds do not simply take long positions in investment-grade sovereign and credit securities, but may also invest in high-yield or emerging markets debt, often including leverage and short positions.

Nontraditional or unconstrained bond funds do not simply take long positions in investment-grade sovereign and credit securities, but may also invest in high-yield or emerging markets debt, often including leverage and short positions.

Market-defensive funds of funds tend to have underlying and unhedged short positions.

Conservative funds of funds have underlying hedged positions.

Diversified funds of funds represent a broad mix of funds.

Strategic funds of funds tend to have underlying directional bets.
An equity kicker is an option for some type of equity participation in the firm (e.g., options to buy shares of common stock) that is packaged with a debt financing transaction.

Venture capital (VC), the best known of the private equity categories, is early-stage financing for young firms with high potential growth that do not have a sufficient track record to attract investment capital from traditional sources, like public markets or lending institutions.

The cash burn rate of a business describes the speed with which cash is being depleted through time and can be used to project when the organization will deplete its cash and require outside funding.

Venture capital securities are the privately held stock, or equity-linked securities, that venture capitalists obtain when investing in business ventures that are striving to become larger and to go public.

Investment structures used by venture capitalists include convertible preferred equity, convertible notes, or debentures that provide for the conversion of the principal amount of the note or bond into either common or preferred shares at the option of the venture capitalist, or other positions such as warrants.

VC exits typically focus on going public (i.e., conducting an initial public offering of the company’s securities), but can also include sales to acquiring firms or even a leveraged recapitalization, where the proceeds from the debt are paid to the venture capitalist.

Convertible preferred stock is used by VC investors to provide higher priority than common stock along with an implicit call option to share in upside potential similar to the upside potential of equity.

The prudent person standard is a requirement that specifies levels of care that should be exercised in particular decision-making roles, such as investment decisions made by a fiduciary.

The terminology 20-bagger indicates a company that appreciates in value 20-fold compared to the cost of the VC investment.

Angel investing refers to the earliest stage of venture capital, in which investors fund the first cash needs of an entrepreneurial idea.

The seed capital stage is the first stage where VC firms invest their capital into a venture and is typically prior to having established the viability of the product.

The change in the prudent person standard was to base analysis on a portfolio basis (rather than a standalone basis) and to test for prudence based on analysis (rather than outcome), allowing U.S. pension funds for the first time to wholly endorse and engage in venture capital investing.

Alpha testing is the process of analyzing a product or service to determine its ability to perform its tasks, potentially under laboratory-like conditions, to generate feedback for developers.
Second or late-stage (i.e., expansion stage) venture capital fills the cash flow deficiency once commercial viability is established.

This is often referred to as beta testing, in which a prototype is sent to potential customers free of charge to get their input into the product’s viability, design, and user-friendliness.

The first-stage, start-up stage, or early-stage of venture capital begins when the start-up company has a viable product that has been beta tested and involves testing of the second-generation prototype with potential end users and funding after seed capital but before commercial viability has been established.

Mezzanine venture capital, or pre-IPO financing, is the last funding stage before a start-up company goes public or is sold to a strategic buyer.

Enterprise value is the total value of the company, which adds the equity value of the firm to its outstanding debt and subtracts the cash on the firm’s balance sheet.

EBITDA is a firm’s earnings or operating income before interest, taxes, depreciation, and amortization and is therefore used as a measure of before-tax cash flow rather than being a net-of-debt measure.

EBITDA multiples are general levels of the perceived ratio between the enterprise value of a firm’s assets and its estimated or projected earnings before income taxes, depreciation, and amortization.

The exit plan describes how venture capitalists can liquidate their investment in the start-up company to realize a gain for themselves and their investors.

A compound option is an option on an option. In other words, a compound option allows its owner the right but not the obligation to pay additional money at some point in the future to obtain an option.

The venture capital business plan should clearly state the business strategy, identify the niche that the new company will fill, and describe the resources needed to fill that niche, including the expenses, personnel, and assets. It must be comprehensive, coherent, and internally consistent.

A milestone is a set of goals that must be met to complete a phase and usually denotes when the entrepreneur will be eligible for the next round of financing.
The **two keys to successful VC investing**: (1) identifying underpriced options by locating potentially valuable projects for which substantial information regarding likely profitability can be obtained prior to commitment of substantial capital, and (2) abandoning worthless out-of-the-money options when they are expiring by ignoring sunk costs and judiciously assessing likely outcomes of success based on the objective analysis of new information.

**Growth equity securities** are newly originated securities that have a minority position in terms of control but a relatively high position in terms of liquidation priority, such as convertible preferred equities or debt.

**Protective provisions in growth equity** provide operational control such as investor consent rights on key transactions, with key growth transactions including changes in capital structure, major assets, tax or accounting policies, key employees, and significant operational activities.

**Redemption rights** grant powers to investors to redeem their position in the company by specifying the triggers and actions that demark the remedies available to the investors.

A **growth equity redemption value** is typically set as the maximum of one of the following or the maximum of two or more of the following: (1) the original issuance price plus a preferred return, (2) a multiple of the original issuance price, and (3) the fair market value of the equity interest.

**Growth equity redemption sources** can be required to include: (1) all "legally available funds," (2) undertaking a "forced sale" or other capital raising transaction, (3) issuing a promissory note for the redemption value, and/or (4) using all other available means in order to effect a required redemption.

**Growth equity default remedies** include springing board remedies and forced sales.

A **springing board remedy** occurs when the investor designates a majority of the defaulting issuer’s board of directors.

A **forced sale remedy** occurs when an investor compels a liquidating transaction, such as sale of the entire company or other transactions, to generate cash to meet the redemption obligation.

The **times revenue method** values an enterprise as the product of its projected annual revenue and a multiple derived from analysis of the value of similar firms.
Four principal considerations in redemption rights are (1) redemption triggers, (2) redemption value, (3) sources of funds, and (4) remedies for defaulted redemption.

A buyout occurs when capital, often as a mix of debt and equity, is used to acquire an entire existing company (private or publicly traded) from its current shareholders and to operate the company as an independent organization—as opposed to an acquisition, in which the acquired company is folded into the buyer’s existing company.

In the context of private equity, buyouts are the purchase of a public company by an entity that has a private ownership structure.

A leveraged buyout (LBO) is distinguished from a traditional investment by three primary aspects: (1) an LBO buys out control of the assets, (2) an LBO uses leverage, and (3) an LBO itself is not publicly traded.

A management buy-in (MBI) is a type of LBO in which the buyout is led by an outside management team.

A buyout of a private company is a form of private equity that is often executed in lieu of an IPO exit, from the perspective of the shareholders who are selling the company.

A management buyout (MBO) is a buyout that is led by the target firm’s current management.

A buy-in management buyout is a hybrid between an MBI and an MBO in which the new management team is a combination of new managers and incumbent managers.

In a secondary buyout, one private equity firm typically sells a private company to another private equity firm.

Rescue capital (or turnaround capital) refers to a strategy in which capital is provided to help established companies recover profitability after experiencing trading, financial, operational, or other difficulties.

Replacement capital (also called secondary purchase capital) refers to a strategy in which capital is provided to acquire existing shares in a company from another PE investment organization.

Segmentation in this context denotes the grouping of market participants into clienteles that focus their activities within specific areas of the market, rather than varying their range of activities more broadly throughout all available opportunities.
A generous compensation scheme, known as a **golden parachute**, is often given to top managers whose careers are being negatively affected by a corporate reorganization.

An **evolution of the buyout market** has occurred that has been driven by substantial buyout activity and has resulted in a less segmented market that has grown into a more efficient, auction-driven asset market, in which greater competition has reduced abnormal profit opportunities.

**Conglomerates** have many different divisions or subsidiaries, often operating in completely different industries.

**Efficiency buyouts** are LBOs that improve operating efficiency.

**Entrepreneurship stimulators** are LBOs that create value by helping to free management to concentrate on innovations.

A **buy-and-build strategy** is an LBO value-creation strategy involving the synergistic combination of several operating companies or divisions through additional buyouts.

A **turnaround strategy** is an approach used by LBO funds that look for underperforming companies with excessive leverage or poor management.

A **buyout-to-buyout deal** takes place when a private equity firm sells one of its portfolio companies to another buyout firm.

**Merchant banking** is the practice whereby financial institutions purchase nonfinancial companies as opposed to merging with or acquiring other financial institutions.

A **winner-take-all market** refers to a market with a tendency to generate massive rewards for a few market participants that apparently provide products or services that are only marginally better than their competitors.

A **unicorn** is a VC-backed firm that soars to $1 billion or more in private market capitalization over a relatively short period of time.

**Private equity funds** are investment pools created to hold portfolios of private equity securities.
21.1 VC fund

A venture capital fund is a private equity fund that pools the capital of large sophisticated investors to fund new and start-up companies.

21.1.4 blind pool

A blind pool is when investors don’t know the underlying portfolio companies before committing capital.

21.1.4 dry powder

The amount committed but not yet called is an undrawn commitment or dry powder.

21.1.4 undrawn commitment

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21.2.4 co-investment

Co-investment involves investors being invited by GPs to make direct investments in portfolio companies, as is discussed in detail in the second level of the CAIA program.

21.2.5 vintage year

The year a particular private equity fund commences operations is known as its vintage year.

21.2.5 sourcing investments

Sourcing investments is the process of locating possible investments (i.e., generating deal flow), reading business plans, preparing intense due diligence on start-up companies, and determining the attractiveness of each start-up company.

21.2.5 reinvestment provision

A reinvestment provision is where the proceeds of realizations within the investment period or a similar time frame may be reinvested in new opportunities and not distributed to investors.

21.2.5 in-kind distributions

Distributions to investors can also take the form of securities of a portfolio company, known as in-kind distributions.

21.2.6 cash flow J-curve

The cash flow J-curve is a representation of the evolution of the net accumulated cash flows from the investors to the fund, which are negative during the early years of existence before making a U-turn and becoming positive in the later years of the fund’s life.

21.2.6 NAV J-Curve

The NAV J-curve is a representation of the evolution of the NAV of a fund versus the net paid in (NPI), which first decreases during the early years of the fund’s existence and then improves in its later years.

21.2.7 commitment risk

Commitment risk describes the situation in which an LP may become a defaulting investor if the proceeds of exiting funds are not sufficient to pay the capital calls of newly committed funds. Commitment risk is discussed in the next section.

21.4.1 committed capital

Committed capital is the cash investment that has been promised by an investor but not yet delivered to the fund.
Capital calls are options for the manager to demand, according to the subscription agreement, that investors contribute additional capital.

There is often a clawback escrow agreement, in which a portion of the manager’s incentive fees are held in a segregated account until the entire fund is liquidated.

Subscription lines are lines of credit used by the GP to make investments in portfolio companies.

A key personnel clause is a provision that allows investors to withdraw their assets from the fund, immediately and without penalty, when the identified key personnel are no longer making investment decisions for the fund.

The capital contribution by GPs is known as hurt money.

LPs may include a bad-leaver clause, which is a for-cause removal of the GP that, if exercised (normally following a simple majority vote of the LPs), causes investments to be suspended until a new fund manager is elected or, in the extreme, the fund is liquidated.

A good-leaver clause enables investors to cease additional funding of the partnership with a vote requiring a qualified majority (generally more than 75% of LPs).

An auction process involves bidding among several private equity firms, with the deal going to the highest bidder.

In a club deal, two or more LBO firms work together to share costs, present a business plan, and contribute capital to the deal.

Business development companies (BDCs) are publicly traded funds with underlying assets typically consisting of equity or equity-like positions in small, private companies. BDCs use a closed-end structure and trade on major stock exchanges, especially the NASDAQ.
Private investments in public equity (PIPE) transactions are privately issued equity or equity-linked securities that are placed outside of a public offering and are exempt from registration.

The large majority of PIPE transactions are traditional PIPEs, in which investors can buy common stock at a fixed price.

Structured PIPEs include more exotic securities, like floating-rate convertible preferred stock, convertible resets, and common stock resets.

A toxic PIPE is a PIPE with adjustable conversion terms that can generate high levels of shareholder dilution in the event of deteriorating prices in the firm's common stock.

The process of lower conversion prices, greater dilution, and lower share prices repeats in a downward death spiral.

Publicly traded PE firms offer investors exposure to PE and earning carried interest from high returns to those firms generated from the underlying fund investments.

Interval funds are semi-liquid, semi-illiquid closed-end funds that do not trade on the secondary market but offer the opportunity for investors to redeem or exit their investments at regularly scheduled intervals.

A drawdown fund is a type of private equity fund (that can be used for private credit) in which investor commitments are called as needed (e.g., to fund investments or meet expenses), in essence providing partnership-like liquidity features in a fund structure.

A loan-to-own investment occurs when the investor focuses on the value of the borrower's assets and the value of the company that could be repossessed if the borrower was unable to service the loan, not necessarily evaluating the ability of the company to pay back the principal and interest as scheduled.

Fulcrum securities are the more junior debt securities that are most likely to be converted into the equity of the reorganized company.

A credit spread is the excess of the yield on a debt security with credit risk relative to the yield on a debt security of similar maturity but no credit risk.

Covenant-lite loans are loans that place minimal restrictions on the debtor in terms of loan covenants.
Incurrence covenants typically require a borrower to take or not take a specific action once a specified event occurs.

Maintenance covenants are stricter than incurrence covenants in that they require that a standard be regularly met to avoid default.

Negative covenants are promises by the debtor not to engage in particular activities, such as paying dividends or issuing new debt.

An indenture is the contract between the borrower and the lender that sets out the terms of the borrowing.

Affirmative covenants are requirements for the borrower to comply with, such as to maintain a debt service coverage ratio that requires a minimum income relative to the size of the current year principal and interest payments.

The recovery rate is the percentage of the credit exposure that the lender ultimately receives through the bankruptcy process and all available remedies.

A unitranche is when a large piece of debt is issued that includes both senior and junior debt at a blended interest rate in a single debt issue.

In finance, the term haircut usually refers to a percentage reduction applied to the value of securities in determining their value as collateral.

Chapter 11 bankruptcy attempts to maintain operations of a distressed corporation that may be viable as a going concern.

Chapter 7 bankruptcy is entered into when a company is no longer viewed as a viable business and the assets of the firm are liquidated. Essentially, the firm shuts down its operations and parcels out its assets to various claimants and creditors.

A plan of reorganization is a business plan for emerging from bankruptcy protection as a viable concern, including operational changes.

A blocking position exists when a creditor or group of creditors holds more than one-third of the dollar amount of any class of claimants and utilizes those holdings to prevent a plan of reorganization.

An absolute priority rule is a specification of which claims in a liquidation process are satisfied first, second, third, and so forth in receiving distributions.

A cramdown is when a bankruptcy court judge implements a plan of reorganization over the objections of an impaired class of security holders.
When secured lenders extend additional credit to the debtor company, it is commonly known as debtor-in-possession financing (DIP financing).

Leveraged loans are syndicated bank loans to non-investment grade borrowers.

The term syndicated refers to the use of a group of entities, often investment banks, in underwriting a security offering or, more generally, jointly engaging in other financial activities.

Direct lending (also called market-based lending, shadow banking, or nonbank lending) is a transaction in which investors extend credit to borrowers outside of the traditional banking system.

Peer-to-peer lending is originating loans directly to consumers and is done by both institutional and retail investors who have an opportunity to originate consumer loans, often through an Internet-based underwriting and brokerage platform.

A warrant is a call option issued by a corporation on its own stock.

The weighted average cost of capital for a firm is the sum of the products of the percentages of each type of capital used to finance a firm times its annual cost to the firm.

A PIK toggle allows the underlying company to choose whether it will make required coupon payments in the form of cash or in kind, meaning with more mezzanine bonds.

Many private credit funds participate in sponsored lending, whereby the borrowing firm is backed by an investment from a private equity fund or buyout fund sponsor.

Bridge financing is a form of gap financing—a method of debt financing that is used to maintain liquidity while waiting for an anticipated and reasonably expected inflow of cash.

In stretch financing, a bank lends more money than it believes would be prudent with traditional lending standards and traditional lending terms.

An intercreditor agreement is an agreement with the company’s existing creditors that places restrictions on both the senior creditor and the mezzanine investor.

Acceleration is a requirement that debt be repaid sooner than originally scheduled, such as when the senior lender can declare the senior debt due and payable immediately.
A **blanket subordination** prevents any payment of principal or interest to the mezzanine investor until after the senior debt has been fully repaid.

A **springing subordination** allows the mezzanine investor to receive interest payments while the senior debt is still outstanding.

A **takeout provision** allows the mezzanine investor to purchase the senior debt once it has been repaid to a specified level.

The annual **default rate** is the annual portion of debt issues that default by failing to pay principal and interest as scheduled or that experience a technical default when a company is unable to comply with the covenants, or terms of the loan outside the payment of principal and interest.

The annual **loan loss rate** is the annual default rate multiplied by the losses on the debt that aren’t recovered through bankruptcy.

**Vulture investors** help the economy by cleaning up after bankruptcies, recycling bad debt and turning poorly run companies into new investments with greater potential profits and job growth.

In the context of alternative investments, **structuring** is the process of engineering unique financial opportunities from existing asset exposures.

A **complete market** is a financial market in which enough different types of distinct securities exist to meet the needs and preferences of all participants.

A **state of the world**, or state of nature (or state), is a precisely defined and comprehensive description of an outcome of the economy that specifies the realized values of all economically important variables.

A **tranche** is a distinct claim on assets that differs substantially from other claims in such aspects as seniority, risk, and maturity.

The **sequential-pay collateralized mortgage obligation** is the simplest form of CMO.
**Contraction risk** is dispersion in economic outcomes caused by uncertainty in the longevity—especially decreased longevity—of cash flow streams.

**Extension risk** is dispersion in economic outcomes caused by uncertainty in the longevity—especially increased longevity—of cash flow streams.

**Interest-only (IO)** tranches receive only interest payments from the collateral pool.

**Planned amortization class (PAC) tranches** receive principal payments in a more complex manner than do sequential pay CMOs.

**Principal-only (PO) tranches** receive only principal payments from the collateral pool.

**Targeted amortization class (TAC) tranches** receive principal payments in a manner similar to PAC tranches but generally with an even narrower and more complex set of ranges.

**Floating-rate tranches** earn interest rates that are linked to an interest rate index, such as the London Interbank Offered Rate (LIBOR), and are usually used to finance collateral pools of adjustable-rate mortgages.

An **inverse floater tranche** offers a coupon that increases when interest rates fall and decreases when interest rates rise.

**Structural credit risk models** use option theory to explicitly take into account credit risk and the various underlying factors that drive the default process, such as (1) the behavior of the underlying assets, and (2) the structuring of the cash flows (i.e., debt levels).

The **call option view of capital structure** views the equity of a levered firm as a call option on the assets of the firm.
The **put option view of capital structure** views the equity holders of a levered firm as owning the firm’s assets through riskless financing and having a put option to deliver those assets to the debt holders.

A **cap** is a series of caplets, and its price is equal to the sum of the prices of the caplets, which, in turn, can be valued using various term-structure models and a procedure similar to the Black-Scholes option pricing model.

A **caplet** is an interest rate cap guaranteed for only one specific date.

In an **interest rate floor**, one party agrees to pay the other when a specified reference rate is below a predetermined rate (known as the floor rate, which is analogous to the strike price of a European put option).

A **floor** is a series of floorlets, and its price is equal to the sum of the prices of the floorlets.

A **floorlet** is an interest rate floor guaranteed for only one specific date.

A **collateralized debt obligation (CDO)** applies the concept of structuring to cash flows from a portfolio of debt securities into multiple claims; these claims are securities and are referred to as tranches.

The **equity tranche** has lowest priority and serves as the residual claimant.

A **mezzanine tranche** is a tranche with a moderate priority to cash flows in the structured product and with lower priority than the senior tranche.

The **senior tranche** is a tranche with the first or highest priority to cash flows in the structured product.

The first percentage loss in the collateral pool that begins to cause reduction in a tranche is known as the **lower attachment point**, or simply the **attachment point**.
The higher percentage loss point at which the given tranche is completely wiped out is known as the upper attachment point, or the detachment point.

The first percentage loss in the collateral pool that begins to cause reduction in a tranche is known as the lower attachment point, or simply the attachment point.

The higher percentage loss point at which the given tranche is completely wiped out is known as the upper attachment point, or the detachment point.

A bull call spread has two calls that differ only by strike price, in which the long position is in the lower strike price and the short position is in the higher strike price.

A bull put spread has two puts that differ only by strike price, in which the long position is in the lower strike price and the short position is in the higher strike price.

Credit risk is dispersion in financial outcomes associated with the failure or potential failure of a counterparty to fulfill its financial obligations.

Default risk is the risk that the issuer of a bond or the debtor on a loan will not repay the interest and principal payments of the outstanding debt in full.

Reduced-form credit models focus on default probabilities based on observations of market data of similar-risk securities.

Exposure at default (EAD) specifies the nominal value of the position that is exposed to default at the time of default.

Loss given default (LGD) specifies the economic loss in case of default.

Probability of default (PD) specifies the probability that the counterparty fails to meet its obligations.

A risk-neutral approach models financial characteristics, such as asset prices, within a framework that assumes that investors are risk neutral.

A risk-neutral investor is an investor that requires the same rate of return on all investments, regardless of levels and types of risk, because the investor is indifferent with regard to how much risk is borne.
To **calibrate a model** means to establish values for the key parameters in a model, such as a default probability or an asset volatility, typically using an analysis of market prices of highly liquid assets.

Credit derivatives transfer credit risk from one party to another such that both parties view themselves as having an improved position as a result of the derivative.

**Derivatives** are cost-effective vehicles for the transfer of risk, with values driven by an underlying asset.

**Hazard rate** is a term often used in the context of reduced-form models to denote the default rate.

Price revelation, or price discovery, is the process of providing observable prices being used or offered by informed buyers and sellers.

Multiname instruments, in contrast to single-name instruments, make payoffs that are contingent on one or more credit events (e.g., defaults) affecting two or more reference entities.

**Single-name credit derivatives** transfer the credit risk associated with a single entity. This is the most common type of credit derivative and can be used to build more complex credit derivatives.

Unfunded credit derivatives involve exchanges of payments that are tied to a notional amount, but the notional amount does not change hands until a default occurs.

Funded credit derivatives require cash outlays and create exposures similar to those gained from traditional investing in corporate bonds through the cash market.

In a plain vanilla **interest rate swap**, party A agrees to pay party B cash flows based on a fixed interest rate in exchange for receiving from B cash flows in accordance with a specified floating interest rate.

The fixed rate of an interest rate swap is referred to as the **swap rate**.

The **swap rate curve** displays the relationship between swap rates and the maturities of their corresponding contracts, having a concept analogous to that of the yield curve.
A credit default swap (CDS) is an insurance-like bilateral contract in which the buyer pays a periodic fee (analogous to an insurance premium) to the seller in exchange for a contingent payment from the seller if a credit event occurs with respect to an underlying credit-risky asset.

In a CDS, the credit protection buyer pays a periodic premium on a predetermined amount (the notional amount) in exchange for a contingent payment from the credit protection seller if a specified credit event occurs.

The credit protection seller receives a periodic premium in exchange for delivering a contingent payment to the credit protection buyer if a specified credit event occurs.

In a total return swap, the credit protection buyer, typically the owner of the credit risky asset, passes on the total return of the asset to the credit protection seller in return for a certain payment.

The CDS spread or CDS premium is paid by the credit protection buyer to the credit protection seller and is quoted in basis points per annum on the notional value of the CDS.

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The standard ISDA agreement serves as a template to negotiated credit agreements that contains commonly used provisions used by market participants.

In a cash settlement, the credit protection seller makes the credit protection buyer whole by transferring to the buyer an amount of cash based on the contract.

Under physical settlement, the credit protection seller purchases the impaired loan or bond from the credit protection buyer at par value.

The referenced asset (also called the referenced bond, referenced obligation, or referenced credit) is the underlying security on which the credit protection is provided.

The process of altering the value of a CDS in the accounting and financial systems of the CDS parties is known as a mark-to-market adjustment.

A novation or an assignment is when one party to a contract reaches an agreement with a third party to take over all rights and obligations to a contract.

A novation or an assignment is when one party to a contract reaches an agreement with a third party to take over all rights and obligations to a contract.
American credit options are credit options that can be exercised prior to or at expiration.

Binary options (sometimes termed digital options) offer only two possible payouts, usually zero and some other fixed value.

European credit options are credit options exercisable only at expiration.

Credit-linked notes (CLNs) are bonds issued by one entity with an embedded credit option on one or more other entities.

CDS indices are indices or portfolios of single-name CDSs.

Bankruptcy remote means that if the sponsoring bank or money manager goes bankrupt, the CDO trust is not affected.

The ramp-up period, is the first period in a CDO life cycle, during which the CDO trust issues securities (tranches) and uses the proceeds from the CDO note sale to acquire the initial collateral pool (the assets).

The second phase in the CDO life cycle is normally called the revolving period, during which the manager of the CDO trust may actively manage the collateral pool for the CDO, potentially buying and selling securities and reinvesting the excess cash flows received from the CDO collateral pool. The sponsor of the trust establishes the trust and bears the associated administrative and legal costs.

The underlying portfolio or pool of assets (and/or derivatives) held in the SPV within the CDO structure is also known as the collateral or reference portfolio.

The weighted average rating factor (WARF), as described by Moody's Investors Service, is a numerical scale ranging from 1 (for AAA-rated credit risks) to 10,000 (for the worst credit risks) that reflects the estimated probability of default.

During the amortization period, the manager of the CDO stops reinvesting excess cash flows and begins to wind down the CDO by repaying the CDO's debt securities.

Arbitrage CDOs are created to attempt to exploit perceived opportunities to earn superior profits through money management.
Balance sheet CDOs are created to assist a financial institution in divesting assets from its balance sheet.

A diversity score is a numerical estimation of the extent to which a portfolio is diversified.

The tranche width is the percentage of the CDO's capital structure that is attributable to a particular tranche.

The weighted average spread (WAS) of a portfolio is a weighted average of the return spreads of the portfolio's securities in which the weights are based on market values.

A cash-funded CDO involves the actual purchase of the portfolio of securities serving as the collateral for the trust and to be held in the trust.

In a synthetic CDO, the CDO obtains risk exposure for the collateral pool through the use of a credit derivative, such as a total return swap or a CDS.

In a cash flow CDO, the proceeds of the issuance and sale of securities (tranches) are used to purchase a portfolio of underlying credit-risky assets, with attention paid to matching the maturities of the assets and liabilities.

In a market value CDO, the underlying portfolio is actively traded without a focus on cash flow matching of assets and liabilities.

An internal credit enhancement is a mechanism that protects tranche investors and is made or exists within the CDO structure, such as a large cash position.

Subordination is the most common form of credit enhancement in a CDO transaction, and it flows from the structure of the CDO trust.

Overcollateralization refers to the excess of assets over a given liability or group of liabilities.

A reserve account holds excess cash in highly rated instruments, such as U.S. Treasury securities or high-grade commercial paper, to provide security to the debt holders of the CDO trust.

An external credit enhancement is a protection to tranche investors that is provided by an outside third party, such as a form of insurance against defaults in the loan portfolio.
A distressed debt CDO uses the CDO structure to securitize and structure the risks and returns of a portfolio of distressed debt securities, in which the primary collateral component is distressed debt.

A collateralized fund obligation (CFO) applies the CDO structure concept to the ownership of hedge funds as the collateral pool.

In a single-tranche CDO, the CDO may have multiple tranches, but the sponsor issues (sells) only one tranche from the capital structure to an outside investor.

Financial engineering risk is potential loss attributable to securitization, structuring of cash flows, option exposures, and other applications of innovative financing devices.

Risk shifting is the process of altering the risk of an asset or a portfolio in a manner that differentially affects the risks and values of related securities and the investors who own those securities.

A copula approach to analyzing the credit risk of a CDO may be viewed like a simulation analysis of the effects of possible default rates on the cash flows to the CDO’s tranches and the values of the CDO’s tranches.

Equity-linked structured products are distinguished from structured products by one or more of the following three aspects: (1) They are tailored to meet the preferences of the investors and to generate fee revenue for the issuer; (2) they are not usually collateralized with risky assets; and (3) they rarely serve as a pass-through or simple tranching of the risks of a long-only exposure to an asset, such as a risky bond or a loan portfolio.

A wrapper is the legal vehicle or construct within which an investment product is offered.

Tax deferral refers to the delay between when income or gains on an investment occur and when they are taxed.

Tax deduction of an item is the ability of a taxpayer to reduce taxable income by the value of the item.

Although there is no universally accepted definition of an exotic option, a useful definition is that an exotic option is an option that has one or more features that prevent it from being classified as a simple option, including payoffs based on values prior to the expiration date, and/or payoffs that are nonlinear or discontinuous functions of the underlying asset.
A simple option has (1) payoffs based only on the value of a single underlying asset observed at the expiration date, and (2) linear payoffs to the long position of the calls and puts based on the distance between the option’s strike price and the value of the underlying asset. A principal-protected structured product is an investment that is engineered to provide a minimum payout guaranteed by the product’s issuer (counterparty).

A structured product without exotic options has a payoff diagram defined exclusively in terms of the payoff to the value of a single underlier at termination and is (1) a continuous relationship, (2) a one-to-one relationship, and (3) a relationship composed entirely of two linear segments. Thus, a structured product based on exotic options violates one or more of the three properties.

The participation rate indicates the ratio of the product’s payout to the value of the underlying asset.

A cash-and-call strategy is a long position in cash, or a zero-coupon bond, combined with a long position in a call option.

An Asian option is an option with a payoff that depends on the average price of an underlying asset through time.

A path-dependent option is any option with a payoff that depends on the value of the underlying asset at points prior to the option’s expiration date.

An active option in a barrier option is an option for which the underlying asset has reached the barrier.

A barrier option is an option in which a change in the payoff is triggered if the underlying asset reaches a prespecified level during a prespecified time period.

A knock-in option is an option that becomes active if and only if the underlying asset reaches a prespecified barrier.

A knock-out option is an option that becomes inactive (i.e., terminates) if and only if the underlying asset reaches a prespecified barrier.
A spread option has a payoff that depends on the difference between two prices or two rates.

A look-back option has a payoff that is based on the value of the underlying asset over a reference period rather than simply the value of the underlying asset at the option’s expiration date.

A quanto option is an option with a payoff based in one currency using the numerical value of the underlying asset expressed in a different currency.

An absolute return structured product offers payouts over some or all underlying asset returns that are equal to the absolute value of the underlying asset’s returns.

A principal protected absolute return barrier note offers to pay absolute returns to the investor if the underlying asset stays within both an upper barrier and a lower barrier over the life of the product.

The EUSIPA (European Structured Investment Products Association) was founded in 2009 as a nonprofit association "to promote the interests of the structured retail investment products market."

The EUSIPA Derivative Map categorizes structured products with two major classifications: investment products and leverage products.

The Investment Products in the EUSIPA Derivative Map includes three major sub-categories: capital protection products, yield enhancement products, and participation products.

Capital protection structured products tend to offer long call-option-like payoffs: downside protection, upside potential, and below-market interest income.

Yield enhancement structured products tend to offer short put-option-like payoffs with full downside exposure, capped upside potential, and above-market interest income (i.e., yield enhancement).

Participation structured products tend to offer exposures (bull or bear) to the underlying index (or assets) that are not capped in terms of potential profits or losses (i.e., in either the bull or bear scenarios).

Leverage structured products have three subcategories: leverage without knock-outs, leverage with knock-outs, and constant leverage.

At its core, in a power reverse dual-currency note (PRDC), an investor pays a fixed interest rate in one currency in exchange for receiving a payment based on a fixed interest rate in another currency.
Dynamic hedging is when the portfolio weights must be altered through time to maintain a desired risk exposure, such as zero risk.

A boundary condition of a derivative is a known relationship regarding the value of that derivative at some future point in time that can be used to generate a solution to the derivative's current price.

The partial differential equation approach (PDE approach) finds the value to a financial derivative based on the assumption that the underlying asset follows a specified stochastic process and that a hedged portfolio can be constructed using a combination of the derivative and its underlying asset(s).

The solution is analytical because the model can be exactly solved using a finite set of common mathematical operations.

Numerical methods for derivative pricing are potentially complex sets of procedures to approximate derivative values when analytical solutions are unavailable.

The building blocks approach (i.e., portfolio approach) models a structured product or other derivative by replicating the investment as the sum of two or more simplified assets, such as underlying cash-market securities and simple options.

A static hedge is when the positions in the portfolio do not need to be adjusted through time in response to stochastic price changes to maintain a hedge.

The payoff diagram level determines the amount of money or the percentage return that an investor can anticipate in exchange for paying the price of the product.

The payoff diagram shape indicates the risk exposure of a product relative to an underlier.

An overconfidence bias is a tendency to overestimate the true accuracy of one's beliefs and predictions.
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