What, Exactly, *Is* a Factor?

Melissa R. Brown
Axioma

Dieter Vandenbussche
Axioma

Esther Mezey
Axioma

Ipek Onat
Axioma

According to BlackRock, $1.9 trillion was invested in factor-based strategies as of June 2018 — a figure expected to grow by nearly 80% to $3.4 trillion by 2022.[1] There is no question that these strategies have moved to the forefront of investing, but their growing popularity begs the basic question: what do we mean by the term "factor?"[2]

When we refer to factor returns, we mean the return to a long-short portfolio with unit exposure to the factor in question, and no exposure to any other model factor. The portfolio encompasses the model's investment universe, is rebalanced daily, and has hundreds or thousands of small positions. While we consider these "Factor-Mimicking Portfolios" (FMPs) to be the purest expression of a factor's return, we recognize that other practitioners may have different definitions – and that those different definitions can produce very different results.

For example, factor returns can impact the decision to use a factor in an investment process, and can help explain the performance of a portfolio when attribution is run using the factor. In addition, a long-only manager may find that using long-short FMPs can give unintuitive results, especially if much of the factor performance comes from the short side. We will address both issues in this paper, part 1 of a 2-part series. Here we will cover the differences that result from factor-construction choices; part 2 will compare long-only portfolio-construction alternatives.

We set out to create a set of portfolios that represent a number of ways one could construct a factor portfolio. The differences in exposures and returns were often quite substantial. All portfolios in the study are designed to be FMPs. In other words, they are meant to represent exposure to the chosen factor, but they have varying degrees of "purity" of that exposure, with some allowing other bets, such as industry and other risk-model style factors. To illustrate our points, we chose a few criteria on which to base our analysis, but left out numerous other possible scenarios. This study is therefore hardly comprehensive, but we hope it conveys a sense of how difficult it is to narrow the criteria for defining a factor. Factor investors may ultimately want to choose a factor that best represents their investment process, and should avoid misleading definitions that muddle numerous factors together.

Factor Portfolio Choices.

With the exception of the portfolios that are long the top quintile and short the bottom according to a given factor (Top-Bottom), all factor portfolios described below use mean-variance optimization for portfolio construction. The objective term seeks to maximize exposure to the factor, subject to various constraints, which are what distinguishes each portfolio. "Pure" in the portfolio descriptions refers to the presence of constraints on all other factors (in this case style and sector).[3,4] For our long-short portfolios, an active risk limit of 3% is imposed, and we ran long-only portfolios at varying levels of active risk.[5] We also varied the investment universe and rebalancing frequency. All portfolios and returns discussed in the paper are active. This means that even if an initial portfolio was created to be long only relative to a benchmark, the discussion of results is based on the active portion.

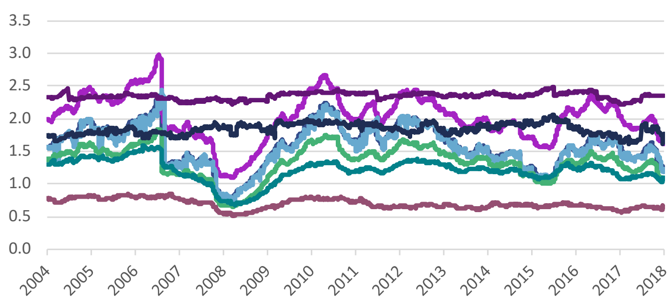
Exhibit 1 describes the investment universe, rebalance frequency, and other restrictions for each portfolio.

The portfolios thus constructed have widely varying levels of volatility and exposure to the desired factor. They can have quite different returns driven by the investment universe, frequency of rebalancing and exposure to other factors. In addition, because the "long only" portfolios may be limited in how much they can short (only up to the benchmark weight), they typically have much lower exposures to the desired factor. At the same time, they may be a better representation of the return that could be achieved from that factor when it is used to evaluate a portfolio that does not permit shorting. Correlations show that the farther you move away from long-short, the broad universe and/or other constraining factors, the more different the portfolios' holdings and returns become.

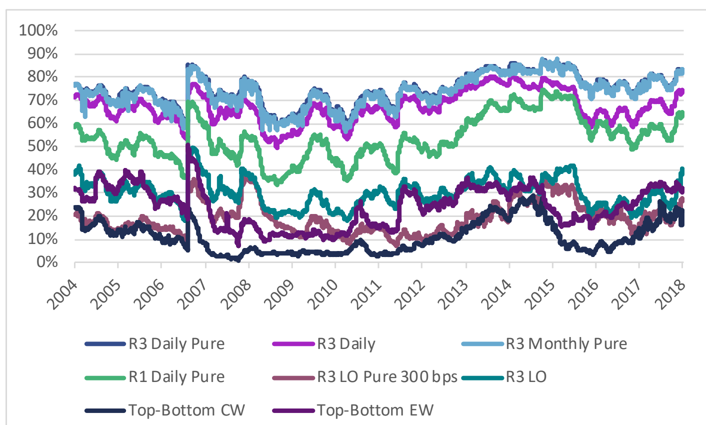
We tested these variations on some of Axioma's traditional factors and, as expected, found substantial differences in exposures, use of risk budget, holdings, etc.

To illustrate the differences, we will focus on the profitability factor, where we focus on a company's return on equity, return on assets, cash flow to assets, cash flow to income, gross margin and sales to assets. Exhibit 2 shows the portfolios' exposures and how much of the risk budget is used up by the factor, both of which highlight the substantial differences from one portfolio to the next.

Profitability Exposure



Profitability Percent of Risk



Note: the R3 Daily Pure line is almost exactly the same as the R3 Monthly Pure line, with the later largely obscuring the former in the charts.

Exhibit 2: Exposures and Percent of Risk

Source: FTSE Russell, Axioma

	Rebalance Frequency	Other Factors Constrained	Benchmark/ Universe	Short Constraints	Tracking Error
Factor*	Daily	Yes	US Estimation Unvierse	No	NA
R3 Daily Pure	Daily	Yes	Russell 3000	No	300 bps
R3 Daily	Daily	No	Russell 3000	No	300 bps
R3 Monthly Pure	Monthly	Yes	Russell 3000	No	300 bps
R1 Daily Pure	Daily	Yes	Russell 1000	No	300 bps
R3 LO Pure	Daily	Yes	Russell 3000	Yes**	300 bps
R3 LO Daily	Daily	No	Russell 3000	Yes**	300 bps
Top-Bottom CW	Daily	No	Russell 3000	No	NA
Top-Bottom EW	Daily	No	Russell 3000	No	NA

* Axioma's Factor-Mimicking Portfolio
** Short active positions only allowed up to benchmark weight

Exhibit 1: Portfolio Options

When we look at exposures and percent of risk used by the Profitability tilt in the same chart, we also see some interesting results. When other factor exposures are allowed in the portfolio, the risk allocation for the desired factor can vary widely. On the Upper chart of Exhibit 3 we show the Top-Bottom Cap-Weighted portfolio data. In this case the factor exposure (the pink line plotted against the left scale) remains fairly steady, as we would expect, given that factor definitions are standardized and the portfolio is always long and short the same proportion of stocks. However, Profitability's contribution to the overall active risk of the portfolio is low and quite variable (because other factors are eating up the risk budget).[6] For the R3 Daily portfolio, which does not constrain exposures to other factors, the risk contribution from the desired factor is higher and fairly steady, but the factor exposure ranges from 1 to 3—quite a wide range.[7] Given this, can either of these portfolios really be described as representing true exposure to Profitability? Probably not.

Exhibit 4 shows a scatter plot of daily returns and correlations between various iterations of the portfolio and the Profitability FMP (Factor). As we remove factor constraints, reduce the ability to short, and change the universe, it is clear that the results move farther and farther away from the FMP. Adding on the capitalization weighting for the Top-Bottom portfolio led to the lowest correlation — in fact, the correlation almost looks as if it is between different factors. Again, this suggests that quintile- (or decile- or other) sorts are not good reflections of the returns that could be generated from a more purely defined factor.

One surprising finding is the high correlation between the returns of the R3 Daily Pure and R3 Monthly Pure portfolios. This suggests, at least for this factor, that a manager's portfolio that is rebalanced less frequently than daily can still effectively use this factor in attribution. Over the course of the test, however, while returns were highly correlated, the daily-rebalanced R3 Pure portfolio did fare slightly better than the monthly version (see Exhibit 5, on the next page).

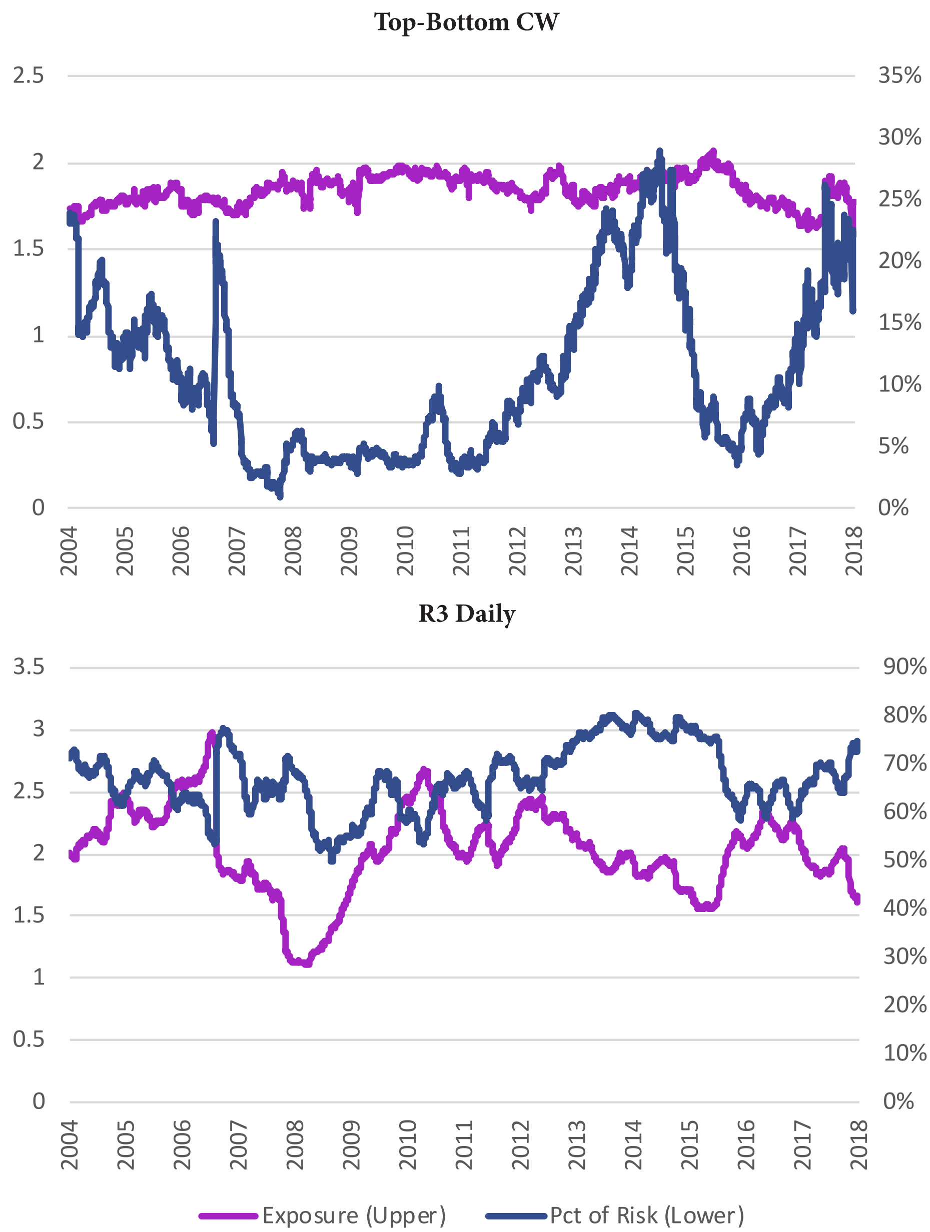
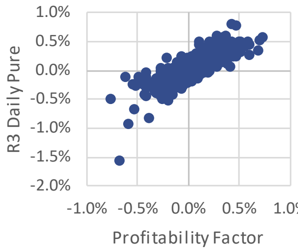
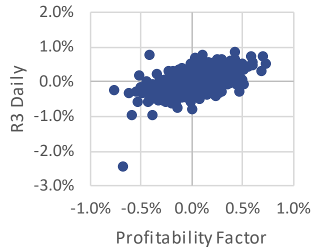


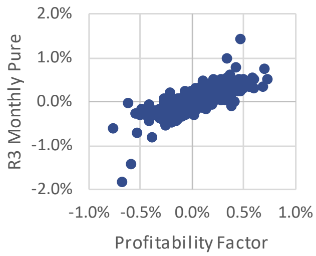
Exhibit 3: Profitability Factor Exposure vs. Percent of Risk
Source: FTSE Russell, Axioma



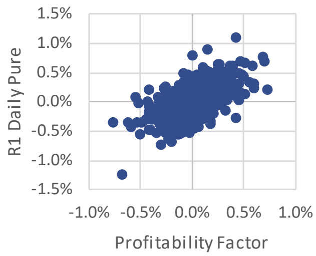
Correlation 0.83



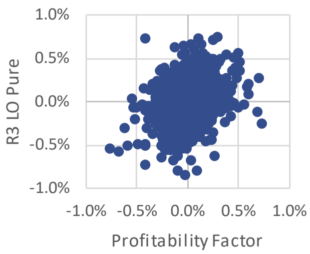
Correlation 0.58



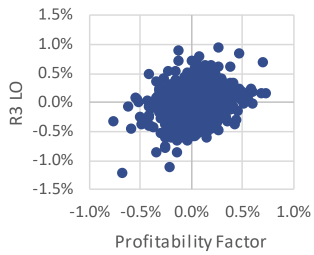
Correlation 0.82



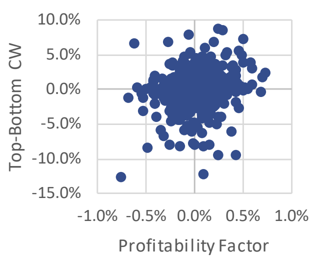
Correlation 0.55



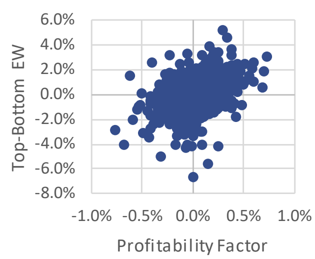
Correlation -0.03



Correlation 0.24



Correlation 0.21



Correlation 0.42

Exhibit 4: Correlations of Portfolio Daily Active Returns with Profitability Factor, 2005-2018
Source: FTSE Russell, Axioma

Portfolio Returns

As shown, the portfolios have very different levels of factor exposure (for example, the exposure is almost 2.5 for the equal-weighted Top-Bottom portfolio, but less than 1 for the pure Long Only portfolio portfolios) and, therefore, their returns are not directly comparable. We have chosen to show these portfolios' returns in their "raw" form because they are likely part of the factor lexicon that is out there, and it is important for users to understand their characteristics. However, to make performance results comparable with each other and to our Factor-Mimicking Portfolio, we have also produced returns that re-scale the factor exposure for each portfolio to 1. While this has a large impact on portfolio returns, it had minimal effect on correlations. These differences lead to our major conclusion:

Beware of how the portfolio used to generate returns is exposed to the factor. A standard "off-the-shelf factor" may not be providing the expected exposure, could therefore overstate or understate achievable returns, and may not be suitable for using in true factor-based attribution.

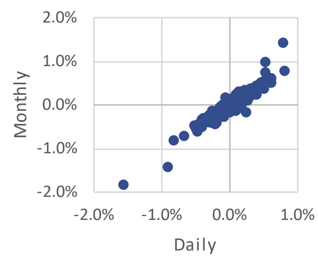
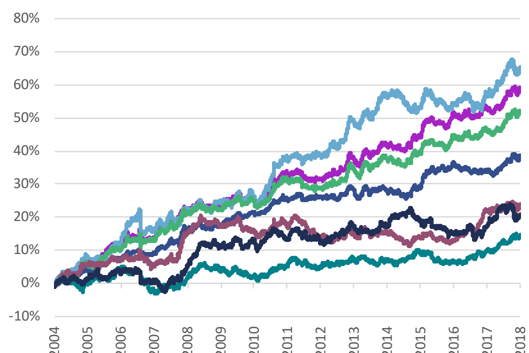


Exhibit 5: Portfolio Correlations with Each Other, Daily Returns of Portfolios Rebalanced Daily Versus Once a Month
Source: FTSE Russell, Axioma

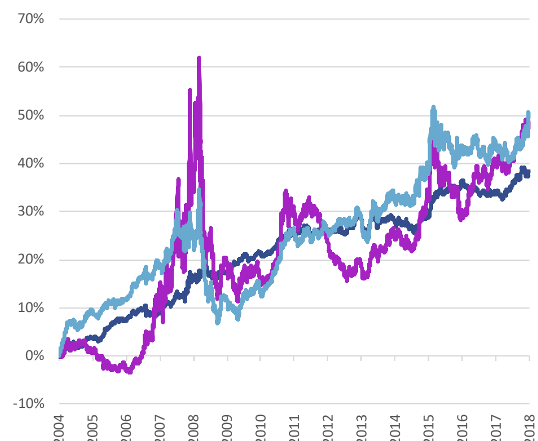
	Factor	R3 Daily Pure	R3 Daily	R3 Monthly Pure	R1 Daily Pure	R3 LO Pure	R3 LO	Top-Bottom CW
R3 Daily Pure	0.83	1						
R3 Daily	0.58	0.73	1					
R3 Monthly Pure	0.82	0.97	0.70	1				
R1 Daily Pure	0.55	0.64	0.47	0.62	1			
R3 LO Pure	0.26	0.24	0.19	0.24	0.35	1		
R3 LO	0.24	0.31	0.45	0.31	0.38	0.43	1	
Top-Bottom CW	0.21	0.18	0.21	0.20	0.15	0.12	0.17	1
Top-Bottom EW	0.42	0.40	0.35	0.41	0.16	0.05	0.05	0.70

Exhibit 6: Full Correlation Matrix, Active Daily Returns, 2005-2018
Source: FTSE Russell, Axioma

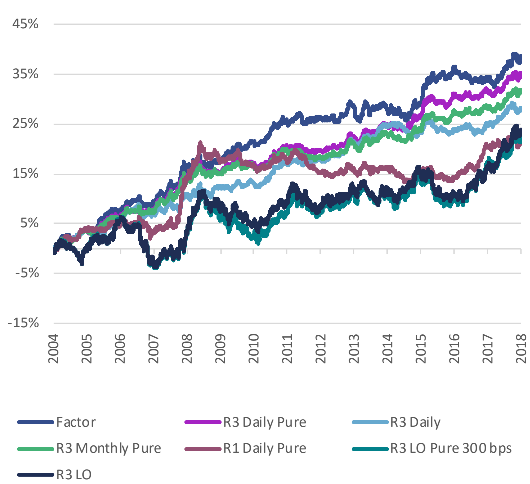
Raw



Raw



Scaled to Exposure of 1



Scaled to Exposure of 1

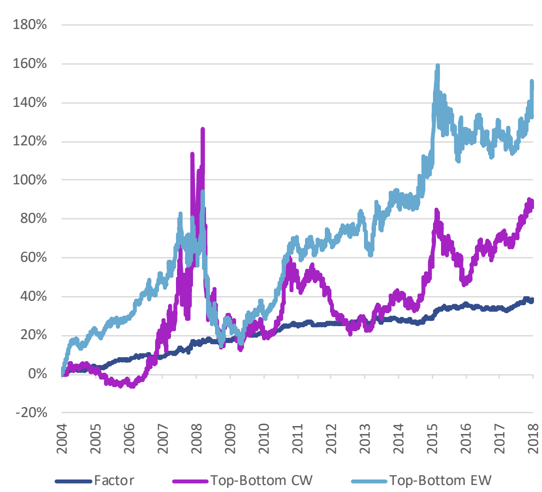
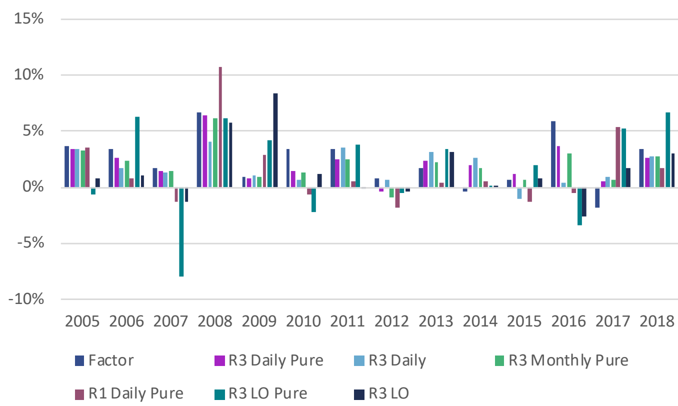


Exhibit 7: Cumulative Active Returns
Source: FTSE Russell, Axioma

Active return differences for portfolios with the same factor exposures are even more pronounced year-by-year. For example, in 2016 the Russell Daily 3000 Pure portfolio returned almost 5%, whereas the pure portfolio built using the Russell 1000 universe was down 53 basis points, and the pure, long-only, 3% tracking error Russell 3000 portfolio fell more than 2%. This suggests the factor fared better among small stocks (since the broader-universe portfolio did better than the one limited to large-cap stocks) and that factor returns were driven by the shorts (since the portfolio that allowed full shorting had a much higher return than the one that limited shorting to the weight in the index). While differences appeared particularly big in that year, the average spread between the highest and lowest return for the optimized portfolios was a substantial 6%.

Even more striking was the magnitude of returns of the top-bottom quintile portfolios. While it was typically far bigger than that of the optimized portfolios (hence the separate chart), it was also sometimes in the opposite direction, most notably in 2009. Finally, there were clearly periods in which the weighting scheme for the top-bottom portfolios (capitalization or equal) made a substantial difference, with one positive and the other negative, as in 2006, 2012, 2013 and 2016.



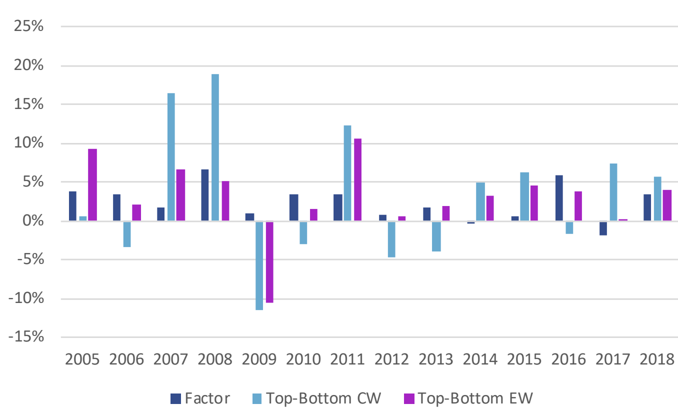


Exhibit 8: Annual Returns, Scaled Portfolios
Source: FTSE Russell, Axioma

Attribution

We ran factor-based performance attribution on selected portfolios for the five years ended December 2018 to highlight the impact of 1) unconstrained factors, and 2) specific returns. Exhibit 9 shows the attribution for three of our portfolios: the long-only portfolios (LO) run at 3% tracking error; one with other factors constrained (the "pure" portfolio); and the one without the other constraints, along with the capitalization-weighted portfolio that is long the top quintile of stocks based on the profitability factor and short the bottom quintile (TBCW). The latter is likely one of the most common alternatives for calculating factor returns to a factor-mimicking portfolio. For this part of the study the portfolios were not scaled to have an exposure of 1 to Profitability; the exposure averaged 0.65 for the constrained long-only portfolio, 1.2 for the unconstrained long-only and 1.8 for TBCW over this time period.

This was a very good period for the Profitability factor, and the benefits of tilting on the factor were apparent in two scenarios. The LO Pure portfolio produced an information ratio (IR) of 0.58 and the TBCW portfolio scored an IR of 0.9 — even with its much higher level of realized risk (10%, versus 2.5% for the LO portfolio). In contrast, the LO portfolio that allowed for other exposures was dragged down by them, most notably the positive exposure to Volatility, as well as a high level of specific return. Those issues reduced its IR to just 0.22.

Since the LO portfolio was "pure", or restricted from taking bets on other factors, most of the return for the LO portfolio was the result of its exposure to the Profitability factor, but that was offset by 81 basis points of drag from specific return. The TBCW portfolio's strong performance was, to be sure, largely attributable to its exposure to the Profitability factor, but many other factors also contributed, including a positive exposure to Earnings Yield and negative exposures to Market Sensitivity and Volatility. In addition, underweight positions in Energy and Financials boosted return. And specific return cut into the overall return by almost 4% annually.

Using this return in attribution would clearly be misleading and very likely overstate the return a manager could have achieved. Although no other factors had a very large impact on return over this period, one could imagine that exposures big enough to lead to these returns (e.g. an average 32% underweight in Financials or -0.87 exposure to Value) could easily have had the opposite impact. In fact, in 2009, when the Profitability factor fared well (and the optimized variations produced positive returns), the unscaled TBCW portfolio lagged the market by more than 18%. A number of factors contributed to that shortfall: negative exposures to Leverage, Liquidity, Market Sensitivity, Value and Volatility, and positive exposures to Size and Medium-Term Momentum each detracted at least 3% from return (with Market Sensitivity contributing more than -9% and Momentum almost -12%). Some of that was offset by a huge underweight in Financials and overweight in Information Technology, but clearly not enough. And specific return was a 3% drag.

So, does the strong performance of TBCW indicate that profitability was a strong factor in 2013-2018? Or a terrible one in 2009? We would argue that it does not, because there were too many other contributors. And a manager who may impose constraints on risk factors, such as sectors, may not have been able to achieve those returns. *A corollary of this finding is that only a "pure" factor, one with no other active exposures, is appropriate for attribution. Otherwise, factors will be double-counted, and results will therefore be too muddled.*

Source of Return	R3 LO Pure 300 bp	R3 LO	Top-Bottom CW
Porfolio	9.36%	8.59%	17.01%
w Benchmark	7.93%	7.93%	7.93%
Active	1.43%	0.67%	9.08%
Specific Return	-0.81%	-2.16%	-3.94%
Factor Contribution	2.24%	2.83%	13.02%
Style	2.22%	2.85%	11.85%
Dividend Yield	0.00%	-0.06%	-0.16%
Earnings Yeild	0.00%	0.23%	2.04%
Exchange Rate Sensitivity	0.00%	0.02%	-0.01%
Growth	0.00%	-0.06%	-0.36
Leverage	0.00%	-0.87%	-0.03%
Liquidity	0.00%	0.02%	-0.01%
Market Sensitivity	0.00%	0.07%	1.06%
Medium-Term Momentum	0.00%	0.21%	-0.08%
MidCap	0.00%	-0.16%	0.12%
Profitability	2.22%	3.97%	6.44%
Size	0.00%	0.36%	-0.33%
Value	0.00%	0.36%	0.47%
Volatility	0.00%	-1.24%	2.69%
Sectors	**0.01%**	**-0.03%**	**1.17%**
Consumer Discretionary	0.00%	0.12%	-0.32%
Consumer Staples	0.00%	-0.06%	-0.21%
Energy	0.00%	0.10%	1.13%
Financials	0.00%	0.11%	1.18%
Health Care	0.00%	-0.22%	-0.55%
Industrials	0.00%	-0.03%	-0.01%
Information Technology	0.00%	-0.03%	-0.01%
Materials	0.00%	0.08%	0.08%
Real Estate	0.00%	-0.05%	-0.26%
Telecommunications Services	0.00%	0.10%	-0.04%
Utilities	0.00%	-0.16%	0.03%

Exhibit 9: Annualized Attribution, 2013-2018
Source: FTSE Russell, Axioma

Conclusion

To reiterate, factors can be defined in a number of ways. The underlying investment universe, frequency of rebalancing, presence or absence of exposures to other factors, and ability to short — in other words, elements of portfolio construction — are all important drivers of the returns of a "factor" portfolio. Some may be far better representations of the factor return an investor may be able to achieve. To repeat:

Beware of how the portfolio used to generate returns is exposed to the factor. A standard "off-the-shelf factor" may not be providing the expected exposure, could therefore overstate or understate achievable returns, and is not suitable for using in true factor-based attribution. Only a "pure" factor, one with no other active exposures, is appropriate for attribution. Otherwise, factors will be double-counted, and results will therefore be too muddled.

Factor investors may ultimately want to choose a factor that best represents their investment process, and should avoid misleading definitions that muddle numerous factors together.

Endnotes

1. This work is adapted from a series of presentations given by Dieter Vandenbussche, in collaboration with Rob Stubbs and Yilin Dai, all of Axioma, entitled "Factor Attribution: A Framework to Align Attribution with Your Investment Strategy."
2. https://www.blackrock.com/investing/investment-ideas/what-is-factor-investing/factor-commentary/andrews-angle/factor-growth.
3. No transaction costs are considered in the creation of any of the portfolios.
4. A regional or global model would also include country and currency constraints.
5. This paper concentrates on the long-only portfolios run with 3% tracking error. A subsequent paper will look at varying levels of portfolio tracking error to highlight the impact of the no-shorting constraint.
6. Profitability is based on a company's return on equity, return on assets, cash flow to assets, cash flow to income, gross margin and sales to assets.
7. A risk analysis shows that over the course of the study about half the active risk came from style factors, with a negative bet on Volatility the second-highest contributor (about 13%) after Profitability (about 22%). Industries contributed another 10%, and stock-specific active risk was about 40% of the risk budget. In contrast, the R3 Daily Pure portfolio gets about 83% of its active risk from its Profitability bet.
8. The exposure range in this case is the result of the varying level of factor risk over time.

Authors Bios'

Melissa R. Brown, *CFA*
Axioma

As the Head of Applied Research, Melissa Brown generates unique insights into risk trends by consolidating and analyzing the vast amount of data on market and portfolio risk maintained by Axioma. Melissa's perspectives help both clients and prospects to better understand and adapt to the constantly changing risk environment. As an author of *Axioma Insight: Quarterly Risk Review*, Melissa reports on the state of risk in publicly traded equity markets around the globe. In addition, she produces periodic special reports on a broad range of topics of interest to investors and asset owners, is a frequent speaker on the subject of market risk and is often quoted by the financial media. Prior to joining Axioma in 2011, Brown was Managing Director and head of the institutional business at Wintrust Capital Management. Before that she spent 10 years at Goldman Sachs Asset Management, most recently as a Partner in the Quantitative Investment Strategies (QIS) Group. At Goldman Sachs Asset Management, Brown worked closely with clients as the senior portfolio manager for GSAM's US Equity Strategy, before becoming co-head of Client Portfolio Management in the QIS Group. She was previously Director of Quantitative Research at Prudential Securities, where among other things she popularized the idea of the "cockroach theory" of earnings surprise and appeared on Institutional Investor's "All-Star" list for 10 straight years. Brown is a Chartered Financial Analyst. She holds a BS in economics from The Wharton School of the University of Pennsylvania and an MBA in finance from New York University.

Dieter Vandenbussche, PhD
Axioma

Dieter's work has led to numerous enhancements to Axioma's portfolio construction, analytics, and risk model products and has been published in the Journal of Portfolio Management and other peer reviewed publications.

Prior to joining Axioma in 2006, Dieter was a professor at the University of Illinois Urbana-Champaign where his research focused on theoretical and computational aspects of solving mathematical optimization problems. Much of this research was published in academic journals such as *Mathematical Programming, SIAM Journal on Optimization, Journal of Global Optimization, and Computational Optimization and Applications.*

Dieter holds an undergraduate and PhD degree in industrial engineering (operations research) from the Georgia Institute of Technology.

Esther Mezey, *PhD*
Axioma

Esther is a Director of Research at Axioma, where she is responsible for developing factor models used for risk management, risk and return attribution, optimized portfolio construction, and stress testing. She develops fundamental, statistical, and macroeconomic factor models (for equities), as well as factor models derived from cash-flow data (for alternative investments). Prior to joining Axioma, Esther earned a PhD in Economics from Cornell University, where her research focused on statistical and dynamical modeling of high-dimensional data using spatio-temporal and time series analysis and regularized approaches.

Ipek Onat
Axioma

As part of the Portfolio Solutions Team, Ipek's main focus is to provide expertise on Axioma products and translate client use cases into implementation projects. She mainly assists EMEA clients in portfolio construction processes and portfolio analytics generation. She has experience with a variety of use cases, such as building hedge portfolios, multi-portfolio optimization and asset allocation.

Ipek joined Axioma in 2015 and has been working closely with various teams since then. Previously she worked in HSBC Treasury in Istanbul, focusing on structured treasury products. As a Fulbright scholar, Ipek received her master's degree in Financial Engineering from UCLA Anderson School of Management and her bachelor's degree in Engineering from University of Galatasaray.